Classification of Radar Targets using Features Based on Warped Discrete Fourier Transform

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Abstract— In this research the signals from ground surveillance radar audio output are classified using hierarchical nonlinear classifier based on Support Vector Machine. Central Doppler frequency and spectral width around it are used as features. These features are obtained based on spectrogram calculated using discrete Fourier transform and spectrogram calculated using warped discrete Fourier transform. The performance of used features for classification are analyzed through classification accuracy. Obtained results show that features extracted from the spectrogram calculated using warped discrete Fourier transform provide higher classification accuracy.

Keywords— ground surveillance radar, classification, spectrogram, Warped Discrete Fourier Transform (WDFT), Support Vector Machine (SVM)

I. INTRODUCTION

The situational awareness on the battlefield is one of the main task of sensors application for military purposes [1]. The complexity of the battlefield implies usage of sensors that work in the different part of the electromagnetic spectrum: radar, visible cameras, thermal cameras, acoustic sensors, etc. Each sensor utilization has its advantages and shortcomings. Comparing to radars, visible and thermal cameras provide higher resolution and they are passive sensors. On the other hand, the possibility of working in complex atmospheric conditions (rain, fog, snow, etc.) and the higher detection range, are advantages of radar sensors.

The main tasks of ground surveillance radar are detection and classification of radar targets. In typical radar systems, target detection is fully automatized, while the classification is performed based on operator experience [2]. The pulse-Doppler radar is often used for ground surveillance due its relatively simple construction. The signal of a certain frequency is emitted by radar and this signal reflects to the radar receiver after scattering from a moving object. The frequency of the received signal differs from the emitted signal frequency due to the Doppler effect. The Doppler frequency is the difference of these two frequencies and it is linearly dependent on the target velocity. The received Doppler signal has additional frequency modulations if other parts of the radar targets are moving additionally regarding bulk motion. These modulations are named micro-Doppler signature and they can be used for radar target classification

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[3]. The signals from the output of the ground surveillance radar are time varying. Due this property, time-frequency techniques are often used for these signals analysis [4]. In [5] the S-method is used for tracking central Doppler frequency using Viterbi algorithm. Authors in [6] propose real radar signals Doppler frequency tracking using modified Bdistribution. The optimal time-frequency distribution is determined in [7] in order to achieve a high energy concentration in the time-frequency plane. The features calculated using bispectrum are used in [8] for radar target classification and the results are compared to the classification using features obtained using standard time-frequency distributions.

The spectrogram-based features are often used for ground radar target classification. In [9] the signals from the audio output of the ground surveillance radar are classified using spectrogram projections on time and frequency axis. The method for estimation of the human movement using Short-Time Fourier Transform (STFT) and chirplet transform are proposed in [10]. The central Doppler frequency and spectral width around it are used as an input in fuzzy expert system projected for radar target classification in [11]. The concatenated convolutional neural network (CNN) model has been proposed that takes the radar signal data and the geolocation type as its inputs and performs a binary classification to categorize animals and humans in [12]. Authors in [13] proposed a novel human activity classification method based on motion orientation determining using multistatic micro-Doppler signals. A pre-trained CNN is utilized as feature extractor whereas the output features were used to train a multiclass Support Vector Machine (SVM) classifier in [14]. In [15] the features extracted from the Doppler spectrogram are used for human activity classification using decision-tree structure formed using SVM. A novel feature extraction method based on micro-Doppler signature is proposed to classify ground moving radar targets in [16]. These features are central Doppler frequency and spectral width around it and these two features are obtained using spectrogram. In [17] the AlexNet-inspired CNN model are trained on the real radar targets spectrograms and the algorithm is implemented in real-time with classification accuracy of 90%. A deep 3-layer convolutional encoder is used in [18] for classification of 12 different human activities with correct classification rate of 94.2%. Authors in [19] use Toeplitz matrix of real radar target signal as the input of the CNN with 38 layers and a 1.6 million trainable parameters with 99.7% classification accuracy.

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The spectrogram is based on the Discrete Fourier Transform (DFT), while the points where it is calculated are equidistantly distributed on the unit circle. The Warped Discrete Fourier Transform (WDFT) as the special case of non-uniform discrete Fourier transform is introduced in [20]. In [21], it is shown that the WDFT has some advantages in solving the problem of multi-tone detection and classification in comparison to the standard DFT. The nonlinear twodimensional WDFT amplitude demodulation method that allows higher-resolution measurements on specific twodimensional regions of interest is proposed in [22]. Authors in [23] proposed the WDFT utilization for improvement of the Doppler beam sharpening method for stripmap syntheticaperture radar image. The signals from the audio output of the ground surveillance radar are analyzed using WDFT in [24], and it is shown that spectrogram calculated using WDFT provides narrower width around central Doppler frequency and suppresses noise.

In this research the real radar signals that origin from five radar target classes (person walking, person running, group of persons walking, group of persons running and vehicle) are classified. The hierarchical classifier based on SVM with polynomial kernel of third degree is projected. The central Doppler frequency and spectral width around it are used as features. These features are extracted from spectrogram calculated using WDFT. The classification performance with these features is compared with the features extracted from spectrogram calculated using DFT. The obtained results show that classifier accuracy is improved when the features extracted from spectrogram calculated using WDFT (88.29%) than features calculated using DFT (80%).

The rest of the paper is organized as follows: section II considers the representation of spectrograms of radar echo signals using DFT and the WDFT, while in section III, the process of features extraction from spectrogram and spectrogram calculated using WDFT is described. In section IV the results of radar target classification is presented. Finally, some concluding remarks are provided, as well as some ideas for further research.

II. RADAR SIGNAL ANALYSIS USING WDFT

Time-frequency analysis methods are often used for signals analysis from the audio output of the ground surveillance radar since these signals are time varying. Spectrogram, $S_{DFT}[n,k]$, is one of the most frequently used methods for these analysis and it is defined as, [25]:

$$S_{DFT}[n,k] = \left| \sum_{r=-\infty}^{+\infty} x[r] w[r-n] e^{-j2\pi k/N} \right|, k = 0, 1, \dots, N-1, \quad (1)$$

where x[n] is analyzed sequence, w[n] is the window function, n is the discrete time index, k is the discrete frequency index and N is the number of points used for the DFT calculation. In [24] is presented modified spectrogram calculated using WDFT instead of DFT. This modified spectrogram is defined as:

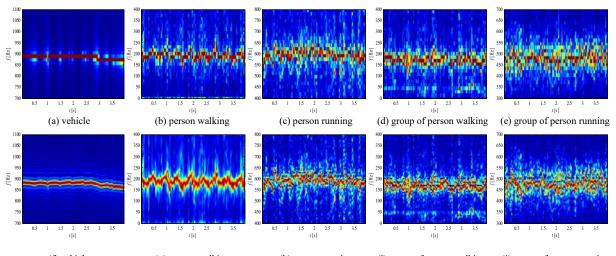
$$S_{WDFT}[n,k] = \left| \sum_{r=-\infty}^{+\infty} x[r] w[r-n] z_k^{-r} \right|, k = 0, 1, \dots, N-1, \quad (2)$$

where z_k are points on the unit circle in which the discrete Fourier transform is calculated after frequency transformation [23]:

$$\omega = \hat{\omega} + 2 \arctan \frac{b_m \sin(\phi_b - \hat{\omega})}{1 + b_m \cos(\phi_b - \hat{\omega})},$$
(3)

where ω is original frequency, $\hat{\omega}$ is the warped frequency and $b = b_m e^{j\phi_b}$ is the complex parameter of the frequency warping. In [23] is shown that the warping of the original frequency axis is more significant with the increase of the complex parameters modules b_m , while the phase of the complex parameter ϕ_b determines the frequency where the points in which the WDFT is calculated are accumulated. Due this, the selection of the ϕ_b should be done as:

$$\phi_b(n) = \frac{f(\arg\max_k S_{DFT}(n,k))}{f_s} 2\pi$$
(4)



(f) vehicle(g) person walking(h) person running(i) group of person walking(j) group of person runningFig. 1. Analyzed radar targets spectrograms calculated using DFT (a-e) and modified spectrograms calculated using WDFT for b_m =0.5 (f-j).

where f_s is sampling frequency, $f(\arg \max_k S_{DFT}(n,k))$ is

the frequency that corresponds to the maximum of spectrogram calculated using DFT $S_{DFT}(n,k)$ in each time bin. In order to provide unique frequency resolution, the frequency axis is divided as in [24]. The new frequency resolution corresponds to the minimal difference of warped frequencies. In this process, the values of this new representation is the values of the WDFT in warped frequency bins, while in the rest of these bins, the value of this representation is zero. After this, the upper envelope of the WDFT is calculated and this represents the new transform with high frequency resolution. Fig. 1 shows the spectrograms parts of 400 Hz around central Doppler frequency calculated using DFT and WDFT of some real radar signals from the database [26]. For spectrograms calculation, it is used rectangular window whose width is 256 samples with overlap 50%, while the number of points where DFT is calculated is 256. Analyzing of Fig. 1 it can be conclude that spectrograms calculated using WDFT provides higher resolution around central Doppler frequency than spectrograms calculated using DFT.

III. RADAR SIGNAL FEATURE EXTRACTION

In this research central Doppler frequency and width around it are used as features for ground surveillance radar target classification [27]. These two features are calculated from modified spectrogram using WDFT. In the first step of this procedure modified spectrogram is normalized regards to the maximum in every time bin. In the second step the segmentation (binarization) of this modified spectrogram is obtained using entropy-based method [28]. Further, the morphological operations of erosion and dilatation are done on the binarized image and the region of the maximal area is determined. In the final step for every time bin there is determined frequency that corresponds of the maximal value of the calculated WDFT, together with up and down cutoff frequencies. The difference of these two frequencies is spectral width around central Doppler frequency.

Central Doppler frequency, F_{CD} , is determined as mean value of all frequencies that corresponds to the maximum of the one modified spectrogram time bins:

$$F_{CD} = \frac{1}{M} \sum_{i=1}^{M} f\left(\arg\max_{k} S_{WDFT}(i,k)\right), \tag{5}$$

while mean value of the utilized spectral widths in each time bin is spectral width (W) around central Doppler frequencies

$$W = \frac{1}{M} \sum_{i=1}^{M} \left(F_{up}(i) - F_{dn}(i) \right), \tag{6}$$

where *M* is the number of time bins, and $F_{up}(i)$ and $F_{dn}(i)$ are up and down cutoff frequencies of each time bin *i*, respectively. After implementing threshold obtained using entropy-based method, in each time bin the maximal value of nonzero frequency is up cutoff frequency, while the minimal value of nonzero frequency is down cutoff frequency. The process of calculating central Doppler frequency and spectral width around it is illustrated in Fig. 2. The spectrogram of the real radar signal that originates from walking person is shown in Fig. 2(a). Spectrogram projection on the frequency axis (Fig. 2(b)) shows that mean central Doppler frequency of the walking human is around 200 Hz. The result of implementing threshold calculated using entropy-based method on spectrogram is shown in Fig. 2(c). Central Doppler frequency together with up and down cutoff frequencies for each time

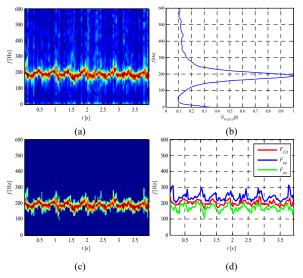


Fig. 2. Real radar signal from person walking: (a) normalized spectrogram calculated using WDFT, (b) spectrogram projection on the frequency axis, (c) binarized spectrogram calculated using WDFT after utilizing entropy-based threshold, (d) central Doppler, up and down cutoff frequencies of the analyzed signal

bin is shown in Fig. 2(d). From this figure it can be noticed the periodical changes of up and down cutoff frequencies due to the periodical moving of arms and legs.

IV. RESULTS

The signals from the audio output of the ground surveillance radar are analyzed in this research. The used radar works in the Ku-band with the emitted signal carrier frequency of f_t =16.9 GHz, while the average emitted signal power is P_a =5 mW. The pulse width of the emitted signal is τ =14.63 µs and the pulse repetition frequency is PRF=34.18 kHz. Range resolution of the used radar is ΔR =150 m, while elevation resolution and azimuth resolution are $\Delta \varepsilon = 7.5^{\circ}$ and $\Delta \varphi = 5^{\circ}$, respectively. Radar use the parabolic antenna with vertical polarization whose gain is $G=32\pm 2$ dB. Detailed description of the used radar can be found in [25]. Signal from radar audio output is brought to the computer audio input and sampled with the frequency f_s =4 kHz. The signals that originate from five radar targets are collected: person walking (pw), person running (pr), group of persons walking (gw), group of persons running (gr) and vehicle (vh). The duration of each sequence is 4 s, while only one radar target is present in one sequence. The database of real radar signals from various classes is available in [26].

In this research the 410 sequences that originate from five radar targets are analyzed: person walking - 96 sequences, person running - 70 sequences, group of persons walking -121 sequences, group of persons running - 50 sequences and vehicles (truck and wheeled) - 73 sequences. For each real radar sequence central Doppler frequency F_{CD} and spectral width around it W are determined. The utilized features are organized as vectors and split into two sets: training and testing set in the ratio 70:30. In Fig. 3 it is shown the training set of features utilized using spectrogram based on WDFT for complex parameters modulus $b_m=0$ and $b_m=0.5$ and used rectangular window of length 256 with 50% overlap. Analyzing Fig. 3 it can be noticed that feature vectors that originate from vehicles characterize lower spectral width around central Doppler frequency. Lower central Doppler frequencies and some wider spectral width around it are utilized from sequences that originate from person walking.

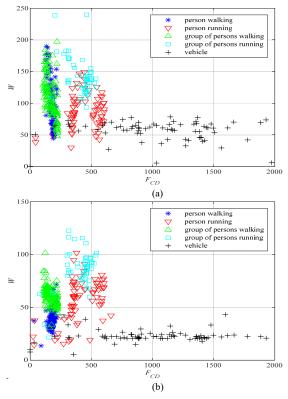


Fig. 3. Features calculated using spectrogram based on WDFT for complex parameter modulus: (a) $b_m=0$, (b) $b_m=0.5$.

Features that originate from group of person walking have more wider spectral width around central Doppler frequency, while the values of these frequencies are similar to ones that originate from class person walking. From Fig. 3 it can be observed that central Doppler frequencies are slightly higher for sequences that origins from person running and group of persons running comparing to the sequences from classes person walking and group of persons walking, but lower than sequences that origins from vehicles. Narrower spectral width around central Doppler frequency is characteristic for sequences that originate from class person running, while for class group of persons running there is some wider spectral width around it. From Fig. 3 it can be noticed that feature vectors that originate from person walking and group of persons walking are separable in spectral width around central Doppler frequency with some overlap. For feature vectors that originate from classes person running and group of persons running it can be similar conclusion, with more overlapping. Analyzing Fig. 3 it can be noticed that features extracted from the modified spectrogram calculated using WDFT with $b_m=0.5$ are more grouped than features extracted from spectrogram ($b_m=0$).

In this research, the features obtained using spectrogram and modified spectrogram using WDFT for complex parameter modulus b_m =0.5 are compared. This comparison is performed according to the criterion [29]:

$$J = S_W^{-1} S_B, (7)$$

where S_W is within-class scatter matrix and S_B is the betweenclass scatter matrix. The higher values of criteria *J* implies that the distance between each class is higher and features are grouped around mean vectors of each class. For features

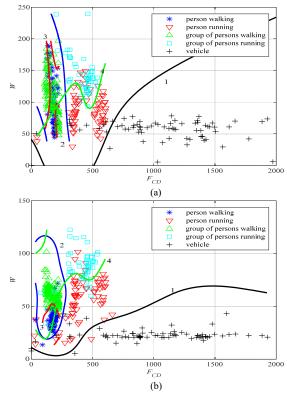


Fig. 4. Projected hierarchical classifier based on SVM: 1–classifier vehicle-other classes, 2–classifier walking-running, 3–classifier person walking-group of persons walking, 4–classifier person running-group of persons running (a) features extracted from spectrogram, (b) features extracted from spectrogram calculated using WDFT

vectors shown in Fig. 3 for spectrogram the value of this criteria is $J_{DFT}=3.25$, while for spectrogram calculated using WDFT this criteria is J_{WDFT} =4.82. From this, it can be expected that features extracted from spectrogram calculated using WDFT provide higher classification accuracy. In this research the hierarchical classifier based on SVM is projected for all extracted features. All projected classifiers are based on polynomial kernel of third degree. The implemented classifier is projected in three layers and it is shown in Fig. 4.In the first layer the classifier between vehicle and all other classes is projected (classifier 1). After that, it is projected the classifier 2 that classifies features into groups walking and running. In the third level, there are two classifiers: in the first one the class of person walking and class group of person walking are classified (classifier 3) and in the second the class person running and class group of persons running are classified (classifier 4). The confusion matrix for features obtained using spectrogram is showed in Table I, while for same features obtained using spectrogram calculated using WDFT is showed in Table II. From the Table I, it can be noticed that the accuracy of the projected classifier is 80%. The most misclassified sequences originate from classes group of persons walking and person walking. This is expected due to feature vector distribution extracted from spectrogram. Analyzing the classification accuracy when the feature vectors extracted from the spectrogram calculated using WDFT (Table II), it can be concluded that accuracy of the projected classifier is improved (88.29%). This is mainly due to the higher proper classification rate for all radar targets, except group of persons running which is slightly smaller.

 TABLE I.
 CONFUSION MATRIX FOR FEATURES EXTRACTED FROM SPECTROGRAM

	рw	pr	gw	gr	vh
pw	79	0	17	0	0
pr	1	58	0	10	1
gw	37	0	84	0	0
gr	1	3	1	45	0
vh	1	10	0	0	62

 TABLE II.
 CONFUSION MATRIX FOR FEATURES EXTRACTED FROM SPECTROGRAM CALCULATED USING WDFT

	рw	pr	gw	gr	vh
pw	82	1	12	1	0
pr	0	62	1	6	1
gw	8	0	113	0	0
gr	2	7	2	39	0
vh	0	7	0	0	66

V. CONCLUSION

In this research the central Doppler frequency and spectral width around it are used for classification of real ground surveillance radar targets. These features are extracted from the spectrogram obtained using warped discrete Fourier transform. The signals that originate from five classes are analyzed in this research: person walking, person running, group of persons walking, group of persons running and vehicle. The hierarchical nonlinear classifiers based on SVM are projected. The classification performance with features extracted from the modified spectrogram calculated using warped DFT is compared with the classification with features extracted from the spectrogram calculated using DFT. The obtained results show higher classifier accuracy if the features extracted from the modified spectrogram are used (88.29%) than spectrogram calculated using DFT (80%). As the only two features are used for classification, the obtained classification accuracy is high. In future work, the modulus of complex parameter b_m that provides the highest classifier accuracy will be determined, as well as advanced methods for determining the place on the unit circle where points in which the DFT is calculated are accumulated.

References

- T. Damarla, "Hidden Markov Model as a Framework for Situational Awareness," in Proc. 11th Int. Conf. Info. Fus., Cologne, France, pp. 1-7, June 30 - July 3, 2008.
- [2] I. Bilik, J. Tabrikian, and A. Cohen, "GMM-Based Target Classification for Ground Surveillance Doppler Radar," IEEE Trans. Aeros. and Elec. Sys., vol. 42, no. 1, pp. 267-278, 2006.
- [3] V. Chen, The Micro-Doppler Effect in Radar, Boston, MA, USA: Artech House, 2011.
- [4] V. Chen and H. Ling, Time-Frequency for Radar Imaging and Signal Analysis, Boston, MA, USA: Artech House, 2002.
- [5] T. Thayaparan, Lj. Stanković, and I. Đurović, "Micro Doppler Based target Detection and Feature extraction in Indoor and Outdoor Environments," J. Franklin Inst., vol. 345, no. 6, pp. 700-722, 2008.
- [6] D. Mikluc, D. Bujaković, M. Andrić, and S. Simić, "Estimation and Extraction of Radar Signal Features Using Modified B Distribution and Particle Filters," Frequenz, vol. 70, no. 9-10, pp. 417-427, 2016.
- [7] D. Bujaković, M. Andrić, B. Bondžulić, S. Mitrović, and S. Simić, "Time-Frequency Distribution Analyses of Ku-band Eadar Doppler Echo Signals," Frequenz, vol. 69, no. 3-4, pp. 119-128, 2015.
- [8] P. Molchanov, J. Astola, K. Egiazarian, and A. Totsky, "Classification of Moving Ground Radar Objects by using Bispectrum-based Features," Telecom. and Radio Engine., vol. 74, no. 6, pp. 527-545, 2015.

- [9] P. Molchanov, J. Astola, K. Egiazarian, and A. Totsky, "Classification of Ground Moving Radar Targets using Joint Time-Frequency Analysis," in Proc. *IEEE Radar Conf.*, Atlanta, Georgia, USA, pp. 1-6, May 7-11, 2012.
- [10] T. Thayaparan, S. Abrol, E. Riseborough, Lj. Stanković, D. Lamothe, and G. Duff, "Analysis of Radar Micro-Doppler Signatures from Experimental Helicopter and Human Data," IET Radar, Sonar and Nav., vol. 1, no. 4, pp. 289-299, 2007.
- [11] M. Andrić, Ž. Đurović, and B. Zrnić, "Ground Surveillance Radar Target Classification based on Fuzzy Logic Approach," in Proc. Int. Conf. EUROCON, Belgrade, Serbia, pp. 1390-1392, November 21-24, 2005.
- [12] A. Tiwari, R. Goomer, S. S. S. Yenneti, S. Mehta, and V. Mishra, "Classification of Humans and Animals from Radar Signals using Multi-Input Mixed Data Model," in Proc. Int. Conf. Comp. Comm. Infor, Coimbatore, India, pp. 1-6, January 27-29, 2021.
- [13] X. Qiao, G. Li, T. Shan, and R. Tao, "Human Activity Classification Based on Moving Orientation Determining Using Multistatic Micro-Doppler Radar Signals," IEEE Tran. Geos. Remote Sens., vol. 60, pp. 1-15, 2022., Art no. 5104415
- [14] E. A. Hadhrami, M. A. Mufti, B. Taha, and N. Werghi, "Ground Moving Radar Targets Classification Based on Spectrogram Images Using Convolutional Neural Networks," in Proc. 19th Int. Radar Symp., Bonn, Germany, pp. 1-9, June 20-22, 2018.
- [15] Y. Kim and H. Ling, "Human Activity Classification Based on Micro-Doppler Signatures Using a Support Vector Machine," IEEE Tran. Geos. Remote Sens., vol. 47, no. 5, pp. 1328-1337, 2009.
- [16] L. Du, L. Li, B. Wang and J. Xiao, "Micro-Doppler Feature Extraction Based on Time-Frequency Spectrogram for Ground Moving Targets Classification With Low-Resolution Radar," IEEE Sensors J., vol. 16, no. 10, pp. 3756-3763, 2016.
- [17] D. Lee, C. Cheung and D. Pritsker, "Radar-Based Object Classification using an Artificial Neural Network," in Proc. Nat. Aero. Elect. Conf., Dayton, USA, pp. 305-310, July 15-19, 2019.
- [18] M. Seyfioglu, A. Ozbayoglu and S. Gurbuz, "Deep Convolutional Autoencoder for Radar-Based Classification of Similar Aided and Unaided Human Activities," IEEE Tran.Aero. Elec. Sys, vol. 54, no. 4, pp. 1709-1723, 2018.
- [19] Y. Li, Y. Rui, and J. Gao, "Ground surveillance radar target classification based on 2D CNN," in Proc. Int. Conf. Sig. Process. Sys., Shanghai, China, p. 117190N, January 20, 2021.
- [20] A. Makur, and S. K. Mitra, "Warped Discrete-Fourier Transform: Theory and Applications," IEEE Tran. Circ and Systems – I Fund. Theory and App., vol. 48, no. 9, pp. 1086-1093, 2001.
- [21] O. Kwon, and F. Taylor, "Multi-tone detection using the warped discrete Fourier transform," in Proc. 51st Midwest Symp. Circ. Sys., Knoxville, TN, USA, pp. 281-284, September 3-6, 2002.
- [22] D. Wei, and K. Takeuchi, "Application of Two-Dimensional Warped Discrete Fourier Transform to Nonlinear Two-Dimensional Amplitude Demodulation," Optics Comm., vol. 510, 127972, 2022.
- [23] D. Bujaković, M. Andrić, S. Simić and G. Antonijević, "Modification of the DBS Algorithm for Stripmap SAR Image Forming Using Warped DFT," in Proc. 2018 New Trends Sig. Process., Liptovski Mikulaš, Slovakia, pp. 1-8, October 10-12, 2018.
- [24] D. Bujaković, M. Andrić, D. Mikluc and B. Bondžulić, "Analysis of Human Radar Echo Signal using Warped Discrete Fourier Transform," in Proc. 2016 New Trends Sig. Process., Liptovski Mikulaš, Slovakia, pp. 1-6, October 12-14, 2016.
- [25] M. Andrić, D. Bujaković, B. Bondžulić, S. Simić, and B. Zrnić, "Analysis of Radar Doppler Signature from Human Data," Radioengineering, vol. 23, no. 1, pp. 11-19, 2014.
- [26] M. Andrić, B. Bondžulić, and B. Zrnić, The Database of Radar Echoes from Various Targets. <u>https://mega.nz/#!L4Nzxa4K!sSes9SnbRVUUn59Z8bUtg782p685Lv</u> 69IVKVx1FzITw
- [27] T. Šević, M. Andrić, B. Bondžulić, D. Bujaković, and D. Ivković, "Radar Target Feature Extraction Using Spectrogram (in Serbian)," in Proc. 22th Int. Conf. INFOTEH, Žabljak, Montenegro, pp. 233-237, February 27-March 4, 2017.
- [28] J. N. Kapur, P. K. Sahoo, and A. K. C. Wong, "A New Method for Gray-level Picture Thresholding using the Entropy of the Histogram," Comp. Vis., Graph. Image Process., vol. 29, no. 3, pp. 273-285, 1985.
- [29] S. Theodoridis, and K. Koutroumbas, Pattern Recognition, Burlington, MA, USA: Academic Press, 2009.