

A Low-Complexity Double EP-Based DFE for Turbo Equalization

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Abstract—In this paper, a new double expectation propagation-based decision feedback equalizer (DEP-DFE) for server inter-symbol interference (ISI) channels employing turbo equalization is proposed. The EP algorithm is used at the equalizer output and the channel decoder output. The proposed DEP-DFE offers a new approach to alleviate error propagation. Additionally, its computational complexity is nearly half of the EP-based minimum mean square error (MMSE)-based linear equalizer (EP-MMSE-LE) proposed by Santos *et al.* The bit error ratio performance of the proposed equalizer is verified through simulation in the well-known severely frequency selective Proakis-C channel for different scenarios. Simulation results demonstrate that the proposed DEP-DFE can achieve similar or better performance than the EP-MMSE-LE. Moreover, it has significant improvement over the double expectation propagation-based MMSE-LE (DEP-MMSE-LE).

Index Terms—Expectation propagation (EP), decision feedback equalizer, turbo equalization, inter-symbol interference

I. INTRODUCTION

Current digital communication systems suffer from inter-symbol interference (ISI), which can be efficiently alleviated by using an appropriate equalizer at the receiver. Similar to turbo codes, the equalizer and the channel decoder can iteratively exchange extrinsic information so as to improve the performance, which is referred to as turbo equalization [1]–[3]. Although the early turbo equalizer in [1] can achieve tremendous performance gains over the standalone equalization, its computational complexity becomes intractable as the channel length and/or the modulation level increase due to the trellis-based equalization algorithm. In this scenario, the filter-type turbo equalizers based on minimum mean squared error (MMSE) criterion have been preferred [3]–[9].

In the MMSE-based linear equalizer (MMSE-LE) [4]–[5], the a priori soft decisions calculated by using the a priori information from the channel decoder are used for soft interference cancellation. The computational complexity is extremely reduced with respect to the trellis-based equalizer. However, the performance loss is significant due to the

inaccurate a priori soft decisions. In [6], the soft decision feedback equalizer (SDFE) is proposed for multi-input multi-output (MIMO) systems, and it is later extended to MIMO-ISI systems [7]. The a priori soft decisions of causal symbols are replaced by the more reliable a posteriori soft decisions at the equalizer output.

The expectation propagation (EP) algorithm has been recently introduced into the turbo equalization [8]–[9], and it can provide a more accurate estimation of the posteriori distributions of transmitted symbols at the equalizer output. In [8], an EP-based MMSE-LE (EP-MMSE-LE) is proposed, which uses an extrinsic symbol-based feedback. The EP-MMSE-LE performs considerably better than the MMSE-LE and the SDFE, but its computational complexity is approximately four times the one of the MMSE-LE due to the inner loop. In order to reduce the number of inner loop iterations, a double EP-based MMSE-LE (DEP-MMSE-LE) is proposed in [9]. In addition to the use of EP in the inner loop at the equalizer output, a second EP procedure is used in the outer loop at the channel decoder output. While the DEP-MMSE-LE has half computational complexity than the EP-MMSE-LE, it considerably suffers from performance loss in the high modulation level.

In this paper, we propose a new double EP-based decision feedback equalizer (DEP-DFE) for turbo equalization over ISI channels. Specifically, the double EP algorithm is extended to the DFE structure, where the extrinsic symbol feedback both in current and previous EP iterations is used to remove the interference. Using the extrinsic symbol feedback, the proposed DEP-DFE can mitigate error propagation which seriously affects the performance of the SDFE. Additionally, the proposed DEP-DFE can efficiently alleviate causal interference with the use of extrinsic symbol-based feedback in the current inner loop EP iteration. In contrast, the DEP-MMSE-LE suffers from residual interference caused by causal symbols, and the noise enhancement issue which can degrade

Algorithm 1: Moment Matching and Damping

Input: a minimum allowed variance δ , damping factor β , $p(x_k)$, $g_k^{[s]}(x_k)$ and $q_E^{[s]}(x_k)$;

Output: $\bar{x}_k^{[s+1]}$, $\bar{v}_k^{2[s+1]}$;

1 Compute the moments $\bar{x}_{p_k}^{[s]}$, $\bar{v}_{p_k}^{2[s]}$ of $\hat{p}^{[s]}(x_k) = q_E^{[s]}(x_k)p(x_k)$ and set $\bar{v}_{p_k}^{2[s]} = \max(\delta, \bar{v}_{p_k}^{2[s]})$;

2 Run moment matching: Set the mean and variance of the unnormalized Gaussian distribution

$$q_E^{[s]}(x_k) \cdot \mathcal{CN}(x_k : \bar{x}_{k,new}^{[s+1]}, \bar{v}_{k,new}^{2[s+1]}) \quad (13)$$

equal to $\bar{x}_{p_k}^{[s]}$ and $\bar{v}_{p_k}^{2[s]}$, yielding

$$\bar{v}_{k,new}^{2[s+1]} = \frac{\bar{v}_{p_k}^{2[s]} \sigma_k^{2[s]}}{\sigma_k^{2[s]} - \bar{v}_{p_k}^{2[s]}} \quad (14)$$

$$\bar{x}_{k,new}^{[s+1]} = \bar{v}_{k,new}^{2[s+1]} \left(\frac{\bar{x}_{p_k}^{[s]}}{\bar{v}_{p_k}^{2[s]}} - \frac{z_k^{[s]}}{\sigma_k^{2[s]}} \right) \quad (15)$$

3 Run damping: Update the values as

$$\bar{v}_k^{2[s+1]} = \left(\beta \frac{1}{\bar{v}_{k,new}^{2[s+1]}} + (1 - \beta) \frac{1}{\bar{v}_k^{2[s]}} \right)^{-1} \quad (16)$$

$$\bar{x}_k^{[s+1]} = \bar{v}_k^{2[s+1]} \left(\beta \frac{\bar{x}_{k,new}^{[s+1]}}{\bar{v}_{k,new}^{2[s+1]}} + (1 - \beta) \frac{\bar{x}_k^{[s]}}{\bar{v}_k^{2[s]}} \right) \quad (17)$$

4 **if** $\bar{v}_k^{[s+1]} < 0$ **then**

5 | $\bar{v}_k^{2[s+1]} = \bar{v}_{p_k}^{2[s]}$, $\bar{x}_k^{[s+1]} = \bar{x}_{p_k}^{[s]}$

6 **end**

demodulator [5], the extrinsic information corresponding to coded bit $b_{k,j}$ can be computed as

$$L_E(b_{k,j}) = \max_{\forall \mathbf{m}_i: m_{i,j}=0} \left(-\varrho_{k,i}^{[S]} + \sum_{\forall j': j' \neq j} \frac{\tilde{m}_{i,j'} L(b_{k,j'})}{2} \right) - \max_{\forall \mathbf{m}_i: m_{i,j}=1} \left(-\varrho_{k,i}^{[S]} + \sum_{\forall j': j' \neq j} \frac{\tilde{m}_{i,j'} L(b_{k,j'})}{2} \right) \quad (12)$$

where

$$\varrho_{k,i}^{[S]} \triangleq \frac{|\alpha_i - z_k^{[S]}|^2}{\sigma_k^{2[S]}}$$
$$\tilde{m}_{i,j} = \begin{cases} +1, & m_{i,j} = 0 \\ -1, & m_{i,j} = 1. \end{cases}$$

B. DEP-DFE: Outer EP loop

At the t th iteration of outer loop, after S iterations the inner loop exploits the extrinsic distribution $q_E^{[t,S]}(x_k)$ to obtain the extrinsic information. Then, the channel decoder can provide the prior of transmitted symbol $p^{[t+1]}(x_k)$. To better initialize

Algorithm 2: DEP-DFE at t th iteration of outer loop

Input: \mathbf{h} , σ_w^2 , y_k , the a priori information $L^{[t]}(b_{k,j})$ and the extrinsic distribution $q_E^{[t-1,S]}(x_k)$ at $t-1$ th iteration of outer loop and S th iteration of inner loop;

Output: $L_E^{[t]}(b_{k,j})$;

1 Initialization: Set $\beta = \min(\exp((t+1)/1.5)/10, 0.7)$;

Compute $p^{[t]}(x_k)$ using $L^{[t]}(b_{k,j})$;

2 EP at the outer loop: Compute $\bar{x}_k^{[t,1]}$, $\bar{v}_k^{2[t,1]}$ with $p^{[t]}(x_k)$ and $q_E^{[t-1,S]}(x_k)$ according to (13)–(15);

3 **if** $\bar{v}_k^{2[t,1]} < 0$ **then**

4 | Set $\bar{x}_k^{[t,1]} = \mathbb{E}_{p^{[t]}}[x_k]$, $\bar{v}_k^{2[t,1]} = \mathbb{E}_{p^{[t]}}[(x_k - \bar{x}_k^{[t,1]})^2]$;

5 **end**

6 EP at the inner loop:

7 **for** $s = 0, \dots, S$ **do**

8 | **for** $k = 1, \dots, K$ **do**

9 | | Compute the k th extrinsic distribution, $q_E^{[t,s]}(x_k)$, as in (8);

10 | | Run Algorithm 1 with $p^{[t]}(x_k)$, $g_k^{[t,s]}(x_k)$ and $q_E^{[t,s]}(x_k)$ to obtain $\bar{x}_k^{[t,s+1]}$, $\bar{v}_k^{2[t,s+1]}$;

11 | | **if** $s = S$ **then**

12 | | | Compute the extrinsic information, $L_E^{[t]}(b_{k,j})$, as in (12);

13 | | **end**

14 | **end**

15 **end**

the inner loop, at the $t+1$ th iteration, EP₂ block is exploited to obtain $g_k^{[t+1,1]}(x_k)$. As described in Fig. 2, by matching the moments of

$$\hat{p}^{[t+1]}(x_k) = q_E^{[t,S]}(x_k)p^{[t+1]}(x_k) \quad (18)$$

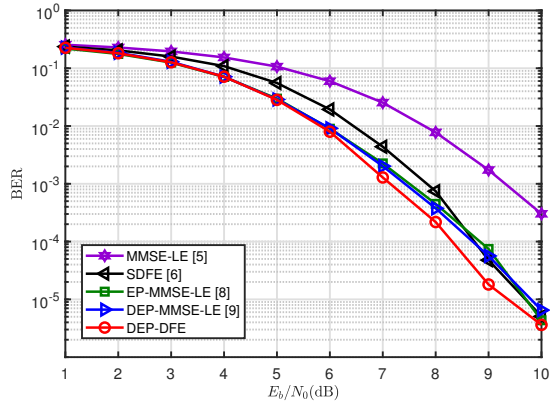
with ones of the new approximate posterior, $q_E^{[t,S]}(x_k)g_k^{[t+1,1]}(x_k)$, the parameters (mean $\bar{x}_k^{[t+1,1]}$ and variance $\bar{v}_k^{2[t+1,1]}$) of $g_k^{[t+1,1]}(x_k)$ can be obtained. The whole procedure is described in Algorithm 2. The complexity of the DEP-DFE is dominated by the computation of the inverse matrix in (4), which is repeated over $S+1$ times per turbo iteration. Therefore, the complexity is in order of $\mathcal{O}((S+1)KN^2)$.

IV. SIMULATION RESULTS

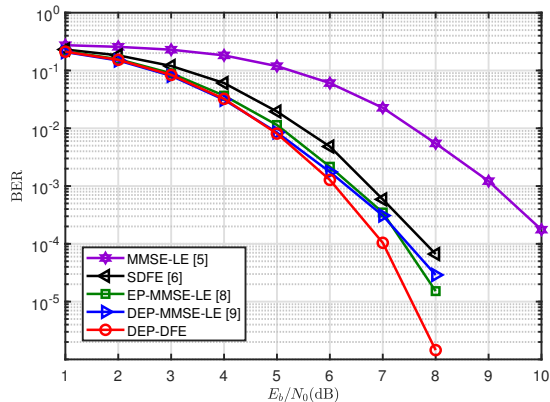
In this section we analyze the bit error rate (BER) performance of the MMSE-LE [5], the SDFE [6], the EP-MMSE-LE [8], the DEP-MMSE-LE [9] and the proposed DEP-DFE for different scenarios. We employ a LDPC of rate 1/3 with the length of 576 bits. The Proakis-C channel with channel response $\mathbf{h} = [0.227, 0.46, 0.688, 0.46, 0.227]^T$ which can incur severe ISI is used [4], [8]–[9]. The block length is set to $K = 32$, and the filter parameters are set to $N_1 = 9$ and $N_2 = 5$. The parameters of EP algorithm are set to $\delta = 10^{-8}$ and $\beta = \min(\exp((t+1)/1.5)/10, 0.7)$ in order to enhance the robustness of the EP algorithm. In addition, the number

TABLE I
COMPUTATIONAL COMPLEXITY COMPARISON BETWEEN EQUALIZERS

Equalizer	Complexity per turbo iteration
MMSE-LE [5]	KN^2
SDFE [6]	KN^2
EP-MMSE-LE [8]	$4KN^2$
DEP-MMSE-LE [9]	$2KN^2$
DEP-DFE	$2KN^2$



(a) after one turbo iteration

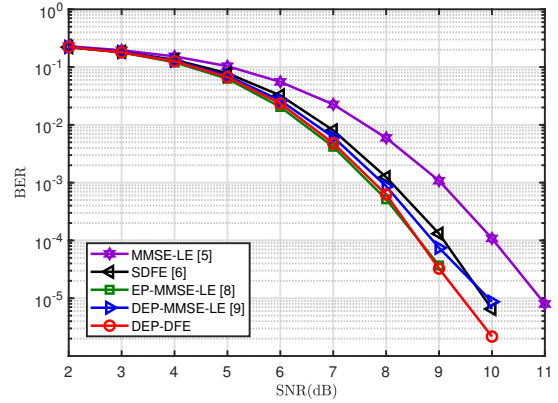


(b) after three turbo iterations

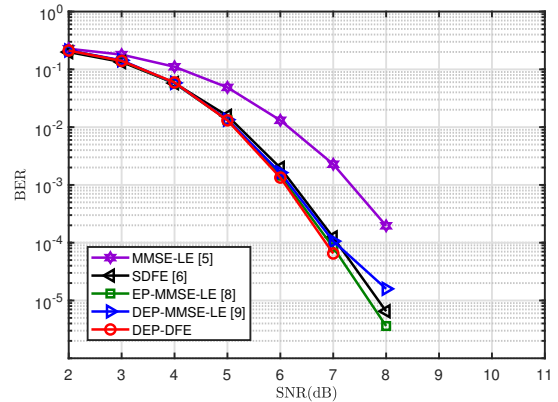
Fig. 2. BER performance of the proposed DEP-DFE with BPSK modulation.

of inner loop iterations is set to $S = 3$ for the EP-MMSE-LE, $S = 1$ for the DEP-MMSE-LE and $S = 1$ for the proposed DEP-DFE. A detailed complexity comparison of all equalizers per turbo iteration is presented in Table I.

In Fig. 2, we depict BER curves of various equalizers after one and three turbo iterations, considering BPSK modulation. From this figure, the DEP-MMSE-LE can achieve a performance similar to that of the EP-MMSE-LE. The proposed DEP-DFE has nearly half complexity than the EP-MMSE-LE. Whereas, its BER performance is much better. This result makes sense since the proposed DEP-DFE can efficiently remove the residual interference caused by causal symbols with the help of the extrinsic symbol feedback in the current



(a) after one turbo iteration



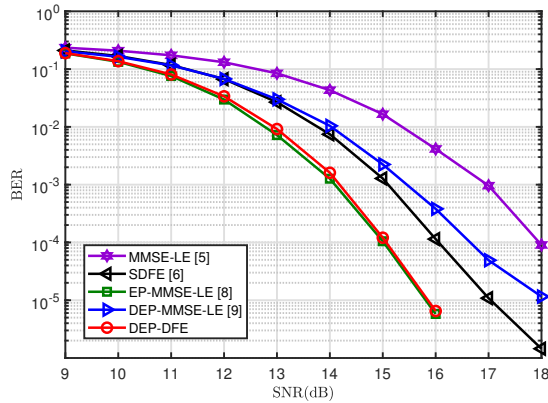
(b) after three turbo iterations

Fig. 3. BER performance of the proposed DEP-DFE with QPSK modulation.

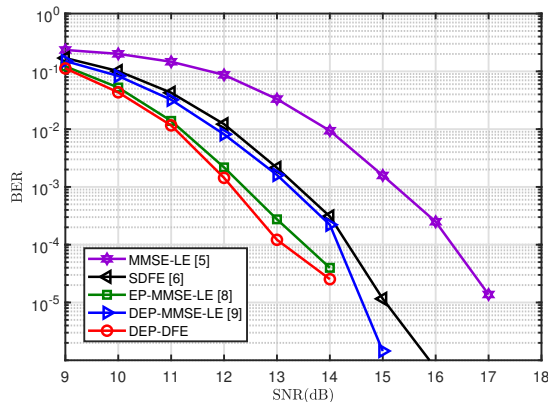
EP iteration. After three iterations, the proposed DEP-DFE offers 0.39 dB (respectively 0.47 dB) gain compared to the EP-MMSE-LE (respectively the DEP-DFE) at the BER level of 10^{-4} .

The BER performance of QPSK modulation with various equalizers is presented in Fig. 3. The proposed DEP-DFE exhibits the same performance as the EP-MMSE-LE after one turbo iteration. However, after three turbo iterations, its performance is slightly better than that of the EP-MMSE-LE. It's clear from the figure that the proposed DEP-DFE consistently outperforms the DEP-MMSE-LE for all turbo iterations since the residual interference from causal symbols is alleviated.

In Fig. 4 we depict the BER performance after one and three turbo iterations for 8PSK modulation. It can be observed that the performance loss of the DEP-MMSE-LE with respect to the EP-MMSE-LE is particularly serious. With the increase of turbo iterations, the proposed DEP-DFE outperforms the EP-MMSE-LE. At the BER level of 10^{-3} , it can outperform the EP-MMSE-LE (respectively the DEP-MMSE-LE) by around 0.23 dB (respectively 0.72 dB).



(a) after one turbo iteration



(b) after three turbo iterations

Fig. 4. BER performance of the proposed DEP-DFE with 8PSK modulation.

CONCLUSION

A double EP-based DFE has been proposed for turbo equalization. The EP algorithm was applied at both the channel decoder output and the equalizer output, and the extrinsic symbol feedback was used to mitigate error propagation. The simulation results have shown that the proposed DEP-DFE achieves similar or better performance than the high-complexity EP-MMSE-LE. In addition, the proposed DEP-DFE provides significant performance improvement compared to the DEP-MMSE-LE with similar complexity. Motivated by the promising BER performance, future work could extend the proposed algorithm to MIMO-ISI channels. The bidirectional equalization [3] using the proposed DEP-DFE also remains unexplored.

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REFERENCES

[1] C. Douillard, M. Jezequel, C. Berrou, A. Picart, P. Didier, and A. Glavieux, "Iterative correction of intersymbol interference: Turbo-

equalization," *Euro Trans. Telecommun.*, vol. 6, no. 5, pp. 507–511, Sept./Oct. 1995.

[2] R. Koetter, A. C. Singer and M. Tuchler, "Turbo equalization," *IEEE Signal Process. Mag.*, vol. 21, no. 1, pp. 67–80, Jan. 2004.

[3] C. Yin, W. Feng, J. Li, X. Bao and G. Li, "Soft-input soft-output block decision feedback equalization for ISI channels," *IEEE Trans. Commun.*, vol. 69, no. 9, pp. 6213–6224, Sept. 2021.

[4] M. Tuchler, A. C. Singer and R. Koetter, "Minimum mean squared error equalization using a priori information," *IEEE Trans. Signal Process.*, vol. 50, no. 3, pp. 673–683, Mar. 2002.

[5] L. Xiao et al., "Time-domain turbo equalization for single-carrier generalized spatial modulation systems," *IEEE Trans. Wireless Commun.*, vol. 16, no. 9, pp. 5702–5716, Sept. 2017.

[6] J. W. Choi, A. C. Singer, J. Lee and N. I. Cho, "Improved linear soft-input soft-output detection via soft feedback successive interference cancellation," *IEEE Trans. Commun.*, vol. 58, no. 3, pp. 986–996, Mar. 2010.

[7] J. Tao, J. Wu, Y. R. Zheng and C. Xiao, "Enhanced MIMO LMMSE turbo equalization: Algorithm, simulations, and undersea experimental results," *IEEE Trans. Signal Process.*, vol. 59, no. 8, pp. 3813–3823, Aug. 2011.

[8] I. Santos, J. J. Murillo-Fuentes, E. Arias-de-Reyna and P. M. Olmos, "Turbo EP-based equalization: A filter-type implementation," *IEEE Trans. Commun.*, vol. 66, no. 9, pp. 4259–4270, Sept. 2018.

[9] I. Santos, J. J. Murillo-Fuentes and E. Arias-de-Reyna, "A double EP-based proposal for turbo equalization," *IEEE Signal Process. Lett.*, vol. 27, pp. 121–125, Dec. 2020.