Coherence-based Subspace Packings for MIMO Noncoherent Communications

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***Abstract*—In this paper, we propose a new algorithm for designing unstructured Grassmannian constellations for noncoherent MIMO communications over Rayleigh block-fading channels. The algorithm minimizes the maximum coherence between subspaces, which is shown to be equivalent to the diversity product previously proposed in the literature. The coherence criterion is optimized by means of a gradient ascent algorithm on the Grassmann manifold. The method is generalized to optimize a weighted cost function that takes into account several neighboring codewords. Simulation results suggest that the constellations designed with the proposed algorithm achieve better SER performance than existing algorithms for unstructured Grassmannian constellation designs.**

***Index Terms*—Noncoherent, MIMO communications, Grassmannian constellations, coherence.**

I. Introduction

In multiple-input multiple-output (MIMO) communications systems, it is usually assumed that the channel state information (CSI) is typically estimated at the receiver side by periodic transmission of a few known pilots and then it is used for decoding at the receiver and/or for precoding at the transmitter. This is known as the coherent approach. The channel capacity for coherent MIMO systems is known to increase linearly with the minimum number of transmit and receive antennas at high signal-to-noise (SNR) ratio [1], [2] when the channel remains approximately constant over a long coherence time (slowly fading scenarios). However, in fast fading scenarios, to obtain an accurate channel estimate would require pilots to occupy a disproportionate fraction of communication resources. This CSI acquisition by orthogonal pilot-based schemes can result in significant overheads in massive MIMO systems [3] even in slowly-varying channels, and the performance of coherent massive MIMO systems can be degraded by channel aging [4]. These scenarios motivate the use of noncoherent MIMO communications schemes in which neither the transmitter nor the receiver have any knowledge about the instantaneous CSI (although they might have some knowledge about the statistical or long-term CSI such as its fading distribution).

Yet the receiver not having CSI, a significant fraction of the coherent capacity can be achieved in noncoherent MIMO communication systems at high SNR, as shown in [5]–[7]. These works proved that at high SNR under additive Gaussian noise, assuming a Rayleigh block-fading channel and when the coherence interval, T, is larger than or equal to twice the number of transmit antennas M ($T \geq 2M$), the optimal strategy achieving the capacity is to transmit isotropically distributed unitary matrices. The pre-log factor in the high-SNR capacity expression is $M^*(1 - M^*/T)$, where $M^* = \min\{M, N\}$ is the minimum between the transmit and receive number of antennas, so the noncoherent multiplexing gain approaches the coherent multiplexing gain as $T \to \infty$. Codebooks are usually formed by isotropically distributed unitary space-time matrices corresponding to optimal packings in Grassmann manifolds [7], [8]. Therefore, in noncoherent MIMO communication systems the information is carried by the column span of the transmitted $T \times M$ matrix, $\mathbf{X}$, which is not affected by the MIMO channel $\mathbf{H}$. In other words, the column span of $\mathbf{X}$ is identical to the column span of $\mathbf{XH}$.

An extensive research has been conducted on the design of noncoherent constellations as optimal packings on the Grassmann manifold [9]–[14]. Some experimental evaluation of Grassmannian constellations in noncoherent communications using over-the-air transmission has been reported in [15]. Existing constellation designs can be generically categorized into two groups: structured or unstructured. The former impose some kind of structure on the constellation points through algebraic constructions such as the Fourier-based constellation in [16], designs based on group representations [17], [18], parameterized mappings of unitary matrices such as the exp-map design in [11] or structured partitions of the Grassmannian like the recently proposed cube-split constellation [12]. The constellation structure of these designs facilitates low complexity constellation mapping and demapping, but the packing efficiency is lower than that achieved by unstructured codes, which in turn translates into poorer performance in terms of symbol error rate (SER). Since our goal is to design quasi-optimal constellations in terms of SER, in this paper

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we focus on unstructured constellations designed through numerical optimization methods.

Among the unstructured designs we can mention the alternating projection method [13] and the numerical methods in [9], [10], [19]. For example, [10] uses the spectral distance (the cosine square of the minimum principal angle), while [9] and [19] employ as a suitable distance metric the chordal distance between subspaces. Other design criteria that emerge from the analyses of pairwise error probability (PEP) [6], [20] are the maximization of the so-called diversity product [21], which can be interpreted as the minimum product of the squares of the sines of the M principal angles, and the minimization of the asymptotic union bound (UB) [22], [23], which is a sum of all diversity products between pairs of codewords.

In this paper, we propose a new algorithm that directly maximizes the minimun diversity product, which as shown in this paper is equivalent to minimizing the maximum coherence between subspaces [24]–[27]. The method proposed in [21] applies a computationally complex simulated annealing algorithm to optimize the coherence criterion. Alternatively, in this paper we perform the optimization of the coherence function of by means of a gradient ascent algorithm on the Grassmann manifold that uses an adaptive step-size. The method is generalized to optimize a weighted cost function that takes into account several neighboring codewords.

II. System Model

We consider a transmitter with M antennas communicating in a noncoherent MIMO system with a receiver equipped with N antennas, over a frequency-flat block-fading channel with coherence time T symbol periods, such that $T \geq 2M$. Hence, the channel matrix $\mathbf{H} \in \mathbb{C}^{M\times N}$ stays constant during each coherence block of T symbols, and changes in the next block to an independent realization. The MIMO channel $\mathbf{H}$ is assumed to be Rayleigh with entries $h_{ij} \sim \mathcal{CN}(0,1)$ and unknown to both the transmitter and the receiver. Within a coherence block, the transmitter sends a unitary matrix $\mathbf{X} \in \mathbb{C}^{T\times M}$, $\mathbf{X}^H\mathbf{X} = \mathbf{I}_M$, that is a unitary basis for the linear subspace $[\mathbf{X}]$ within $\mathbb{C}^T$. The signal at the receiver $\mathbf{Y} \in \mathbb{C}^{T\times N}$ is

$$\mathbf{Y} = \mathbf{X}\mathbf{H} + \sqrt{\frac{M}{T\rho}}\mathbf{W}, \tag{1}$$

where $\mathbf{W} \in \mathbb{C}^{T\times N}$ represents the additive Gaussian noise, modeled as $w_{ij} \sim \mathcal{CN}(0,1)$, and ρ represents the signal-to-noise-ratio (SNR).

The optimal Maximum Likelihood (ML) detector that minimizes the probability of error, assuming equiprobable codewords, is given by

$$\tilde{\mathbf{X}} = \arg\max_{\mathbf{X}\in\mathcal{C}} \operatorname{tr}\left(\mathbf{Y}^H\mathbf{P}_{\mathbf{X}}\mathbf{Y}\right), \tag{2}$$

where $\operatorname{tr}(\mathbf{X})$ denotes trace of $\mathbf{X}$, $\mathcal{C}$ represents the codebook of K codewords and $\mathbf{P}_{\mathbf{X}} = \mathbf{X}\mathbf{X}^H$ is the projection matrix to the subspace $[\mathbf{X}]$. Each codeword carries $\log_2(K)$ bits of information.

III. Proposed codebook design

A. Preliminaries

The high-SNR capacity for noncoherent MIMO systems is achieved by transmitting isotropically distributed unitary space-time matrices $\mathbf{X}$, [5]–[7], that is, transmitting subspaces of $\mathbb{C}^T$ represented by points uniformly distributed on a suitable matrix manifold. The Stiefel manifold parametrizes the set of all M-dimensional orthonormal frames in $\mathbb{C}^T$

$$\mathbb{S}t(M,\mathbb{C}^T) = \{\mathbf{X} \in \mathbb{C}^{T\times M} : \mathbf{X}^H\mathbf{X} = \mathbf{I}_M\}, \tag{3}$$

and the Grassmann manifold $\mathbb{G}(M,\mathbb{C}^T)$ the set of M-dimensional subspaces on $\mathbb{C}^T$. Each subspace $[\mathbf{X}]$ is represented by an $\mathbf{X} \in \mathbb{S}t(M,\mathbb{C}^T)$, up to the unitary right-action $\mathbf{X}\mathbf{U}$, with $\mathbf{U} \in \mathcal{U}(M)$. Note that, in the noiseless case, Grassmannian signaling guarantees error-free detection without CSI because $\mathbf{X}$ and the noise-free observation $\mathbf{X}\mathbf{H}$ represent the same point in $\mathbb{G}(M,\mathbb{C}^T)$. When noise is present, the span of the received signal $\mathbf{Y}$ deviates from that of $\mathbf{X}$ with respect to a distance measure, producing a detection error if $\mathbf{Y}$ is not in the decision region of the transmitted symbol. Different functions of the principal angles between subspaces ultimately yield the different distance metrics between subspaces. For example, optimizing the minimal chordal distance is of particular importance in the context of non-coherent communications [9], [19]. For simplicity, in order to design codebooks $\mathcal{C} = \{\mathbf{X}_1,\ldots,\mathbf{X}_K\}$ that minimize the SER, a pairwise error probability bound is used instead [20], which leads us to use a non-metric cost function described next.

B. Cost function

The symbol error probability for equiprobable symbols can be bounded by

$$P_e \leq \frac{1}{K}\sum_{i=1}^{K}\sum_{j=i+1}^{K} P_e\left(\mathbf{X}_i,\mathbf{X}_j\right) \tag{4}$$

where $P_e\left(\mathbf{X}_i,\mathbf{X}_j\right)$ is the asymptotic PEP at high-SNR. The right hand side in (4) is the union bound (UB). Up to a constant, this is (cf. [20])

$$\mathrm{UB}(\mathbf{X}_1,\ldots,\mathbf{X}_K) = \sum_{i<j}\det\left(\mathbf{I}_M - \mathbf{X}_i^H\mathbf{X}_j\mathbf{X}_j^H\mathbf{X}_i\right)^{-N}. \tag{5}$$

The UB is the best proxy for the actual SER which is the ultimate performance metric in practice. Therefore, minimizing the UB is the most principled design criterion for unstructured Grassmanian constellations. However, the calculation of its gradient involves high computational cost. The design criterion based on optimizing the dominant term of the UB is the so-called diversity product defined in [21] as $DP = \min_{i\neq j}\det\left(\mathbf{I}_M - \mathbf{X}_i^H\mathbf{X}_j\mathbf{X}_j^H\mathbf{X}_i\right)$. It can be shown that $\det(\mathbf{I}_M - \mathbf{X}_i^H\mathbf{X}_j\mathbf{X}_j^H\mathbf{X}_i)$ can be written as $1 - \nu([\mathbf{X}_i],[\mathbf{X}_j])^2$ where $\nu([\mathbf{X}_i],[\mathbf{X}_j])$ is the so-called *coherence* between subspaces [24]–[27], so maximizing the diversity product is equivalent to minimizing the coherence between subspaces. Therefore, the minimum coherence criterion or *coherence criterion* solves:

$$\underset{[\mathbf{X}_1],\ldots,[\mathbf{X}_K]}{\operatorname{argmax}} \quad \min_{k \neq j} \det(\mathbf{I}_M - \mathbf{X}_k^H \mathbf{X}_j \mathbf{X}_j^H \mathbf{X}_k). \tag{6}$$

Thus, the goal is to make the coherence between subspaces as small as possible or, equivalently, $\det(\mathbf{I}_M - \mathbf{X}_i^H \mathbf{X}_j \mathbf{X}_j^H \mathbf{X}_i)$ as large as possible. Recall that the SVD of $\mathbf{X}_i^H \mathbf{X}_j$, i.e. $\mathbf{U}\mathbf{D}\mathbf{V}^H$, is given in terms of the cosines of the principal angles between the subspaces $[\mathbf{X}_i]$ and $[\mathbf{X}_j]$, $\cos\theta_1, \ldots, \cos\theta_M$, cf. [21], so

$$\det\left(\mathbf{I}_M - \mathbf{X}_i^H \mathbf{X}_j \mathbf{X}_j^H \mathbf{X}_i\right) = \det\left(\mathbf{I}_M - \mathbf{D}^2\right) = \prod_{i=1}^{M} \sin^2\theta_i. \tag{7}$$

It is also evident from the coherence criterion that to achieve full diversity no pair of subspaces should have nontrivial intersection, i.e. we must have $[\mathbf{X}_i] \cap [\mathbf{X}_j] = \{\mathbf{0}\}$, $i \neq j$. In other words, to achieve full diversity the cosines of the principal angles between all pairs of subspaces must not be equal to one. Full-diversity codebooks for which this condition holds attain the maximum slope of the SER vs. SNR curve, which is NM.

Unlike other criteria like [19] which use the chordal distance optimization to design Grassmannian constellations, the function $\det(\mathbf{I}_M - \mathbf{X}_k^H \mathbf{X}_j \mathbf{X}_j^H \mathbf{X}_k)$ is not a mathematical distance between subspaces (for example, it does not satisfy the triangle inequality). The following lemma provides the main technical result to optimize the coherence criterion on the Grassmann manifold by means of a gradient ascent algorithm.

Lemma 1 *Let $\mathbf{X}_j \in \mathbb{S}t(M, \mathbb{C}^T)$ represent some fixed element in $\mathbb{G}(M, \mathbb{C}^T)$, and let $\varphi : \mathbb{G}(M, \mathbb{C}^T) \to [-\infty, \infty)$ be given by $\varphi(\mathbf{X}_k) = \log\det(\mathbf{I}_M - \mathbf{X}_k^H \mathbf{X}_j \mathbf{X}_j^H \mathbf{X}_k)$. Then, φ is smooth whenever it is not $-\infty$ and the gradient of φ at $\mathbf{X}_k$ is*

$$\nabla\varphi(\mathbf{X}_k) = -2(\mathbf{I}_T - \mathbf{X}_k \mathbf{X}_k^H)\mathbf{X}_j \mathbf{X}_j^H \mathbf{X}_k \cdot (\mathbf{I}_M - \mathbf{X}_k^H \mathbf{X}_j \mathbf{X}_j^H \mathbf{X}_k)^{-1}. \tag{8}$$

Using the singular value decomposition (SVD) $\mathbf{X}_j^H \mathbf{X}_k = \mathbf{U}\mathbf{D}\mathbf{V}^H$, this can be written as

$$\nabla\varphi(\mathbf{X}_k) = -2(\mathbf{I}_T - \mathbf{X}_k \mathbf{X}_k^H)\mathbf{X}_j \mathbf{U}\mathbf{D}(\mathbf{I}_M - \mathbf{D}^2)^{-1}\mathbf{V}^H. \tag{9}$$

PROOF. The smoothness of the logarithm and determinant yield that φ is smooth unless $\varphi(\mathbf{X}_k) = -\infty$. Its gradient is

$$\nabla\varphi(\mathbf{X}_k) = (\mathbf{I}_T - \mathbf{X}_k \mathbf{X}_k^H) D\tilde{\varphi}(\mathbf{X}_k), \tag{10}$$

where $D\tilde{\varphi}(\mathbf{X}_k)$ is the unconstrained gradient of the function $\tilde{\varphi}(\mathbf{X}_k) = \log\det(\mathbf{I}_M - \mathbf{X}_k^H \mathbf{X}_j \mathbf{X}_j^H \mathbf{X}_k)$ defined for $\mathbf{X}_k \in \mathbb{C}^{T \times M}$, and the parenthesis is the projector onto the Grassmannian tangent space. The directional derivative of this function follows from Jacobi's formula for the derivative of the determinant:

$$\begin{aligned}
D\tilde{\varphi}(\mathbf{X}_k)(\dot{\mathbf{X}}_k) &= \left.\frac{d}{dt}\right|_{t=0} \left(\tilde{\varphi}(\mathbf{X}_k + t\dot{\mathbf{X}}_k)\right) \\
&= \left.\frac{d}{dt}\right|_{t=0} \left[\log\det(\mathbf{I}_M - (\mathbf{X}_k + t\dot{\mathbf{X}}_k)^H \mathbf{X}_j \mathbf{X}_j^H (\mathbf{X}_k + t\dot{\mathbf{X}}_k))\right] \\
&= -\operatorname{tr}\left((\mathbf{I}_M - \mathbf{X}_k^H \mathbf{X}_j \mathbf{X}_j^H \mathbf{X}_k)^{-1} \cdot (\dot{\mathbf{X}}_k^H \mathbf{X}_j \mathbf{X}_j^H \mathbf{X}_k + \mathbf{X}_k^H \mathbf{X}_j \mathbf{X}_j^H \dot{\mathbf{X}}_k)\right) \\
&= -\Re\langle 2\mathbf{X}_j \mathbf{X}_j^H \mathbf{X}_k (\mathbf{I}_M - \mathbf{X}_k^H \mathbf{X}_j \mathbf{X}_j^H \mathbf{X}_k)^{-1}, \dot{\mathbf{X}}_k\rangle_F,
\end{aligned}$$

the last step follows from the circularity of the trace operator and the definition of Frobenius Hermitian product. Thus, the Euclidean gradient of $\tilde{\varphi}$ at $\mathbf{X}_k$ is

$$D\tilde{\varphi}(\mathbf{X}_k) = -2\mathbf{X}_j \mathbf{X}_j^H \mathbf{X}_k (\mathbf{I}_M - \mathbf{X}_k^H \mathbf{X}_j \mathbf{X}_j^H \mathbf{X}_k)^{-1}, \tag{11}$$

and the lemma follows from (10) and (11). □

C. *Coherence-NN optimization*

Focusing on the dominant term of the UB in Eq. (5) leads naturally to explore improved optimization approaches that consider not only the closest codeword but a number L of nearest neighbors. Here we understand "closeness" between codewords as those with higher PEP, i.e. those with lower values of the determinant function (and consequently higher coherence values). Although a rigorous treatment of this problem seems difficult to address, it is possible to imagine different heuristic mechanisms that consider a number of nearest neighbors in the optimization process. Here we present one that provides a good compromise between performance and computational complexity.

Because all the pairwise determinantal terms of the coherence criterion appear in the union bound, we propose using the gradients of additional PEP terms and not just the dominant one, so that moving the codeword in a weighted direction improves several of those terms at the same time. That is, we consider moving every codeword in an averaged direction by weighting the gradient ascent directions of the L-nearest neighbors with weights inversely proportional to the order (see step 8 in Algorithm 1).

This is motivated by the fact that separating a codeword from its nearest neighbor may bring it closer to another one, so the ideal optimization step would be to move a codeword in a direction that separates it, if possible, from its L nearest neighbors. Clearly, the essential aspect of the method is how to weight the gradients on the manifold corresponding to the L neighbors. The proposed solution is to use weights according to the harmonic series. That is, the gradient of the nearest codeword is weighed by 1, that of the next nearest by 1/2 and so on up to the gradient of the Lth nearest codeword whose gradient is weighed by $1/L$. Although there is no theoretical basis for the optimality of this choice of weights, in practice it provides good results. The proposed Grassmannian constellation design is summarized in Algorithm 1 and is termed *Coherence-NN*.

The method includes a line-search procedure to speed up convergence, with adaptive step-size μ, and three stopping criteria: a maximum number of iterations, N_{max}, a minimum value of the step-size, μ_{min}, and a minimum improvement of the value of the coherence criterion, δ_{min}.

Algorithm 1: Coherence-NN Algorithm

Input: T, M, K, initial step-size μ_{ini}, μ_{min}, adaptation rate α, N_{max}, δ_{min}, L

1 Generate K random subspaces $[\mathbf{X}]$ in $\mathbb{G}\left(M, \mathbb{C}^T\right)$
2 Obtain initial minimum coherence d_0.
3 Initialize $\mu = \mu_{ini}$ and $n = 1$
4 **do**
5   **for** $k = 1 : K$ **do**
6   Find the L closest elements $\mathbf{X}_{j_1}, \ldots, \mathbf{X}_{j_L}$ to codeword $\mathbf{X}_k$ ordered by coherence
7   Construct the matrices $\mathbf{\Delta}_{kj_1}, \ldots, \mathbf{\Delta}_{kj_L}$ that yield the best direction to get $\mathbf{X}_k$ away from each $\mathbf{X}_{j_1}, \ldots, \mathbf{X}_{j_L}$ using the corresponding gradient (e.g. normalizing Eq. 8)
8   Compute the weighted direction defined by these neighbors as

$$\mathbf{\Delta}_k = \mathbf{\Delta}_{kj_1} + \frac{1}{2}\mathbf{\Delta}_{kj_2} + \frac{1}{3}\mathbf{\Delta}_{kj_3} + \cdots + \frac{1}{L}\mathbf{\Delta}_{kj_L}$$

9   Move $\mathbf{X}_k$ in the normalized weighted direction $\tilde{\mathbf{X}}_k = \mathbf{X}_k + \mu\mathbf{\Delta}_k/||\mathbf{\Delta}_k||_F$
10   Retract $\tilde{\mathbf{X}}_k$ to the manifold by computing the Q factor in its reduced QR decomposition, which will be the new $\mathbf{X}_k$
11   **end for**
12   Obtain new minimum coherence d_n
13   **if** $d_n > d_{n-1}$ **then**
14   Update codebook $\mathcal{C}$ with the new codewords $\tilde{\mathbf{X}}_k$, $k = 1 : K$
15   **if** $d_n - d_{n-1} < \delta_{min}$ **then**
16   End optimization
17   **end if**
18   Increase step-size $\mu = \alpha\mu$
19   Move to next iteration $n = n + 1$
20   **else**
21   Decrease step-size $\mu = \mu/\alpha$
22   *(note that in this case lines 6, 7 and 8 will not be computed again)
23   **end if**
24 **while** ($n \leq N_{max}$ **and** $\mu \geq \mu_{min}$)

IV. Performance Evaluation

In this section, we assess the performance of the proposed algorithm (labeled as Coherence-NN) and compare it to other numerical optimization algorithms for designing unstructured Grassmannian constellations. Fig. 1 depicts the histograms of the diversity product or coherence function $\det\left(\mathbf{I}_M - \mathbf{X}_k^H\mathbf{X}_i\mathbf{X}_i^H\mathbf{X}_k\right)$ for Coherence-NN codebooks using $L \in \{1, 2, 10, 64\}$ neighbors for a MIMO system with coherence time $T = 4$ symbol periods, $M = 2$ transmit antennas, and $K = 64$ codewords. Here we can clearly observe that the minimum value of the diversity product increases as the number of neighbors L in the cost function grows. This improvement in the minimum diversity product will translate into an improvement of the PEP and, consequently, the SER. In addition, we see that a small number of neighbors (e.g., $L = 10$) brings most of the possible improvement without a significant increase in computational cost.

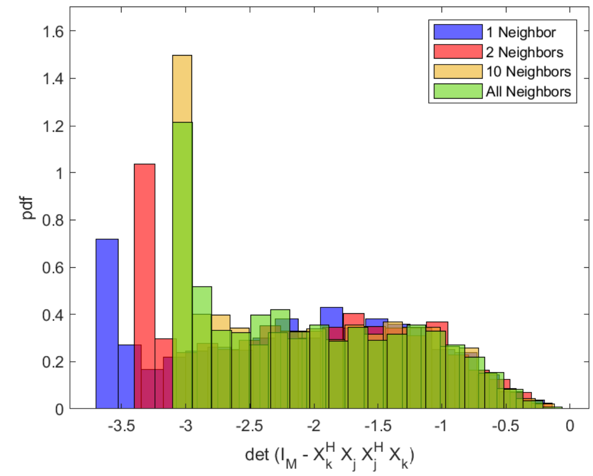

Fig. 1. Improvement of the distribution of pairwise coherence values by using an increasing number of neighboring points for $K = 64$ codewords, $T = 4$ and $M = 2$.

Fig. 2 shows the SER for the proposed Coherence-NN codebooks compared to the method proposed in [19] (which maximizes the minimum chordal distance using a gradient ascent approach on the Grassmannian, and hence is labeled as GMO-Chordal), and the alternating projection (AP) method [13] for $T = 4$, $M = 2$, $N = \{1, 2\}$ and $K = 64$ codewords. In this case, the Coherence-NN algorithm uses 10 neighbors for the cost function. We can see that the Coherence-NN constellation clearly outperforms the other two methods, and this gain in SER performance becomes more significant as we increase the number of receive antennas N.

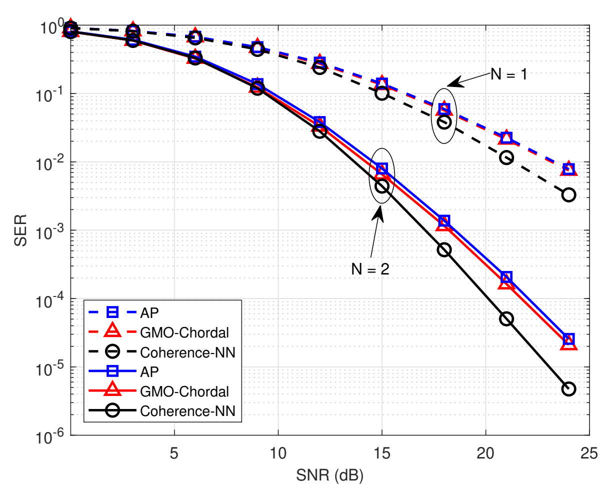

Fig. 2. SER curves for $K = 64$ codewords, $T = 4$, $M = 2$ and $N \in \{1, 2\}$.

Finally, in Fig. 3 we show the SER curves for the

Coherence-NN constellation compared to the GMO-Chordal for $T = 4$, $M = 2$, $N = 1$, and different constellation sizes $K \in \{16, 64, 256\}$. For this simulation, Coherence-NN uses around 15 % of the total constellation size as neighbors for the cost function. As it can be observed, in all cases the proposed Coherence-NN constellations clearly outperforms the GMO-Chordal in terms of SER.

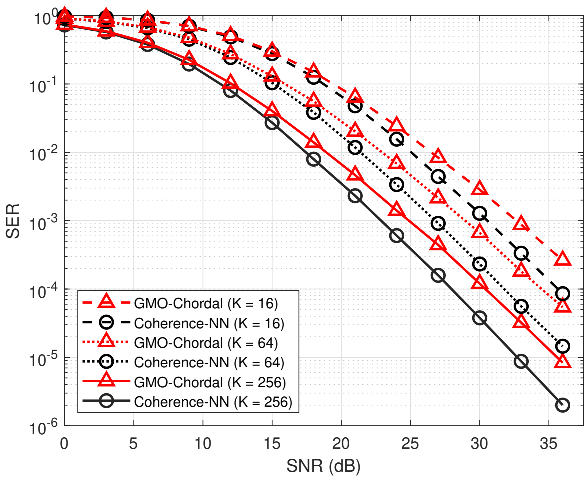

Fig. 3. SER curves for $K \in \{16, 64, 256\}$ $T = 4$, $M = 2$ and $N = 1$.

V. Conclusions

We have proposed a new algorithm for designing unstructured Grassmannian constellations for noncoherent communications, which is based on a gradient ascent approach that operates directly on the Grassmann manifold. The proposed design criterion minimizes the maximum coherence between subspaces, which is equivalent to the diversity product previously proposed in the literature. A natural extension of the method leads us to employ a weighted sum of diversity products (or coherences) for L nearest neighbors. Simulation results show that Coherence-NN outperforms other numerical optimization methods, such as AP or GMO-Chordal, in terms of symbol error rate.

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