SPARSE-AWARE APPROACH FOR COVARIANCE CONVERSION IN FDD SYSTEMS

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ABSTRACT

This paper proposes a practical way to solve the Uplink-Downlink Covariance Conversion (UDCC) problem in a Frequency Division Duplex (FDD) communication system. The UDCC problem consists in the estimation of the Downlink (DL) spatial covariance matrix from the prior knowledge of the Uplink (UL) spatial covariance matrix without the need of a feedback transmission from the User Equipment (UE) to the Base Station (BS). Estimating the DL sample spatial covariance matrix is unfeasible in current massive Multiple-Input Multiple-Output (MIMO) deployments in frequency selective or fast fading channels due to the required large training overhead. Our method is based on the application of sparse filtering ideas to the estimation of a quantized version of the so-called *Angular Power Spectrum* (APS), being the common factor between the UL and DL spatial channel covariance matrices.

***Index Terms*— Covariance Conversion, Sparse-aware processing, Basis Pursuit denoising, FDD, Sparse filtering**

I. INTRODUCTION

The current trend in communications applications is an increasing demand for transmission rate, low latency, more connectivity and reliability on the communication. To meet these requirements, it is necessary to explore the millimeter Wave (mmWave) band [1]. This band allows for more connectivity than lower frequency bands and permits the transceivers to feasibly incorporate a much larger number of antennas such that it can be called massive MIMO [2]. As it is known in the massive MIMO literature, having Channel State Information (CSI) is critical to achieving the potential benefits of this technology. The general motivation of this work is the estimation of the channel spatial second-order statistics (statistical CSI) in the context of the mmWave band and massive MIMO technologies in FDD systems where channel reciprocity does not hold, although other applications are benefited by the ideas presented in this work such as in Ultra-Wideband communications [3].

In Time Division Duplex (TDD) systems, the UL and DL channels are reciprocal, as long as the total transmission period is shorter than the channel coherence time. This is the reason why in typical TDD realizations the BS can use the CSI inferred from the UE.

On the contrary, channel reciprocity does not hold in general FDD schemes. In common FDD systems, the ratio between carriers is $\frac{\lambda_u}{\lambda_d} \approx 1$, being λ_u and λ_d the UL and DL carrier wavelengths respectively. However, even if this value is approximately 1, the increased number of antennas in the antenna arrays accentuates the fact that both wavelengths are not exactly equal. Consequently, the UL and DL channels are more likely to be uncorrelated in massive MIMO than in previous MIMO configurations [4].

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Given that channel reciprocity does not hold and considering the necessity of a large channel overhead to estimate the CSI in FDD systems with massive MIMO configurations, several methods have been explored to estimate the DL CSI using the prior knowledge of the UL CSI. An example of these approaches are the well-studied frequency correction techniques [5].

This work is focused on the family of algorithms that involve second-order statistics transformations from the UL to the DL channel, usually labeled as Covariance Conversion techniques, to eliminate the non-reciprocity property of channels in FDD systems. The general Covariance Conversion problem is depicted as:

UL-DL Covariance Conversion (UDCC) problem: *Given the prior knowledge of the UL spatial channel covariance matrix,* $\mathbf{R}_u$*, or an estimated version,* $\hat{\mathbf{R}}_u$*, estimate the DL spatial channel covariance matrix,* $\hat{\mathbf{R}}_d$*.*

The main assumption that motivates this family of techniques is the reciprocity on the angular power distribution, also called *Angular Power Spectrum* (APS), considered in the 3GPP spatial correlation model [6]. Furthermore, there are some applications where the power distribution along the angular domain is shown to become sparse, such as in mmWave band communications [7] or in Ultra-Wideband communications. The sparse assumption motivates the use of Sparse-Aware ideas [8] in this work. Known methods for Covariance Conversion in the literature are:

- Transforming $\mathbf{R}_u$ to $\mathbf{R}_d$ by employing a change of basis in the Hilbert subspaces spanned by the UL and DL spatial signatures [9]. This method exhibits the best performance in the UDCC problem among all the state-of-the-art approaches, but suffers from high computational complexity and is non-reconfigurable.
- The interpolation of measured pairs $(\mathbf{R}_d, \mathbf{R}_u)$ [10] is an approach to solve the UDCC in a well-characterized scenario. However, if the given scenario changes suddenly, the offered solution in [10] becomes unreliable.
- Spline interpolation to different carrier wavelengths [11] suffers from a lower performance than other state-of-the-art approaches, but has low computational complexity.

Our work shows better performance and more configurability than previously proposed methods in the literature, making it a more preferable choice in a wider range of applications. Furthermore, the novelty of our work lies in the fact that our approach to Covariance Conversion incorporates the prior information of a sparse APS in the covariance transformation, while earlier approaches do not. In contrast to previous state-of-the-art sparsity approaches, the ideas present in this work focus on the estimation of statistical CSI (estimating covariance matrices) whereas previous sparse methods focus on obtaining full CSI in FDD systems (estimating channel vectors/matrices). We refer to [12] for more insights on full sparse CSI estimation in FDD systems. There are four ideas that motivate our work:

- In multiuser coherent communications, the knowledge of statistical CSI is needed to obtain optimal beamforming directions between the BS and UE, in the case of SIMO/MISO communications [13].

- In non-coherent communications, it is possible to conform a more efficient beam by knowing the spatial channel second-order statistics. This technique is called eigen-beamforming [14].
- The spatial second-order statistics of the channel, as in the model in [6], changes much slower than the channel coherence time.
- In FDD, UDCC requires the same channel overhead as TDD systems. Therefore, solving the UDCC problem reduces the overall latency in an FDD scheme. It also transfers the computational complexity to the BS which is a desired feature in mobile cellular deployments.

This paper is structured in the following way: in Section II we state the considerations and assumptions necessary to understand the derivation of our approach. Then, we derive our ideas in Section III. Finally, in Section IV, we show some results comparing our method to the best performing algorithm in the state-of-the-art [9].

II. PROBLEM STATEMENT

The algorithm presented in this work is general to any kind of communications scheme that takes benefit from the solution of the UDCC problem. This means that the ideas presented in this manuscript suit those applications where the correlation between receiving antennas is useful and both the UL and DL channels are not reciprocal. Taking the possible applications into consideration, the derivation and the problem setting that we present in this work are general for any kind of array configuration between the BS and UE, i.e. SIMO/MISO or MIMO configurations, where the duplexing between the UL and DL channels is done by an FDD scheme. Let the UL channel be:

$$\mathbf{y}(n) = \mathbf{H}_u(n)\mathbf{x}(n) + \mathbf{w}(n), \tag{1}$$

where $\mathbf{x}(n) \in \mathbb{C}^{M_u}$ are the transmitted symbols, $\mathbf{H}_u(n) \in \mathbb{C}^{M_d \times M_u}$ is the channel matrix and $\mathbf{w}(n) \in \mathbb{C}^{M_d}$ is the additive white noise term. M_d and M_u denote the BS and UE number of antennas respectively. In this setting, the correlation between the receiving antennas is defined as the expected value of the next matrix product:

$$\mathbf{R}_u = E[\mathbf{H}_u(n)\mathbf{H}_u^H(n)]. \tag{2}$$

The DL counterpart of (2) is defined in a similar way with the respective DL channel matrix. The main goal of the UDCC protocol is to obtain an estimation of $\mathbf{R}_u \in \mathbb{C}^{M_d \times M_d}$, so it can be transformed to its DL counterpart $\mathbf{R}_d \in \mathbb{C}^{M_d \times M_d}$. Note that both matrices have the same dimensions because they contain information about the correlation in the BS antennas, but are valid on different carrier frequencies. From those two matrices, the DL spatial covariance matrix is the one that is useful for the BS to transmit to the UE through the DL channel.

The estimation of $\mathbf{R}_u$ is done by means of a training phase where the BS obtains the second-order statistics information from the UL channel. The training phase consists on the transmission of K independent pilot symbols from the UE to the BS, which are required to be zero mean and uncorrelated. The required pilot symbols, $\mathbf{x}(n)$, must fulfill:

$$E[\mathbf{x}(n)] = \mathbf{0}, \tag{3}$$

and

$$E[\mathbf{x}(n)\mathbf{x}^H(n)] = \sigma_x^2\mathbf{I}, \tag{4}$$

where $M_u\sigma_x^2$ is the total received power at the BS.

Then, the UL spatial covariance matrix is obtained by the sample covariance matrix with $K > M_d$ independent realizations as:

$$\hat{\mathbf{R}}_u = \frac{1}{K\sigma_x^2}\sum_{k=1}^{K}\mathbf{y}(k)\mathbf{y}^H(k). \tag{5}$$

In order to be able to convert the UL channel covariance to the DL channel covariance, the channel must fulfill one of two conditions, ensuring that the channel realizations for each k are independent. On the one hand, the independence of channel realizations is accomplished by having a sampling period between channel uses, T_s, greater than the channel coherence time, T_c, or by having such a sufficient interleaving depth that the previous property is obtained virtually [15]. In this way, each channel realization $\mathbf{H}_u(k)$ is independent for each k. On the other hand, in a frequency selective channel the independence between realizations in the sample estimation (5) is obtained due to the large delay spread between symbols [15] or, equivalently, by having a baseband bandwidth, B_s, greater than the channel coherence bandwidth, B_c, obtained virtually by spread spectrum techniques.

The transformation of the UL channel spatial covariance matrix to the DL spatial covariance matrix is achieved by exploiting the common terms in the 3GPP spatial covariance model [6]. According to [6], the UL and DL spatial covariance matrices are described by the following model:

$$\begin{aligned}\mathbf{R}_u &= \int_{-\pi}^{\pi} \rho(\theta)\mathbf{a}_u(\theta)\mathbf{a}_u^H(\theta)d\theta,\\ \mathbf{R}_d &= \int_{-\pi}^{\pi} \rho(\theta)\mathbf{a}_d(\theta)\mathbf{a}_d^H(\theta)d\theta,\end{aligned} \tag{6}$$

where $\mathbf{a}_u(\theta)$ and $\mathbf{a}_d(\theta)$ denote the array responses from the UL and DL channels respectively, also known as the spatial signatures, and $\rho(\theta)$ is the APS or *Angular Spread Function* (ASF) which depicts the average power distribution along the angular domain, $\theta \in [-\pi, \pi]$. The carrier wavelengths, λ_u and λ_d, are taken into consideration in the spatial signatures.

Unlike the array responses, the APS is assumed to be frequency invariant. This property results in $\rho(\theta)$ being the common term in (6), enabling the Covariance Conversion.

As the functions that describe the behaviour of (6) depend entirely on the geometry of the environment, the transmission period where the knowledge of these matrices is still valid is much larger than the channel coherence time, T_c. In the applications that motivated this work, the angular power distribution only contains information of few differentiated scatter clusters, meaning that one may found at most S differentiated peaks in $\rho(\theta)$ [7]. The APS model in which we base the derivation of the Sparse-Aware Covariance Conversion algorithm is the Geometry-based Stochastic Channel Model (GSCM) [16] which describes mathematically the APS as:

$$\rho(\theta) = \sum_{s=1}^{S}\alpha_s f_s(\theta), \tag{7}$$

where $S < M$ is the total number of peaks and $f_s(\cdot)$ is any appropriate Radial Basis Function (RBF) that models the dispersion and location of each scatter cluster s. To model the sparsity in the angular domain inherent to the mmWave band, the scale parameter in each $f_s(\cdot)$ must model a narrow RBF. In (7), the different Angles of Arrival/Departure, θ_s, must be sufficiently separated so every cluster is well differentiated. This sparsity prior on the APS is considered in the derivation of our approach in Section III.

Finally, even though the ideas presented in this paper are general to any kind of spatial signatures, for simplicity, we consider a Uniform Linear Array (ULA) in the BS. Given that the BS array is an ULA, a steering structure is imposed on the spatial signatures, $\mathbf{a}_u(\theta)$ and $\mathbf{a}_d(\theta)$.

III. PROPOSED APPROACH

III-A. Quantization and preliminaries

Our approach is based on the quantization of the angular domain θ, so (6) can be numerically approximated. The equispaced sequence of angles between θ_{min} and θ_{max} is defined as:

$$\theta_i = \theta_{min} + (i-1)\frac{\theta_{max} - \theta_{min}}{N-1}, \quad (8)$$

where θ_{min} and θ_{max} are chosen depending on the specific application and the efficiency of the quantization is tightly related to the choice of these two angles. For instance, in the case of ULAs, it is known that this array configuration cannot differentiate between an arbitrary angle, θ, and its reciprocal angle, $\theta + \pi$. Additionally, in typical cellular networks sectorization configurartion [17] an efficient choice for $[\theta_{min}, \theta_{max}]$ would be $[-\frac{\pi}{3}, \frac{\pi}{3}]$. Having a narrower expected span of angles, θ_{min} and θ_{max}, results in better quantization resolution in the desired angular window for a given complexity (N).

The quantization of the angular domain in (8) is used to draw samples from $\rho(\theta)$ as:

$$\boldsymbol{\rho} = [\rho(\theta_1), ..., \rho(\theta_N)]^T = [\rho_1, ..., \rho_N]^T. \quad (9)$$

A numerical approximation is derived by plugging (9) into (6):

$$\begin{aligned} \mathbf{R}_u(\boldsymbol{\rho}) &= \sum_{i=1}^{N} \rho_i \mathbf{u}_i \mathbf{u}_i^H \Delta\theta, \\ \mathbf{R}_d(\boldsymbol{\rho}) &= \sum_{i=1}^{N} \rho_i \mathbf{d}_i \mathbf{d}_i^H \Delta\theta, \end{aligned} \quad (10)$$

where $\Delta\theta = \frac{\theta_{max} - \theta_{min}}{N-1}$ and for clarity of the notation we define:

$$\begin{aligned} \mathbf{u}_i &\triangleq \mathbf{a}_u(\theta_i), \\ \mathbf{d}_i &\triangleq \mathbf{a}_d(\theta_i). \end{aligned} \quad (11)$$

In this way, one must take into consideration the number of samples, N, when implementing the ideas presented in this manuscript. As a side note, whenever N tends to infinity, it is clear that (10) tends to the original model, as long as the span of visible angles, between θ_{min} and θ_{max}, is chosen correctly and efficiently. Note that (10) defines the Covariance Conversion operation between $\mathbf{R}_u$ and $\mathbf{R}_d$ for a given $\boldsymbol{\rho}$.

Estimating $\boldsymbol{\rho}$ or $\Delta\theta\boldsymbol{\rho}$ from (11) is equivalent as $\Delta\theta$ is a known multiplicative constant. For simplicity, we omit $\Delta\theta$ in the derivation of the presented ideas.

III-B. Sparse-Aware approach on UDCC

Given the UL sample spatial covariance matrix, $\hat{\mathbf{R}}_u$, as a prior estimation, our approach consists on estimating $\boldsymbol{\rho}$ by finding the optimal value for the following convex program:

$$\hat{\boldsymbol{\rho}} = \underset{\boldsymbol{\rho} \succcurlyeq 0}{\arg\min} ||\boldsymbol{\rho}||_1 \text{ s.t. } \frac{||\mathbf{R}_u(\boldsymbol{\rho}) - \hat{\mathbf{R}}_u||_F^2}{||\hat{\mathbf{R}}_u||_F^2} < \epsilon, \quad (12)$$

where the positivity constraint, $\boldsymbol{\rho} \succcurlyeq 0$, is needed because in this way one ensures the power distribution meaning of $\boldsymbol{\rho}$. The rationale behind (12) consists in the incoporation of a sparse prior on the APS in the estimation of $\boldsymbol{\rho}$, being a key difference with previous Sparse-Aware CSI estimation which are generally based on a sparse assumption of the channel matrices, $\mathbf{H}_u(n)$, [12].

In (12), we differentiate two components to remark: the cost function and the main constraint. As for the cost function, we want to promote sparsity on $\boldsymbol{\rho}$. Indeed, $\boldsymbol{\rho}$ is known to be a sparse vector due to two different reasons. Firstly, the actual APS, $\rho(\theta)$, has finite and well-differentiated scatter clusters. Secondly, as the dimension of $\boldsymbol{\rho}$ increases with N, the number of negligible levels of power also increases due to quantization. However, the proportion between zero and non-zero components is still maintained. The main reason to choose the ℓ_1-norm as a sparse inducing function over other choices is that ℓ_1-norm fulfills the regularity property of a sparsity measure: "*Significant concentration of energy in one single coefficient makes the rest negligible*" [18]. The regularity property is not fulfilled by all the sparsity functions (i.e. the ℓ_0-norm does not fulfil this property) and is desired in this context to cope with noise components as N increases.

On the other hand, to achieve a practical solution, we define the constraint to be a normalized Frobenius norm upper bound between the estimated UL spatial covariance matrix, $\hat{\mathbf{R}}_u$, and the matrix that is generated from $\boldsymbol{\rho}$, $\mathbf{R}_u(\boldsymbol{\rho})$.

Considering a normalized metric as a constaint in (12) is useful because ϵ can be chosen independently of matrix dimensions. In addition, the proposed constraint has the property that it is a convex function on $\boldsymbol{\rho}$. It can be easily proven by expanding the Frobenius distance that the constraint in (12) adopts the following form:

$$d_E(\hat{\mathbf{R}}_u, \mathbf{R}_u(\boldsymbol{\rho})) \triangleq \frac{||\mathbf{R}_u(\boldsymbol{\rho}) - \hat{\mathbf{R}}_u||_F^2}{||\hat{\mathbf{R}}_u||_F^2} = \boldsymbol{\rho}^H \mathbf{K}_u \boldsymbol{\rho} - 2\boldsymbol{\rho}^H \mathbf{g} + ||\hat{\mathbf{R}}_u||_F^2, \quad (13)$$

where:

$$\begin{aligned} \mathbf{U} &= [\mathbf{u}_1, ..., \mathbf{u}_N], \\ [\mathbf{K}_u]_{i,j} &= |\mathbf{u}_i^H \mathbf{u}_j|^2, \\ \mathbf{g} &= \text{diag}\left(\mathbf{U}^H \hat{\mathbf{R}}_u \mathbf{U}\right), \end{aligned} \quad (14)$$

where $[\cdot]_{i,j}$ denotes the (i, j)-th component of the input matrix and $\text{diag}(\cdot)$ represents the functional that extracts the main diagonal of the input matrix. Note that $\mathbf{K}_u$ is the second-degree polynomial kernel of the UL spatial signature. The latter property of $\mathbf{K}_u$ ensures that the expression in (13) is convex on $\boldsymbol{\rho}$ since a kernel matrix is known to be semi-definite positive. Consequently, (12) is a convex optimization problem on $\boldsymbol{\rho}$.

We can rewrite (12) in a Disciplined Convex Programing (DCP) form (see the implementations from the authors in [19]) by plugging (13) on (12) to foresee an efficient convex optimization algorithm for this problem:

$$\boxed{\hat{\boldsymbol{\rho}} = \underset{\boldsymbol{\rho} \succcurlyeq 0}{\arg\min} ||\boldsymbol{\rho}||_1 \text{ s.t. } \boldsymbol{\rho}^H \mathbf{K}_u \boldsymbol{\rho} - 2\boldsymbol{\rho}^H \mathbf{g} + (1-\epsilon)||\hat{\mathbf{R}}_u||_F^2 < 0.} \quad (15)$$

It is remarked that, by means of the rationale that we have presented, the initial problem in (12) has been transformed into a quadratic program as in (15).

Having derived the DCP form in (15), another advantage of ℓ_1-norm is that several efficient and fast algorithms that solve ℓ_1-regularized convex programs are known in the literature, such as the family of Iterative Soft-Thresholding Algorithms (ISTA) [20] or the well-known Interior Point Method (IPM) (see [19] and [21] for more details in how to solve (15) using IPM). Each algorithm has its pros and cons. For example, the ISTA family is more robust than IPM, but the latter converges much faster.

Given that ϵ regulates the closeness between $\hat{\mathbf{R}}_u$ and $\mathbf{R}_u(\boldsymbol{\rho})$, there are two points that one must take into consideration to choose a value of ϵ. Firstly, to estimate $\boldsymbol{\rho}$ it does not make sense to require $\mathbf{R}_u(\boldsymbol{\rho})$ to be as close as possible to $\hat{\mathbf{R}}_u$ otherwise the estimation of $\boldsymbol{\rho}$ would be incorporating noise components inherent to the estimation of $\hat{\mathbf{R}}_u$. Secondly, the sparsity of the solution is controlled by ϵ. Small values of ϵ lead to a non-sparse solution and yield a closer $\mathbf{R}_u(\boldsymbol{\rho})$ to $\hat{\mathbf{R}}_u$, but require higher complexity (higher values of N), otherwise (15) becomes unfeasible. There is a clear trade-off between ϵ and N in the sense that it is not possible to reduce the value of both parameters while (15) still being a feasible convex program. As a consequence, one must choose between higher performance (a reasonably small ϵ) or lower complexity (small N).

Finally, given an estimation of $\hat{\boldsymbol{\rho}}$, generated by solving (15), the conversion to the DL spatial covariance matrix is computed as:

$$\hat{\mathbf{R}}_d(\hat{\boldsymbol{\rho}}) = \sum_{i=1}^{N} \hat{\rho}_i \mathbf{d}_i \mathbf{d}_i^H. \quad (16)$$

IV. NUMERICAL RESULTS

In this section we show the performance of our approach to the UDCC problem, as compared to three other estimators: DL sample spatial covariance matrix, UL sample spatial covariance matrix and the basic algorithm from the projection methods approach [9]. We have chosen [9] because it is the best performing state-of-the-art algorithm for UDCC. The performances of each estimator are shown as the statistical mean of the normalized Frobenius distance, $d_E(\hat{\mathbf{R}}_d, \mathbf{R}_d)$, obtained by Monte Carlo simulations.

Following the same methodology as in [9], we base our simulations on the model in (7) [16]. The solution in (15) is labeled as the Sparse-Aware method.

The model in (7) is simulated in such a way that the conditions of the mmWave band are emulated, in particular, the sparse prior on the APS. S is fixed to 5 scatter clusters and $f_s(\cdot)$ are defined as a Gaussian RBF:

$$f_s(\theta) = \exp(-\frac{|\theta - \theta_s|^2}{\sigma_s}), \quad (17)$$

where θ_s is uniformly drawn in each MonteCarlo realization from $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$ and each σ_s is uniformly drawn from $\left[\frac{0.1\pi}{180}, \frac{0.2\pi}{180}\right]$, denoting the location and dispersion of the s-th scatter cluster. Moreover, the weights α_s from (7) are uniformly drawn from $[0, 1]$, the UL and DL carrier frequencies ratio is $\frac{\lambda_d}{\lambda_u} = \frac{1.8}{1.9}$ and the antenna separation is set to be half the UL carrier wavelength, $d = \frac{\lambda_u}{2}$. The Signal to Noise Ratio (SNR) of the UL/DL channel is defined in these simulations as:

$$\gamma = \frac{\text{tr}(\mathbf{R})}{M_d \sigma_w^2}, \quad (18)$$

where $\mathbf{R}$ is either the UL or the DL spatial channel covariance matrix and σ_w^2 is the input noise power. Note that the SNR value is invariant to the covariance matrix dimensions. In each iteration, the UL and DL sample covariance matrices are estimated with an SNR uniformly drawn from $[10, 30]$ dB with $K = 1000$ noisy measurements.

The Sparse-Aware and the Projection-based algorithms are fed with the UL sample covariance matrix to perform the UDCC. As for the Sparse-Aware specific parameters, ϵ is fixed to 0.0075 and $N = 2M^2$ in the simulation depicted in Figure 1. The reasoning behind those values is that they are the combination of ϵ and N such that ϵ is minimized and (15) is still feasible with reasonable complexity. In this way, the Sparse-Aware algorithm can exhibit state-of-the-art performance.

The rationale behind the presented simulations is that we wanted to emulate a setting similar to the one depicted in [9] with the addition of the sparse assumption on the APS. The sparse assumption on the APS is the reason why we have chosen the scale parameters in (17), σ_s, to be uniformly drawn from $\left[\frac{0.1\pi}{180}, \frac{0.2\pi}{180}\right]$. It is remarked that in both simulations M_d is referred as M for notation simplicity.

As it can be observed in Figure 1, using UL sample spatial channel covariance matrix as $\hat{\mathbf{R}}_d$ is not a reliable estimator for $\mathbf{R}_d$ in massive MIMO (large M_d) with FDD. Still, it is shown that there exists some correlation between both channels, especially in the small array regime (small M_d).

In addition, it is remarked in the experiment shown in Figure 1 that the Sparse-Aware algorithm yields a similar performance to the Projection method algorithm. What is more, the scale of the problem is equivalent in both algorithms, being in the order of $2M_d^2$ in this case.

The trade-off between N and ϵ is observed numerically in Figure 2, where the Sparse-Aware algorithm shows a better result than the Projection methods and the DL sample covariance matrix in the case of $\epsilon = 0.005$. However, the increment on computational complexity is larger than the performance improvement.

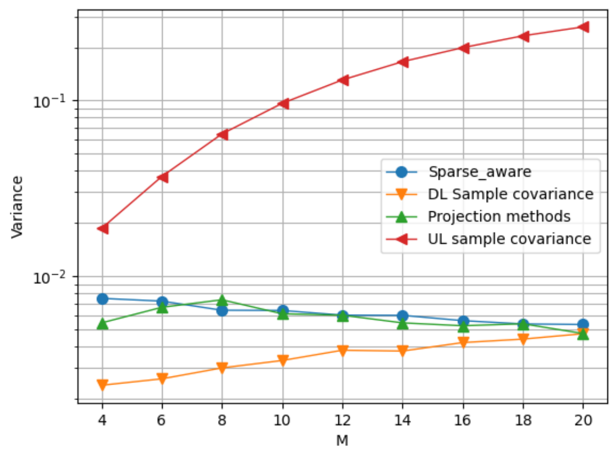

Fig. 1: Simulation results: $\epsilon = 0.0075$ and $N = 2M_d^2$.

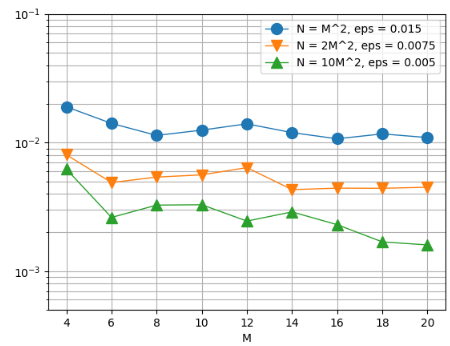

Fig. 2: Simulation results: Trade-off between ϵ and N in the Sparse-Aware algorithm. ($M_d = M$)

It should also be noted that the Projection method has more computational complexity overall than the Sparse-Aware since it is required to compute $2M_d^2$ integrals every time there is a spatial signature reconfiguration or at the start of the communication [9].

V. CONCLUSIONS

All in all, we show that through quantization on the 3GPP spatial covariance matrix model [6] and sparse filtering known results [8] our approach is capable of achieving state-of-the-art performance on the UDCC problem while being a more configurable algorithm than other alternatives.

There still is room for improvement on the ideas presented in this work. For example, it is possible to use the Angle of Arrival/Departure prior knowledge to reduce the complexity of the Sparse-Aware algorithm by allowing a smaller N for a given ϵ. Due to space limitations, this idea is issued for future work.

To conclude, the Sparse-Aware algorithm presented delivers high performance at a reasonable complexity in a massive MIMO scheme, which is the main scope of this work.

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