MIMO Radar Transmit Beampattern Matching Based on Block Successive Upper-bound Minimization

Ehsan Raei, Mohammad Alaee-Kerahroodi, Bhavani Shankar M. R.
Interdisciplinary Centre for Security,Reliability and Trust (SnT), University of Luxembourg
Email: {ehsan.raei@, mohammad.alaee@, bhavani.shankar@}uni.lu

***Abstract*—Waveform design with considering discrete phase constraint at the design stage tends to be pertinent in the emerging radar designs, especially since the digital to analogue converters are limited in the resolution. However, this constraint confines the degree of freedom to be only the waveform phase, which should be selected from a limited alphabet. In this paper, we aim to approximate a desired beampattern closely by designing the transmit waveform while considering the discrete phase constraint at the design stage. To this end, we consider a novel ℓ_p-norm metric to achieve quasi-equiripple beampattern, which reduces the interference from the undesired directions. This problem leads to a NP-hard and non-convex optimization problem, where to efficiently solve it, we utilize the BSUM algorithm which successively optimizes the objective function by optimizing a certain upper bound of the original objective in a coordinate wise manner. In the numerical results, we show the performance of the proposed method and compare it with the state-of-the art.**

***Index Terms*—Waveform Design, MIMO Radar, Beampattern Matching, Block Successive Upper Bound Minimization (BSUM)**

I. INTRODUCTION

Transmit beampattern shaping controls the directionality of the transmission on transmit antennas in Multiple-Input Multiple-Output (MIMO) radar systems. Beampattern shaping by providing a better Signal to Interference plus Noise Ratio (SINR), improves the spectral-special efficiency, better detection probability, target identification, etc. Furthermore, in the coexistence approach with communications systems the MIMO radar systems adjust its transmitter beampattern to mitigate the interference for communication systems [1], [2]. In this regards, adaptive waveform design, plays important role to shape the beampattern effectively. Generally, there are two approaches for beampattern shaping via waveform design are exist, the indirect (two-step) and direct methods [3]. In indirect approach first, the waveform correlation matrix is designed and then the original waveform matrix is obtained through one of the decomposition methods [4], [5]. While in direct method the waveform is designed directly [3], [6], [7]. Besides, there are several metrics (objective functions) to shape the beampattern, such as, beampattern matching, spatial-Integrated Sidelobe Level Ratio (ISLR) minimization and SINR maximization. In beampattern matching the goal is minimizing the difference of the beampattern response of MIMO radar with the desired beampattern [3], [8]–[11]. In Spatial-ISLR minimization approach, the aim is minimizing the ratio of *summation of beampattern response on undesired over desired angles* [12]–[14]. In SINR optimization approaches, the problem does not deal with the beampattern directly. However, kind of beampattern will be shaped as a result of transmit waveform optimization [15]–[18].

In this paper, we consider the ℓ_p-norm beampattern matching problem under discrete phase, i.e. M-ary Phase Shift Keying (MPSK) constraint. Considering the ℓ_p-norm metric for the beampattern matching problem was originally suggested in [11], and it has been shown that it provides quasi-equiripple beampattern comparing with the standard ℓ_2-norm metric. In [11], Peak-to-Average Ratio (PAR) and energy constraints were considered as the optimization constraints in the design stage. It is worth noting that in the aforementioned schemes, high-resolution digital-to-analog converters (DACs) are considered by default. However, it will cause massive power consumption and huge hardware cost when employing MIMO radars, especially for the 4D-imaging cases [14]. This motivates the use of low resolution DACs. Recently, in [19], [20], the low-resolution DACs is utilized to beampattern design. However, the aforementioned methods need several time approximation which can lead some performance loss. Different from the works in the literature, we directly solve the problem of ℓ_p-norm based beampattern matching considering the discrete phase (MPSK) constraint at the design stage. This scheme results in a non-convex, possibly NP-hard problem. Our approach is to design the waveform directly using BSUM which offers a low complexity methodology to a complex problem.

To this end, the paper is organized as follow. Section II introduces the system model and describes the problem formulation. Section III presents the proposed BSUM based method whose performance is numerically assessed in section IV. [1]

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[1]**Notations:** We adopt the notation of using lower case boldface for vectors ($\mathbf{a}$) and capital boldface for matrix ($\mathbf{A}$). The transpose, conjugate transpose, Frobenius norm absolute and Hadamard product operators are denoted by the $(.)^T$, $(.)^H$, $\| . \|_F$, $|.|$ and respectively. The letter j represents the imaginary unit (i.e., $j = \sqrt{-1}$), while the letter (i) is use as step of a procedure.

II. System Model and Problem Formulation

Let $\mathbf{X} \in \mathbb{C}^{M,N}$ be the transmitted waveform in the base-band of a MIMO radar system with M transmitters and the sequence length of N. At time sample n, the waveform transmitted through the M antennas is denoted by $\mathbf{x}_n$, where,

$$\mathbf{x}_n = [x_{1,n}, x_{2,n}, \ldots, x_{M,n}]^T \in \mathbb{C}^M. \tag{1}$$

In (1), $x_{m,n}$ denotes the n^{th} sample of the m^{th} transmitter. Let Uniform Linear Array (ULA) be the configuration of the transmitter, where the distance between the elements are $d_t = \frac{\lambda}{2}$ and λ is the wavelength. Thus, the steering vector at angle θ ($\theta \in [0, 2\pi)$) can be written as [21], $\mathbf{a}(\theta) = [1, e^{j\pi \sin(\theta)}, \ldots, e^{j\pi(M-1)\sin(\theta)}]^T \in \mathbb{C}^M$. In this case the transmit beampattern is given by [4],

$$r(\mathbf{X}, \theta) = \sum_{n=1}^{N} \left| \mathbf{a}^H(\theta)\mathbf{x}_n \right|^2 = \sum_{n=1}^{N} \mathbf{x}_n^H \mathbf{A}(\theta)\mathbf{x}_n, \tag{2}$$

where, $\mathbf{A}(\theta) \triangleq \mathbf{a}(\theta)\mathbf{a}^H(\theta)$.

Let q_k be the desired beampattern, where $k \in \{1, \ldots, K\}$ and K denotes the number of discrete angles. Using the beampattern matching under discrete phase constraint leads us to solve the following optimization problem [11],

$$\begin{cases} \min\limits_{\mathbf{X},\mu} & f(\mathbf{X}, \mu) \triangleq \sum\limits_{k=1}^{K} |r(\mathbf{X}, \theta_k) - \mu q_k|^p \\ s.t. & x_{m,n} = e^{j\phi}, \quad \phi \in \Phi_L, \end{cases} \tag{3}$$

where, $p \geq 2$, μ is a scaling factor [3], and Φ_L indicates the discrete phase alphabet. Precisely, Φ_L indicates the MPSK alphabet, e.g. $\Phi_L = \left\{0, \frac{2\pi}{L}, \ldots, \frac{2\pi(L-1)}{L}\right\}$.

As can be seen the (3) is a multi-variable, non-convex and NP-hard optimization problem. In the following, we propose an algorithm based on BSUM to deal with (3).

III. Proposed Method

The BSUM algorithm generalizes the Block Coordinate Descent (BCD) methods and includes procedure that successively optimize particular upper-bounds or local approximation functions of the original objectives in a block by block manner [22], [23]. One possible choice for the approximation function is Majorization-Minimization (MM) function. This choice is one of the condition which guarantees the convergence of the argument in optimization problem [24], [25]. In this paper, we use the following lemma to obtain the majorizer function.

Lemma III.1. *Let* $|x| \in [0, \tau]$, *for* $p \geq 2$, $|x|^p$ *can be majorized by* $\eta|x|^2 + \psi\Re\left\{x\frac{x^{(i)}}{|x^{(i)}|}\right\} + \nu$, *where,* $\psi \triangleq p|x^{(i)}|^{(p-1)} - 2\eta|x^{(i)}|$, $\eta \triangleq \frac{\tau^p + (p-1)|x^{(i)}|^p - p\tau|x^{(i)}|^{(p-1)}}{(\tau - |x^{(i)}|)^2}$, $\nu \triangleq \eta|x^{(i)}|^2 - (p-1)|x^{(i)}|^p$.

Proof. see [26]. □

Substituting $|r(\mathbf{X}, \theta_k) - \mu q_k|$ in lemma III.1 and considering $r(\mathbf{X}, \theta_k) - \mu q_k$ is a real function, it can be shown that $f(\mathbf{X}, \mu)$ can be majorized by the following,

$$\begin{aligned} u(\mathbf{X}, \mu) = & \sum_{k=1}^{K} \eta_k (r(\mathbf{X}, \theta_k) - \mu q_k)^2 \\ & + \sum_{k=1}^{K} \psi_k (r(\mathbf{X}, \theta_k) - \mu q_k) + \sum_{k=1}^{K} \nu_k. \end{aligned} \tag{4}$$

Defining $g_k^{(i)} \triangleq r(\mathbf{X}^{(i)}, \theta_k) - \mu^{(i)} q_k$ we have,

$$\begin{aligned} \eta_k &\triangleq \frac{\tau^p - |g_k^{(i)}|^p - p|g_k^{(i)}|^{p-1}(\tau - |g_k^{(i)}|)}{(\tau - |g_k^{(i)}|)^2} \\ \psi_k &\triangleq (p|g_k^{(i)}|^{p-2} - 2\eta_k) g_k^{(i)}, \nu_k \triangleq \eta_k |g_k^{(i)}|^2 - (p-1)|g_k^{(i)}|^p \end{aligned} \tag{5}$$

According to III.1, in each iteration, τ should be chosen such that it is a upper bound of $|g_k|^p$. Therefore, one possible choice is $\tau = \left\| g_k^{(i)} \right\|_p$ [26].

The problem (4) depends on $\mathbf{X}$ and μ. One possible solution to tackle this problem is using *alternating optimization* technique [27]. Based on this technique, first we optimize the problem with respect to μ, then in the next step we optimize it with respect to $\mathbf{X}$.

A. Scaling factor optimization

The majorization function (4) has a quadratic form with respect to μ. In this case the problem is convex and the optimum value of μ can be obtain by finding the roots of the derivative of the objective function. It can be shown that the optimum value for μ is given by,

$$\mu^\star = \frac{\sum_{k=1}^{K} 2q_k \eta_k r(\mathbf{X}^{(i)}, \theta_k) + q_k \psi_k}{2\sum_{k=1}^{K} \eta_k q_k^2} \tag{6}$$

B. Waveform Optimization

The BSUM procedure consists of three steps as follows, 1) Select a block. 2) Find a local approximation function that locally approximates the objective function. 3) At every iteration (i), a single block, is optimized by minimizing a approximation function of the selected block. In the smallest case each entry of matrix $\mathbf{X}$ can be considered as a block. In particular at i^{th} iteration, one entry of $\mathbf{X}^{(i)}$ is considered as the only variable while others are held fixed and with respect to this identified variable, the objective function is optimized. Such a methodology is efficient when the objective function can be written in a simplified form with respect to that variable. Let us assume that $x_{t,d}^{(i)} = e^{j\phi_{t,d}^{(i)}}$ is the only variable. Therefore the optimization problem with respect to $\phi_{t,d}^{(i)}$ can be written equivalently as [12],

$$\begin{cases} \min\limits_{\phi_{t,d}^{(i)}} & u(\mu^\star, \phi_{t,d}^{(i)}) = \sum\limits_{n=-2}^{2} c_n^{(i)} e^{jn\phi_{t,d}^{(i)}} \\ s.t. & \phi_{t,d}^{(i)} \in \Phi_L, \end{cases} \tag{7}$$

where the coefficients c_n are given in the Appendix.

Since in discrete phase the phases are chosen from limited alphabet of length L, the objective function can be written with respect to the indices of Φ_L as,

$$u(\mu^\star, l) = e^{j\frac{4\pi l}{L}} \sum_{n=-2}^{2} c_n e^{j\frac{2\pi(n-2)l}{L}} \tag{8}$$

where $l \in \{0, \dots, L-1\}$. As can be seen, the summation part of (8) is the definition of L-points Discrete Fourier Transform (DFT) of sequence $[c_2, \dots, c_{-2}]^T$. Therefore (8) can be written equivalently as,

$$u(\mu^\star, l) = \mathbf{h} \odot \mathcal{F}_L\{c_2, c_1, c_0, c_{-1}, c_{-2}\}, \tag{9}$$

where, $\mathbf{h} = [1, e^{j\frac{4\pi}{L}}, \dots, e^{j\frac{4\pi(L-1)}{L}}]^T \in \mathbb{C}^L$ and $\mathcal{F}_L$ is $L-$point DFT operator. The current function is only valid for $L \geqslant 5$. According to periodic property of DFT, $u(\mu^\star, l)$ can be written as,

$$L = 4 \Rightarrow u(\mu^\star, l) = \mathbf{h}_L \odot \mathcal{F}_L\{c_2 + c_{-2}, c_1, c_0, c_{-1}\},$$
$$L = 3 \Rightarrow u(\mu^\star, l) = \mathbf{h}_L \odot \mathcal{F}_L\{c_2 + c_{-1}, c_1 + c_{-2}, c_0\},$$
$$L = 2 \Rightarrow u(\mu^\star, l) = \mathbf{h}_L \odot \mathcal{F}_L\{c_2 + c_0 + c_{-2}, c_1 + c_{-1}\}.$$

Therefore the optimum solution for discrete phase is, $l^\star = \arg \min_{l=1,\dots,L} \left\{u(\mu^\star, l)\right\}$. Subsequently, the optimum phase is, $\phi_d^\star = \frac{2\pi(l^\star-1)}{L}$.

1) Proposed Algorithm: The proposed method is summarized in **Algorithm 1**. The inputs of this algorithm comprise $\mathbf{X}^{(0)}$ which is a set of random and feasible waveform, and the desired beampattern q_k. In the initialization step the optimization parameters will be initialized with proper values. Then in the first step we obtain the optimum value of μ, subsequently, the variable $x_{t,d}$ will be updated by $e^{j\phi_d^\star}$. This procedure will be continued until the stationary point is obtained. We consider to terminate the algorithm procedure when the argument of the objective convergence to the optimum value, e.g., we consider $\Delta\mathbf{X}^{(i)} \triangleq \left\|\mathbf{X}^{(i)} - \mathbf{X}^{(i-1)}\right\|_F \leq \zeta$ as the stopping criterion.

Algorithm 1 : Waveform Design

Input: $\mathbf{X}^{(0)}$, q_k
Output: Optimized waveform, $\mathbf{X}^\star$

1) **Initialization**
   - Set $i := 0$, $t, d := 1$ and $\mu := 1$;
2) **Optimizing the scaling factor**
   - calculate the coefficients by (11)
   - Obtain the optimum μ by (6);
3) **Optimizing the waveform**
   - Calculate $u(\mu^\star, l)$, using (9);
   - Find the Optimum phase, using $\phi_d^\star = \frac{2\pi(l^\star-1)}{L}$;
   - $\mathbf{X}^{(i)} = \mathbf{X}^{(i-1)}\big|_{x_{t,d}=e^{j\phi_d^\star}}$;
   - If $t = M$ then $t := 1$; otherwise $t := t + 1$;
   - If $d = N$ go to 4); otherwise $d := d + 1$ and go to 2);
4) **Stopping criterion**
   - If $\Delta\mathbf{X}^{(i)} = \left\|\mathbf{X}^{(i)} - \mathbf{X}^{(i-1)}\right\|_F \leq \zeta$, go to 5); otherwise $d := 1$ and go to 2);
5) **Output**
   - Set $\mathbf{X}^\star = \mathbf{X}^{(i)}$

IV. Numerical results

In this section, we provide some representative numerical examples to illustrate the effectiveness of proposed method. We consider the following assumptions. For system parameters we consider ULA configuration with $M = 16$ transmitters with $N = 128$ pulses. For purpose of simulation, we consider an uniform sampling of the regions $\theta = [-90^o, 90^o]$ with a grid size of 5^o. For the **Algorithm 1**, we consider a random MPSK sequences as initial waveform and the stopping condition of algorithm 1 is set at $\zeta = 10^{-3}$.

A. Convergence Behavior

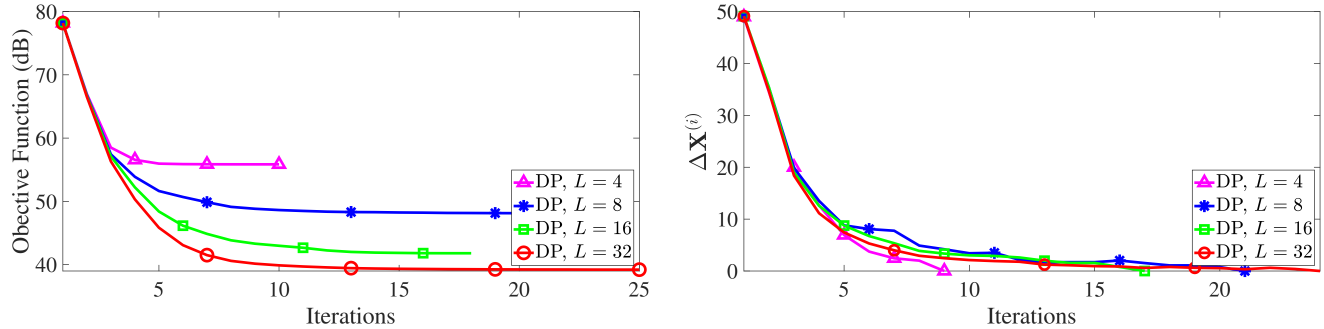
Fig. 1 shows the convergence behavior of the proposed algorithm in two aspects, namely the objective function and the argument. For these figures, we assume that the desired angles are located at $[-15^o, 15^o]$ and the algorithm 1 is initialized with random MPSK sequence with alphabet size of $L = 4$. Fig. 1a shows the convergence behavior of the objective function with different alphabet sizes. As can be seen, in all cases the objective function decreases monotonically. By increasing the alphabet size of the waveform the feasible set of the problem increases, therefore the performance of the proposed method becomes better. Fig. 1b shows the convergence behavior of the argument of the problem. Observe that in all cases the argument converges to the optimum value.

B. The impact of alphabet size

Here we investigate the impact of alphabet size of the waveform on beampattern response. Fig. 2 shows the beampattern response of the proposed method with different alphabet sizes. In this figure, we consider similar simulation setup with Fig. 1. Observe that, increasing the alphabet size cause better beampattern response in terms of the side-lobes. This behavior was expected, because by increasing the alphabet size the feasible set will increase as well.

C. Beampattern Analysis

In this subsection we evaluate the performance of the proposed method in terms of beampattern response. Fig. 3 compares the beampattern response of the proposed method with UNImodular set of seQUEnce design (UNIQUE)-C_4 [12] and the Alternating Direction Method of Multipliers (ADMM) [3] methods. The UNIQUE-C_4 method proposed a Coordinate Descent (CD)-based method to solve the spatial-ISLR problem under discrete phase, while the authors in [3] solved the beampattern matching problem based on ADMM method under continuous phase constraint. Observe that UNIQUE offers the lowest sidelobes in both terms of spatial-ISLR and -Peak Sidelobe Level Ratio (PSLR) among the methods. However, the beampattern response on the mainlobe is imperfect.



(a) objective function

(b) argument

Fig. 1: Convergence behavior of the proposed method with different alphabet size ($M = 16$, $N = 128$, $p = 3$ and $q_k \in [-15^o, 15^o]$)

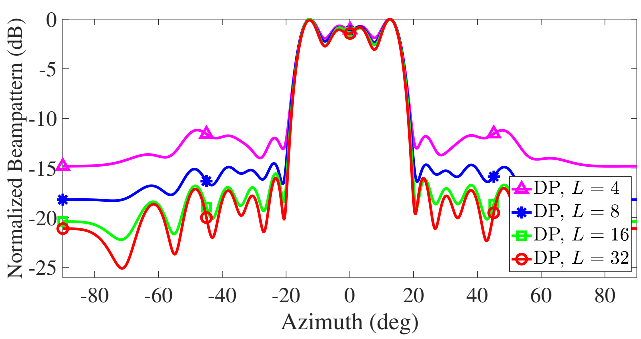


Fig. 2: The impact of alphabet size on the beampattern response of the proposed method ($M = 16$, $N = 128$, $p = 3$ and $q_k \in [-15^o, 15^o]$).

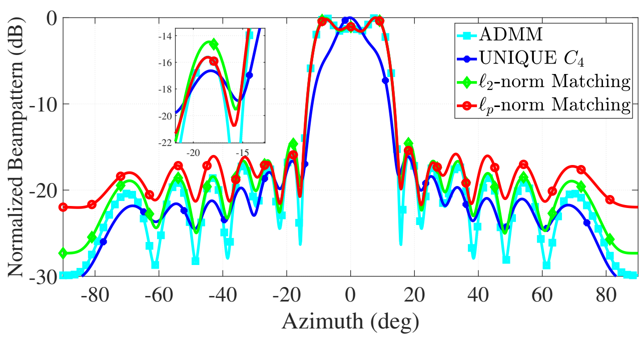


Fig. 3: Comparing the beampattern response of the proposed method with UNIQUE-C_4 and ADMM ($M = 16$, $N = 32$ $p = 64$, $L = 16$ and $q_k \in [-10^o, 10^o]$).

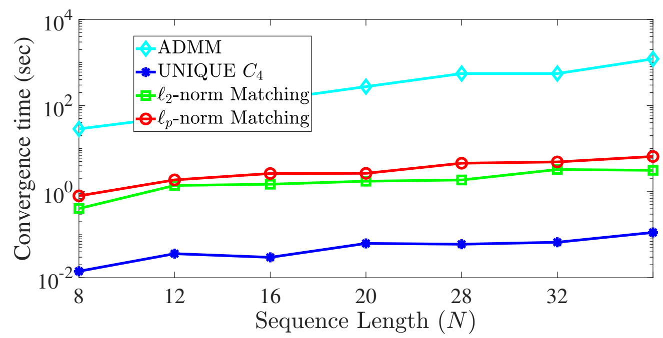


Fig. 4: Comparing the convergence time of the proposed method with UNIQUE-C_4 and ADMM, with different sequence length ($M = 16$, $p = 64$, $L = 16$ and $q_k \in [-10^o, 10^o]$).

The ADMM-based and proposed methods with ℓ_2- and ℓ_p-norm matching have the same mainlobe beampattern response. However, the advantage of the proposed method is designing a discrete phase waveform with finite alphabet size which is more attractive for radar engineers, due to the simplicity.

D. Computational Complexity

Fig. 4 compares the convergence time of the proposed with UNIQUE-C_4 and ADMM, with different sequence length. As can be seen, the ADMM has the highest convergence time which indicates the the high computational complexity of it. The UNIQUE-C_4 method offers the lowest convergence time which shows the efficiency of the algorithm. However, convergence time of the ℓ_2-norm and ℓ_p-norm beampattern matching approach is some how between the two aforementioned methods. Since in the proposed method we do not directly deal with the original problem, this behavior is expected. Furthermore, from Fig. 4 it can be concluded that, approximately the UNIQUE-C_4 is 100 times faster than the ℓ_2-norm and ℓ_p-norm approaches. In similar way the ℓ_2-norm and ℓ_p-norm approach is 100 times faster than ADMM method.

V. Conclusion

In this paper, we devised an efficient algorithm for designing MIMO radar waveform by ℓ_p-norm beampattern matching technique. We proposed a BSUM-based method to solve the multi-variable, non-convex and NP-hard problem. Through the numerical results we have shown that the proposed algorithm decreases the objective function monotonically. Besides we show the performance of the proposed method in terms of beampattern response and we have compared it with the state of the art. We show that, the proposed method designs a beampattern with good mainlobe response with good convergence time.

Appendix

The function $r(\mathbf{X},\theta_k)$ can be written with respect to $x_{t,d}$ as $r(\mathbf{X},\theta_k) = b_{1,k}e^{j\phi_{t,d}} + b_{0,k} + b_{-1,k}e^{-j\phi_{t,d}}$ [12], where,

$$b_{1,k} \triangleq \sum_{\substack{m=1\\m\neq t}}^{M} x_{m,d}^{*} a_{k_{m,t}}, \quad b_{-1,k} \triangleq b_{1,k}^{*},$$
$$b_{0,k} \triangleq a_{k_{t,t}} + \sum_{\substack{n=1\\n\neq d}}^{N} \mathbf{x}_n^H \mathbf{A}(\theta_k)\mathbf{x}_n + \sum_{\substack{m=1\\m\neq t}}^{M}\sum_{\substack{l=1\\l\neq t}}^{M} x_{m,d}^{*} a_{k_{m,l}} x_{l,d}, \tag{10}$$

and $a_{k_{m,l}}$ is the $(m,l)^{th}$ entry of matrix $\mathbf{A}(\theta_k)$. By substituting (10) in (4) and some mathematics manipulation the objective function in (7) can be obtained, as,

$$c_2 \triangleq \sum_{k=1}^{K} \eta_k b_{1,k}, c_1 \triangleq \sum_{k=1}^{K} 2\eta_k b_{1,k}(b_{0,k} - \mu q_k) + \psi_k b_{1,k},$$
$$c_0 \triangleq \sum_{k=1}^{K} 2\eta_k(|b_{1,k}|^2 + (b_{0,k} - \mu q_k)^2) + \psi_k(b_{0,k} - \mu q_k) + \nu_k,$$
$$c_{-1} \triangleq c_1^{*}, c_{-2} \triangleq c_2^{*}. \tag{11}$$

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