# Cooperative Radar-Communications System With Reduced-complexity: A Combined Transmitter Selection and Receiver Deployment Approach

Liming Wang[1,2], Qian He[1,2,*], and Huiyong Li[1,2]
1. Yangtze Delta Region Institute Quzhou, University of Electronic Science and Technology of China, Quzhou, Zhejiang 324000, China
2. University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, China

***Abstract*—The cooperative multiple-input multiple-output (MIMO) radar and MIMO communications system is investigated with the objective of complexity reduction in the presence of radar hardware limitation. A combined transmitter selection and receiver deployment (TSRD) problem is formulated to minimize the Cramer-Rao bound (CRB) on target parameter estimation, where the transmitters can be chosen from a discrete set and the receivers can be deployed over a continuous region. A genetic algorithm (GA)-based method is developed for solving this mix-integer nonlinear programming problem efficiently and approximately. The result shows that the TSRD is capable of assisting in the complexity reduction for receiver processings.**

***Index Terms*—Cramer-Rao bound, joint radar and communications system, receiver deployment, transmitter selection.**

## I. Introduction

The coexistence of radar and communications [1], [2] is a well-known prototype for the joint radar and communications (RadCom) system [3]–[5]. Interference mitigation is a significant issue with coexisting RadCom systems. Instead of treating signals from the other system as interference, the work in [6] proposes an idea of cooperative RadCom system. By cooperating properly to share information with each other, both the multiple-input multiple-output (MIMO) radar and MIMO communications may achieve performance gains.

The reduction of system complexity has garnered great interest in the study of RadCom systems. Some works focus on reducing the computational complexity [7]–[11]. While to further reduce the hardware cost in possibly radio frequency chains and processors etc, some works attempt to reduce the number of antenna elements [12], [13]. Nonetheless, the majority of research on the reduced-complexity RadCom system is focused on the dual-function RadCom system, not the cooperative RadCom system [6], [10], [11], [14], [15].

This paper discusses the design of a reduced-complexity system for estimating target parameters in a cooperative RadCom system with widely separated antennas. In this case, the system complexity is mainly determined by the selection of transmitted signals to process at each of the radar receivers. The CRB is derived to assess the performance of the cooperative system in estimating target location and velocity. We develop a combined antenna selection and receiver deployment (TSRD) problem and show that it is a mix-integer nonlinear programming (MINLP) problem. The GA is a power tool for the MINLP, and it is a robust global optimizer [16]. Thus, a GA-based algorithm is proposed to approximately and efficiently solve the TSRD problem. The performance of the cooperative system designed by the proposed method is analyzed through numerical examples.

*Notations*: Throughout this paper, we use the following notations. The symbol $\odot$ means the Hadamard product, $\otimes$ the Kronecker product, $(\cdot)^T$ and $(\cdot)^H$ the transpose and conjugate transpose respectively, $\boldsymbol{I}_K$ denotes a $K\times K$ identity matrix, $\|\cdot\|_0$ the $\ell_0$-norm and $diag\{\cdot\}$ the diagonal operator.

## II. Signal Model

Assume that the cooperative radar and communications system owns a MIMO radar system with $M_R$ single antenna transmitters and a MIMO communications system with $M_C$ single antenna transmitters in a two-dimensional Cartesian coordinate system. The $m_R$th, $m_R = 1,\ldots,M_R$ radar transmitter and the $m_C$th, $m_C = 1,\ldots,M_C$ communications transmitter are transmitting signals, whose baseband forms are $\sqrt{E_{R,m_R}}s_{R,m_R}(kT_s)$ and $\sqrt{E_{C,m_C}}s_{C,m_C}(kT_s)$ respectively, where $T_s$ is the sampling interval, $E_{R,m_R}$ and $E_{C,m_C}$ denote the transmitted power, all waveforms have been normalized such that $\sum_{k=1}^{K}|s_{R,m_R}(kT_s)|^2 = \sum_{k=1}^{K}|s_{C,m_C}(kT_s)|^2 = 1/T_s$, and $M_R + M_C = M$. The radar has $N$ receivers, where the $n$th, $n = 1,\ldots,N$ single antenna radar receiver locates at $(x_{r,n}, y_{r,n})$. Suppose there is a target at $(x, y)$ moving with speed $(v_x, v_y)$. Thanks to cooperation, the communications and radar signals are assumed to have been perfectly estimated or known to the receivers, so the cooperative system is able to estimate the target parameters by utilizing signal returns from both systems [6]. At the $k$th, $k = 1,\ldots,K$ sampling instant at the $n$th radar receiver, the received signal contributed from all transmitters

*Correspondence: qianhe@uestc.edu.cn
This work was supported by the Municipal Government of Quzhou under Grant Number 2021D005.

is [6]

$$
\begin{aligned}
r_n[k] = &\sum_{m_R=1}^{M_R} \sqrt{E_{R,m_R}}\zeta_{R,nm_R}s_{R,m_R}(kT_s-\tau_{R,nm_R})e^{j2\pi f_{R,nm_R}kT_s} \\
&+\sum_{m_C=1}^{M_C} \sqrt{E_{C,m_C}}\zeta_{C,nm_C}s_{C,m_C}(kT_s-\tau_{C,nm_C})e^{j2\pi f_{C,nm_C}kT_s} \\
&+ w_n[k],
\end{aligned} \tag{1}
$$

where the time delay, Doppler frequency shift and target reflection coefficient associated with the radar system are denoted by $\tau_{R,nm_R}$, $f_{R,nm_R}$ and $\zeta_{R,nm_R}$ respectively, while these terms for the communications system are denoted by $\tau_{C,nm_C}$, $f_{C,nm_C}$ and $\zeta_{C,nm_C}$ respectively, and $w_n[kT_s]$ is the clutter-plus-noise. Assume that $\zeta_{R,nm_R}$ and $\zeta_{C,nm_C}$ from all paths are constant over the observation interval, and they have been estimated through preprocessing.

Receiver's radio frequency chains and processors may be costly. Confronted by such hardware limitation, receivers may only process a certain amount of signals. However, $M_R$ and $M_C$ in the cooperative system could be large, it may be impractical to process all their transmitted signals. This creates a critical issue with the selection of transmitters to enable good target parameter estimation performance.

Define $d_{R,nm_R} \in \{1,0\}$ as an equivalent variable for radar transmitter selection, implying that the $n$th receiver selects ($d_{R,nm_R}=1$) or rejects ($d_{R,nm_C}=0$) the signal transmitted from the $m_R$th radar transmitter. Similar definition can be given to $d_{C,nm_C} \in \{1,0\}$ for the communications transmitter selection. Define $\mathbf{d}_{R,n} = [d_{R,n1},\ldots,d_{R,nM_R}]^T$ and $\mathbf{d}_{C,n} = [d_{C,n1},\ldots,d_{C,nM_C}]^T$, then $\mathbf{d}_n = [\mathbf{d}_{R,n}^T,\mathbf{d}_{C,n}^T]^T$ implies the selection strategy of the $n$th receiver. Assume that the receiver hardware limitation is $\|\mathbf{d}_n\|_0 \le W_n$. The equivalent received signal of (1) under $\mathbf{d_n}$ for the $n$th receiver is

$$
\begin{aligned}
&r_n[k] \mid \mathbf{d}_n = \\
&\sum_{m_R=1}^{M_R} d_{R,nm_R}\sqrt{E_{R,m_R}}\zeta_{R,nm_R}s_{R,m_R}(kT_s-\tau_{R,nm_R})e^{j2\pi f_{R,nm_R}kT_s} \\
&+\sum_{m_C=1}^{M_C} d_{C,nm_C}\sqrt{E_{C,m_C}}\zeta_{C,nm_C}s_{C,m_C}(kT_s-\tau_{C,nm_C})e^{j2\pi f_{C,nm_C}kT_s} \\
&+ w_n[k].
\end{aligned} \tag{2}
$$

Let $\mathbf{u}_{R,n} = [\sqrt{E_{R,1}}\zeta_{R,n1},\ldots,\sqrt{E_{R,M_R}}\zeta_{R,nM_R}]^T$, $\mathbf{u}_{C,n} = [\sqrt{E_{C,1}}\zeta_{C,n1},\ldots,\sqrt{E_{C,M_C}}\zeta_{C,nM_C}]^T$, $\mathbf{s}_{R,n}[k] = [s_{R,1}(kT_s-\tau_{R,n1})e^{j2\pi f_{R,n1}kT_s},\ldots,s_{R,M_R}(kT_s-\tau_{R,nM_R})e^{j2\pi f_{R,nM_R}kT_s}]^T$, and $\mathbf{s}_{C,n}[k] = [s_{C,1}(kT_s-\tau_{C,n1})e^{j2\pi f_{C,n1}kT_s},\ldots,s_{C,M_C}(kT_s-\tau_{C,nM_C})e^{j2\pi f_{C,nM_C}kT_s}]^T$. Then, (2) can be rewritten as

$$
\begin{aligned}
&r_n[k] \mid \mathbf{d}_n = \\
&(\mathbf{d}_{R,n}\odot\mathbf{u}_{R,n})^T\,\mathbf{s}_{R,n}[k] + (\mathbf{d}_{C,n}\odot\mathbf{u}_{C,n})^T\,\mathbf{s}_{C,n}[k] + w[k].
\end{aligned} \tag{3}
$$

Stacking $K$ snapshot observations for $r_n[k]$ leads to

$$
\begin{aligned}
\mathbf{r}_n \mid \mathbf{d}_n &= [r_n[1],\ldots,r_n[K]]^T \\
&= \mathbf{D}_{R,n}\odot\mathbf{U}_{R,n}\mathbf{s}_{R,n} + \mathbf{D}_{C,n}\odot\mathbf{U}_{C,n}\mathbf{s}_{C,n} + \mathbf{w}_n,
\end{aligned} \tag{4}
$$

where $\mathbf{D}_{R,n} = \mathbf{I}_K\otimes(\mathbf{d}_{R,n})^T$, $\mathbf{D}_{C,n} = \mathbf{I}_K\otimes(\mathbf{d}_{C,n})^T$, $\mathbf{U}_{R,n} = \mathbf{I}_K\otimes(\mathbf{u}_{R,n})^T$, $\mathbf{U}_{C,n} = \mathbf{I}_K\otimes(\mathbf{u}_{C,n})^T$, $\mathbf{s}_{R,n} = [(\mathbf{s}_{R,n}[1])^T,\ldots,(\mathbf{s}_{R,n}[K])^T]^T$, $\mathbf{s}_{C,n} = [(\mathbf{s}_{C,n}[1])^T,\ldots,(\mathbf{s}_{C,n}[K])^T]^T$, and $\mathbf{w}_n = [w_n[1],\ldots,w_n[K]]^T$. The observations from all receivers under the selection strategy $\mathbf{d} = [\mathbf{d}_1^T,\ldots,\mathbf{d}_N^T]^T$ form

$$
\begin{aligned}
\mathbf{r} \mid \mathbf{d} &= \left[\mathbf{r}_1^T,\ldots,\mathbf{r}_N^T\right]^T \\
&= \mathbf{F}_R\odot\mathbf{U}_R\mathbf{s}_R + \mathbf{F}_C\odot\mathbf{U}_C\mathbf{s}_C + \mathbf{w} \\
&= \mathbf{\Sigma}_R + \mathbf{\Sigma}_C + \mathbf{w},
\end{aligned} \tag{5}
$$

where $\mathbf{F}_R = diag\{\mathbf{D}_{R,1},\ldots,\mathbf{D}_{R,N}\}$, $\mathbf{F}_C = diag\{\mathbf{D}_{C,1},\ldots,\mathbf{D}_{C,N}\}$, $\mathbf{U}_R = diag\{\mathbf{U}_{R,1},\ldots,\mathbf{U}_{R,N}\}$, $\mathbf{U}_C = diag\{\mathbf{U}_{C,1},\ldots,\mathbf{U}_{C,N}\}$, $\mathbf{s}_R = [(\mathbf{s}_{R,1})^T,\ldots,(\mathbf{s}_{R,N})^T]^T$, $\mathbf{s}_C = [(\mathbf{s}_{C,1})^T,\ldots,(\mathbf{s}_{C,N})^T]^T$, $\mathbf{w} = [\mathbf{w}_1^T,\ldots,\mathbf{w}_N^T]^T$ is assumed to be zero mean Gaussian white with known variance matrix $\mathbf{Q} = \sigma^2\mathbf{I}_{NK}$, and $\mathbf{\Sigma}_R = \mathbf{F}_R\odot\mathbf{U}_R\mathbf{s}_R$ and $\mathbf{\Sigma}_C = \mathbf{F}_C\odot\mathbf{U}_C\mathbf{s}_C$ represent the contribution from the radar and the communications end respectively.

## III. combined transmitter Selection and Receiver Placement

In the cooperative system, the radar task is to jointly estimate the target location and velocity gathered into a vector $\boldsymbol{\theta} = [x,y,v_x,v_y]^T$. For certain selection strategy $\mathbf{d}$, the log-likelihood function is

$$
\begin{aligned}
&L(\mathbf{r}\mid\boldsymbol{\theta},\mathbf{d}) \propto \\
&-(\mathbf{r}-(\mathbf{\Sigma}_R+\mathbf{\Sigma}_C))^H\,\mathbf{Q}^{-1}\,(\mathbf{r}-(\mathbf{\Sigma}_R+\mathbf{\Sigma}_C)) - \ln(\det(\mathbf{Q})).
\end{aligned} \tag{6}
$$

The maximum likelihood (ML) estimate of $\boldsymbol{\theta}$ is

$$
\hat{\boldsymbol{\theta}}_{ML} = \arg\max_{\boldsymbol{\theta}} L(\mathbf{r}\mid\boldsymbol{\theta},\mathbf{d}). \tag{7}
$$

According to [17], the Fisher information matrix (FIM) conditioned on $\mathbf{d}$ is

$$
\begin{aligned}
&\mathbf{J}(\boldsymbol{\theta}) = \\
&2\Re\left\{\left[\mathbf{A}_R\frac{\partial\mathbf{s}_R^H}{\partial\boldsymbol{\tau}_R}(\mathbf{F}_R\odot\mathbf{U}_R)^H + \mathbf{B}_R\frac{\partial\mathbf{s}_R^H}{\partial\boldsymbol{f}_R}\mathbf{F}_R\odot\mathbf{U}_R^H\right.\right. \\
&\qquad\left. +\mathbf{A}_C\frac{(\partial\mathbf{s}_C)^H}{\partial\boldsymbol{\tau}_C}(\mathbf{F}_C\odot\mathbf{U}_C)^H + \mathbf{B}_C\frac{\partial\mathbf{s}_C^H}{\partial\boldsymbol{f}_C}\mathbf{F}_C\odot\mathbf{U}_C^H\right] \\
&\quad\times\mathbf{Q}^{-1}\left[(\mathbf{F}_R\odot\mathbf{U}_R)\frac{\partial\mathbf{s}_R}{\partial\boldsymbol{\tau}_R}\mathbf{A}_R^T + (\mathbf{F}_R\odot\mathbf{U}_R)\frac{\partial\mathbf{s}_R}{\partial\boldsymbol{f}_R}\mathbf{B}_R^T\right. \\
&\qquad\left.\left. +(\mathbf{F}_C\odot\mathbf{U}_C)\frac{\partial\mathbf{s}_C}{\partial\boldsymbol{\tau}_C}\mathbf{A}_C^T + (\mathbf{F}_C\odot\mathbf{U}_C)\frac{\partial\mathbf{s}_C}{\partial\boldsymbol{f}_C}\mathbf{B}_C^T\right]\right\},
\end{aligned} \tag{8}
$$

where $\Re\{\cdot\}$ represents taking the real part of a matrix, the block matrix are

$$
\mathbf{A}_R = \begin{bmatrix}
\frac{\partial\tau_{R,11}}{\partial x} & \ldots & \frac{\partial\tau_{R,1M_R}}{\partial x} & \frac{\partial\tau_{R,21}}{\partial x} & \ldots & \frac{\partial\tau_{R,NM_R}}{\partial x} \\
\frac{\partial\tau_{R,11}}{\partial y} & \ldots & \frac{\partial\tau_{R,1M_R}}{\partial y} & \frac{\partial\tau_{R,21}}{\partial y} & \ldots & \frac{\partial\tau_{R,NM_R}}{\partial y} \\
0 & \ldots & 0 & 0 & \ldots & 0 \\
0 & \ldots & 0 & 0 & \ldots & 0
\end{bmatrix}, \tag{9}
$$

$$
\mathbf{A}_C = \begin{bmatrix}
\frac{\partial\tau_{C,11}}{\partial x} & \ldots & \frac{\partial\tau_{C,1M_C}}{\partial x} & \frac{\partial\tau_{C,21}}{\partial x} & \ldots & \frac{\partial\tau_{C,NM_C}}{\partial x} \\
\frac{\partial\tau_{C,11}}{\partial y} & \ldots & \frac{\partial\tau_{C,1M_C}}{\partial y} & \frac{\partial\tau_{C,21}}{\partial y} & \ldots & \frac{\partial\tau_{C,NM_C}}{\partial y} \\
0 & \ldots & 0 & 0 & \ldots & 0 \\
0 & \ldots & 0 & 0 & \ldots & 0
\end{bmatrix}, \tag{10}
$$

$$
\mathbf{B}_R = \begin{bmatrix} \frac{\partial f_{R,11}}{\partial x} & \cdots & \frac{\partial f_{R,1M_R}}{\partial x} & \frac{\partial f_{R,21}}{\partial x} & \cdots & \frac{\partial f_{R,NM_R}}{\partial x} \\ \frac{\partial f_{R,11}}{\partial y} & \cdots & \frac{\partial f_{R,1M_R}}{\partial y} & \frac{\partial f_{R,21}}{\partial y} & \cdots & \frac{\partial f_{R,NM_R}}{\partial y} \\ \frac{\partial f_{R,11}}{\partial v_x} & \cdots & \frac{\partial f_{R,1M_R}}{\partial v_x} & \frac{\partial f_{R,21}}{\partial v_x} & \cdots & \frac{\partial f_{R,NM_R}}{\partial v_x} \\ \frac{\partial f_{R,11}}{\partial v_y} & \cdots & \frac{\partial f_{R,1M_R}}{\partial v_y} & \frac{\partial f_{R,21}}{\partial v_y} & \cdots & \frac{\partial f_{R,NM_R}}{\partial v_y} \end{bmatrix}, \tag{11}
$$

$$
\mathbf{B}_C = \begin{bmatrix} \frac{\partial f_{C,11}}{\partial x} & \cdots & \frac{\partial f_{C,1M_C}}{\partial x} & \frac{\partial f_{C,21}}{\partial x} & \cdots & \frac{\partial f_{C,NM_C}}{\partial x} \\ \frac{\partial f_{C,11}}{\partial y} & \cdots & \frac{\partial f_{C,1M_C}}{\partial y} & \frac{\partial f_{C,21}}{\partial y} & \cdots & \frac{\partial f_{C,NM_C}}{\partial y} \\ \frac{\partial f_{C,11}}{\partial v_x} & \cdots & \frac{\partial f_{C,1M_C}}{\partial v_x} & \frac{\partial f_{C,21}}{\partial v_x} & \cdots & \frac{\partial f_{C,NM_C}}{\partial v_x} \\ \frac{\partial f_{C,11}}{\partial v_y} & \cdots & \frac{\partial f_{C,1M_C}}{\partial v_y} & \frac{\partial f_{C,21}}{\partial v_y} & \cdots & \frac{\partial f_{C,NM_C}}{\partial v_y} \end{bmatrix}. \tag{12}
$$

The CRBs for the estimation of elements in $\boldsymbol{\theta}$ are $\mathrm{CRB}_x = [\mathbf{J}^{-1}(\boldsymbol{\theta})]_{1,1}$, $\mathrm{CRB}_y = [\mathbf{J}^{-1}(\boldsymbol{\theta})]_{2,2}$, $\mathrm{CRB}_{v_x} = [\mathbf{J}^{-1}(\boldsymbol{\theta})]_{3,3}$, and $\mathrm{CRB}_{v_y} = [\mathbf{J}^{-1}(\boldsymbol{\theta})]_{4,4}$, where $[\cdot]_{p,q}$ denotes taking the $p$th row and $q$th column element of the matrix. For any unbiased estimator $\hat{\boldsymbol{\theta}}$, the mean square error (MSE) matrix satisfies [17]

$$
\mathbb{E}\left\{\left(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}\right)\left(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}\right)^H\right\} \geq \mathbf{J}^{-1}(\boldsymbol{\theta}), \tag{13}
$$

where $\mathbf{A} \geq \mathbf{B}$ means $(\mathbf{A} - \mathbf{B})$ is positive semidefinite. Thus, the CRBs can be used to evaluate the cooperative system target parameter estimation performance after the transmitter selection. To quantify the total performance of localization and velocity estimation, the weighted CRB (WCRB) is calculated,

$$
\mathrm{WCRB} \triangleq \alpha_x \mathrm{CRB}_x + \alpha_y \mathrm{CRB}_y + \alpha_{v_x} \mathrm{CRB}_{v_x} + \alpha_{v_y} \mathrm{CRB}_{v_y}, \tag{14}
$$

where $\alpha_x = w_x A_x$, $\alpha_y = w_y A_y$, $\alpha_{v_x} = w_{v_x} A_{v_x}$, and $\alpha_{v_y} = w_{v_y} A_{v_y}$. The factor $A_{(\cdot)}$ normalizes the CRBs for the location and velocity estimation in different base unit, while the weight $w_{(\cdot)}$ specifies the emphasis on specific parameters according to user needs.

For simplicity, suppose the communications and radar transmitters are fixed. It is assumed that the communications part has been well-designed to ensure the requisite performance, and this paper's primary focus is on optimizing the radar's performance. The FIM in (8) is a function of the transmitter selection variable $\mathbf{d}$ and all receiver locations. Thus, a joint optimization for the transmitter selection and receiver deployment (TSRD) can be formulated as followed,

$$
\begin{aligned}
\min_{\boldsymbol{\beta}} \quad & \mathrm{WCRB} && (15)\\
\text{s.t.} \quad & \boldsymbol{\beta} = \left\{\mathbf{d}^T, x_{r,1}, y_{r,1}, \ldots, x_{r,N}, y_{r,N}\right\}, && (15a)\\
& \mathbf{d} = \left[\mathbf{d}_1^T, \ldots, \mathbf{d}_N^T\right]^T, \mathbf{d}_n = \left[\mathbf{d}_{R,n}^T, \mathbf{d}_{C,n}^T\right]^T, && (15b)\\
& \mathbf{d}_{R,n} = [d_{R,n1}, \ldots, d_{R,nM_R}]^T, && (15c)\\
& \mathbf{d}_{C,n} = [d_{C,n1}, \ldots, d_{C,nM_C}]^T, && (15d)\\
& d_{R,nm_R} \in \{0, 1\}, m_R = 1, \ldots, M_R, && (15e)\\
& d_{C,nm_C} \in \{0, 1\}, m_C = 1, \ldots, M_C, && (15f)\\
& \|\mathbf{d}_n\|_0 \leq W_n, \forall n = 1, \ldots, N, && (15g)\\
& (x_{r,n}, y_{r,n}) \in \mathcal{D}_{xy,n}, \mathcal{D}_{xy,n} \subset \mathbb{R}^2, \forall n = 1, \ldots, N, && (15h)
\end{aligned}
$$

where $\|\mathbf{d}_n\|_0 \leq W_n$ restricts each receiver to process $W_n$ transmitted signals at most, and $\mathcal{D}_n$ is a two-dimensional feasible region to place the $n$th receiver. The optimization variables $\boldsymbol{\beta}$ in (15a) takes discrete (15e)(15f) and continuous (15h) values. The objective (15) is a nonlinear function of $\boldsymbol{\beta}$ from (14). The problem (15) turns out to be an MINLP [18], which is generally difficult to solve. The MINLP is a combinatorial problem that can be efficiently solved using GA [19]. When the problem is MINLP, many popular GA solvers (e.g., [20]) can offer a convenient way to find the solution. Here we use GA to solve the problem approximately. In initializing, the feasible regions for all variables in $\boldsymbol{\beta}$ are passed to the GA solver. Next the WCRB can be computed from the CRBs under a given $\boldsymbol{\beta}$. Finally, the GA solver is called to find the approximate solution $\boldsymbol{\beta}^* = [\mathbf{d}^*, x_{r,1}^*, y_{r1}^*, \ldots, x_{r,N}^*, y_{r,N}^*]$ to (15). The procedure is summarized in Algorithm 1.

**Algorithm 1:** GA-based TSRD algorithm.

**Input:** SCNR, $w_x, w_y, w_{v_x}, w_{v_y}, W_1, \ldots, W_N, \mathcal{D}_1, \ldots, \mathcal{D}_N$.

1 Set the feasible region of $\mathbf{d}, x_{r,1}, y_{r1}, \ldots, x_{r,N}, y_{r,N}$ according to (15e), (15f) and (15h).
2 Compute $\mathbf{J}(\boldsymbol{\theta}) \mid \mathbf{d}$ using (8).
3 Compute $\mathrm{CRB}_x, \mathrm{CRB}_y, \mathrm{CRB}_{v_x}$, and $\mathrm{CRB}_{v_y}$.
4 Derive the WCRB in (14).
5 Use GA to approximately solve $\arg\min_{\boldsymbol{\beta}}$ WCRB in (15).

**Output:** $\boldsymbol{\beta}^* = \left[\mathbf{d}^*, x_{r,1}^*, y_{r1}^*, \ldots, x_{r,N}^*, y_{r,N}^*\right]$.

## IV. Numerical Examples

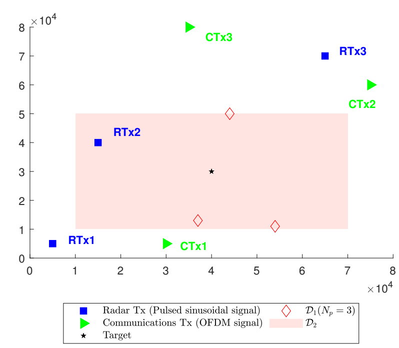


Fig. 1: System setup.

Assume in the cooperative system, there are $M_R = 3$ radar transmitters and $M_C = 3$ communications transmitters. A set of frequency spread pulsed sinusoidal signals are employed by the radar transmission, whose baseband waveform is $s_{R,m_R}[k] = 1/\sqrt{T}\exp\{j2\pi m_R \Delta f_R k T_s\}$, where $\Delta f_R$ is the frequency offset between adjacent radar transmitters, and $T$ is the pulse width. The communications system uses the OFDM signals, whose baseband waveform $s_{C,m_C}[k] = \sum_{i=-\infty}^{\infty} s_{m_C i}(kT_s - iT')$ is composed of $s_{m_C}[kT_s] = \sum_{n=-N_a/2}^{N_a/2-1} a_{m_C}[n] e^{j2\pi n \Delta f k T_s} p_{T'}(kT_s)$, where $a_{m_C}[n]$ is the data symbols, $p_{T'}(kT_s)$ is the rectangular pulse with unit amplitude and width $T'$, $\Delta f$ is the frequency space between successive subcarriers, and $N_a$ is the number of subcarriers. Configurations of the radar and communications signals are $\Delta f_R$ = 1KHz, $T$ = 85ms, $\Delta f$ = 125Hz, $N_a$ = 6,

$T' = 10$ms, and $E_{R,1} = \cdots = E_{C,M_C} = E$. Assume the cooperative system has $N = 2$ radar receivers. The receiver hardware limitation is the same for all receivers, such that $W_1 = W_2 = W$. Let $W = 4$, and the receiver feasible regions are $\mathcal{D}_{xy,1} = \mathcal{D}_{xy,2} = \mathcal{D}$.

Two types of receiver feasible regions are used, where the first region $\mathcal{D}_1$ is composed of $N_p$ randomly generated discrete points, and the second region is a continuous set $\mathcal{D}_2 = \{(x_{r,n}, y_{r_n}) \mid x_{r,n} \in [10000, 70000], y_{r,n} \in [10000, 50000], n = 1, \ldots, N\}$. Suppose the target presents at $(30186, 20097)$m with speed $(20, 20)$m/s, the target reflection coefficient for all paths are identical to be $0.1 + 0.6j$. The variance of the clutter-plus-noise is $\sigma^2 = 0.01$. Define the signal-to-clutter-plus-noise ratio (SCNR) as SCNR$=10\log_{10}(ME/\sigma^2)$. Set the user specified weights to $w_x = w_y = w_{v_x} = w_{v_y} = 1/4$. The system setup is illustrated in Fig. 1.

### A. Verification of the CRBs

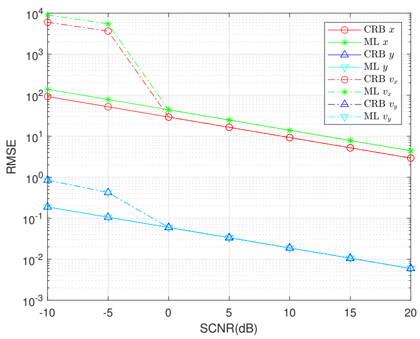


Fig. 2: The CRB versus the RMSE of the ML estimate for a given transmitter selection and receiver deployment.

First, we verify the correctness of the derived CRB. Suppose in the system setup in Fig. 1, the points Rx1=$(37000, 13000)$m and Rx3=$(44000, 50000)$m in $\mathcal{D}_1$ are chosen to place the receivers. Arbitrarily allocate the receivers using $\mathbf{d}_1 = [1, 1, 1, 1, 0, 0]^T, \mathbf{d}_2 = [1, 1, 0, 1, 1, 0]^T$, which indicates that Rx1 selects all radar transmitters and CTx1, and Rx2 selects RTx1, RTx2, CTx1 and CTx2. The root-mean-square errors (RMSEs) of the ML estimates and the corresponding CRBs are compared in Fig. 2. The ML estimator is asymptotically unbiased, whose variance can be numerically derived from (7). The CRB is calculated from (8), which is the achievable lower bound of any unbiased estimators. When SCNR approaches infinity, the ML estimates can attain the CRB asymptotically. By comparing the RMSE of the ML estimates and the corresponding CRBs, it can examine the correctness of the CRB [17]. Fig. 2 shows that the RMSEs of the location and velocity estimation asymptotically approach the corresponding CRBs, which verifies the correctness

### B. Effectiveness of Algorithm 1

With the scenario in Fig. 1, the TSRD problem is solved over $\mathcal{D} = \mathcal{D}_1$, where the set has $N_p = 400$ randomly generated discrete points. In this case, $\beta$ is feasible over a discrete set. Algorithm 1 is used to solve the TSRD problem in (15), where the GA solver is employed. For comparison, an exhaustive search is conducted to determine the optimal solution to the problem (15). The normalized WCRBs of the result obtained by Algorithm 1 and the exhaustive search are illustrated in Fig. 3. Results by Algorithm 1 appears to be capable of achieving almost the same WCRB as the exhaustive search, indicating that Algorithm 1 can approximately solve the TSRD problem. Additionally, Algorithm 1 is faster than the exhaustive search. In the sequel, we will focus exclusively on Algorithm 1 for approximate solution of the TSRD problem.

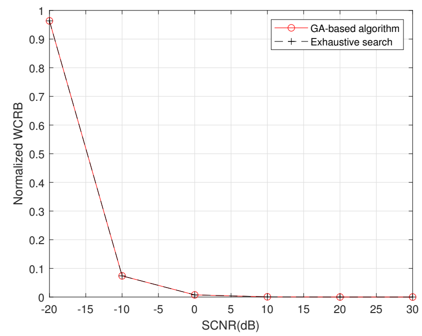


Fig. 3: The normalized WCRB obtained using exhaustive search and Algorithm 1.

### C. System complexity reduction by TSRD

This section shows the TSRD's ability in system complexity reduction for the cooperative RadCom system. A continuous receiver feasible region $\mathcal{D} = \mathcal{D}_2$ with the setup in Fig. 1 is adopted. In this case, the search space is infinite. Due to the difficulty of performing an exhaustive search, we use Algorithm 1 to solve the problem.

The solutions are given in TABLE I. It shows that the antenna selections and receiver deployments change in different SCNR regions. In the results by TSRD, we see that all receivers only need to pick 2 transmitted signals at SCNR =10dB, which is much smaller than $W = 4$. In this case, it is also noticed that the second and the third radar transmitter RTx2 and RTx3 are not elected by any receivers, they can be turned off. It indicates that the TSRD can guide the coexisting system designs to decrease the system complexity by reducing the radar transmissions and/or receiver processings.

| SNR (dB) | -10 | 0 | 10 |
|---|---|---|---|
| $x_{r,1}$ (m) | 39984.478 | 40045.412 | 39895.457 |
| $y_{r,1}$ (m) | 30029.652 | 29975.621 | 30200.896 |
| $x_{r,2}$ (m) | 14950.356 | 14911.864 | 14866.163 |
| $y_{r,2}$ (m) | 30606.843 | 30564.019 | 30137.846 |
| $\mathbf{d}_1$ | $[0, 0, 0, 1, 1, 0]^T$ | $[0, 1, 0, 1, 1, 0]^T$ | $[0, 0, 0, 1, 1, 0]^T$ |
| $\mathbf{d}_2$ | $[1, 0, 0, 0, 1, 0]^T$ | $[0, 0, 1, 0, 1, 0]^T$ | $[1, 0, 0, 0, 1, 0]^T$ |
| CRB$_x$ (m) | 363.989 | 41.403 | 36.398 |
| CRB$_y$ (m) | 234.214 | 33.941 | 23.422 |
| CRB$_{v_x}$ (m/s) | 0.312 | 0.061 | 0.0312 |
| CRB$_{v_y}$ (m/s) | 0.279 | 0.0559 | 0.028 |

TABLE I: Solutions of the TSRD by Algorithm 1.

### D. Cooperation versus Non-cooperation

Finally, the estimation performance of the coexisting system using the GA-based TSRD is investigated for the cooperative and non-cooperative MIMO radar and MIMO communications systems in the system setup in Fig. 1 with feasible region $\mathcal{D}_2$. In the non-cooperation case, the receiver can only use the radar transmitted signals. Two cases are considered for comparison: Case-1: $w_x = w_y = w_{v_x} = w_{v_y} = 1/4$ (Equal weights); Case-2: $w_x = w_y = 1/2$, $w_{v_x} = w_{v_y} = 0$ (All weights on location).

Fig. 4 plots the normalized WCRBs for the cooperative and non-cooperative coexisting system in the two cases designed by the TSRD using Algorithm 1. For Case-1, within the SCNR region of interest, the WCRB of the cooperation coexisting system is always lower than the non-cooperation counterpart. This indicates that the TSRD is able to retain the estimation performance gain [6] inherited from the cooperative coexisting system. Case-2 supports a similar conclusion.

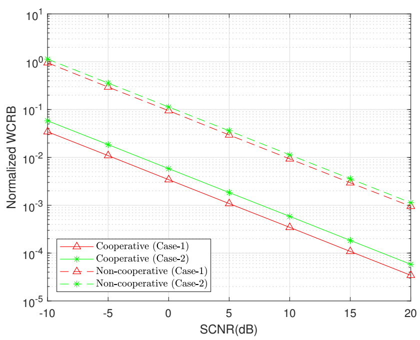


Fig. 4: The normalized WCRBs for the cooperative and the non-cooperative coexisting system designed by the proposed TSRD.

## V. Conlusion

The estimation of target parameters in a cooperative MIMO radar and MIMO communications system was investigated. The CRB for the joint localization and velocity estimation in presence of radar hardware limitations for the cooperative system was derived. A TSRD problem was formulated to determine the transmitter selection and receiver deployment that minimize the WCRB. The problem was solved approximately by a GA-based Algorithm. Through numerical examples, we showed that the proposed TSRD can reduce the complexity on radar transmissions and/or receiver processings. The TSRD can retain the performance advantage brought by cooperation over the non-cooperative systems.

## References

[1] C. Wang, J. Tong, G. Cui, X. Zhao, and W. Wang, “Robust interference cancellation for vehicular communication and radar coexistence,” *IEEE Communications Letters*, vol. 24, no. 10, pp. 2367–2370, Oct. 2020.

[2] F. Liu, C. Masouros, A. Li, T. Ratnarajah, and J. Zhou, “MIMO radar and cellular coexistence: a power-efficient approach enabled by interference exploitation,” *IEEE Transactions on Signal Processing*, vol. 66, no. 14, pp. 3681–3695, July 2018.

[3] Z. Cheng, C. Han, B. Liao, Z. He, and J. Li, “Communication-aware waveform design for MIMO radar with good transmit beampattern,” *IEEE Transactions on Signal Processing*, vol. 66, no. 21, pp. 5549–5562, Nov. 2018.

[4] Marian Bica and Visa Koivunen, “Radar waveform optimization for target parameter estimation in cooperative radar-communications systems,” *IEEE Transactions on Aerospace and Electronic Systems*, vol. 55, no. 5, pp. 2314–2326, Oct. 2019.

[5] F. Liu, L. Zhou, C. Masouros, A. Li, W. Luo, and A. P. Petropulu, “Toward dual-functional radar-communication systems: optimal waveform design,” *IEEE Transactions on Signal Processing*, vol. 66, no. 16, pp. 4264–4279, Aug. 2018.

[6] Q. He, Z. Wang, J. Hu, and R. S. Blum, “Performance gains from cooperative MIMO radar and MIMO communication systems,” *IEEE Signal Processing Letters*, vol. 26, no. 1, pp. 194–198, Jan. 2019.

[7] H. Yuan, K. Dai, Q. Li, H. Li, H. Chen, and H. Zhang, “A low-complexity parameter estimation algorithm for an integrated radar-communication waveform with cross-mode interference,” *IEEE Communications Letters*, pp. 1–1, 2021.

[8] F. Dong, W. Wang, Z. Hu, and T. Hui, “Low-complexity beamformer design for joint Radar and communications systems,” *IEEE Communications Letters*, vol. 25, no. 1, pp. 259–263, Jan. 2021.

[9] X. Liu, T. Huang, N. Shlezinger, Y. Liu, J. Zhou, and Y. C. Eldar, “Joint transmit beamforming for multiuser MIMO communications and MIMO radar,” *IEEE Transactions on Signal Processing*, vol. 68, pp. 3929–3944, 2020.

[10] Leonardo Leyva, Daniel Castanheira, Adão Silva, Atílio Gameiro, and Lajos Hanzo, “Cooperative Multiterminal Radar and Communication: A New Paradigm for 6G Mobile Networks,” *IEEE Vehicular Technology Magazine*, vol. 16, no. 4, pp. 38–47, Dec. 2021.

[11] Owen Ma, Alex R. Chiriyath, Andrew Herschfelt, and Daniel W. Bliss, “Cooperative Radar and Communications Coexistence Using Reinforcement Learning,” in *2018 52nd Asilomar Conference on Signals, Systems, and Computers*, Oct. 2018, pp. 947–951.

[12] A. Ahmed, S. Zhang, and Y. D. Zhang, “Optimized Sensor selection for joint radar-communication systems,” in *Proceedings of 2020 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, May 2020, pp. 4682–4686.

[13] X. Wang, A. Hassanien, and Moeness G. Amin, “Sparse transmit array design for dual-function radar communications by antenna selection,” *Digital Signal Processing*, vol. 83, pp. 223–234, Dec. 2018.

[14] Christ D. Richmond, Prabahan Basu, Rachel E. Learned, James Vian, Andrew Worthen, and Michael Lockard, “Performance bounds on cooperative radar and communication systems operation,” in *2016 IEEE Radar Conference*, May 2016, pp. 1–6.

[15] Daniel W. Bliss, “Cooperative radar and communications signaling: The estimation and information theory odd couple,” in *2014 IEEE Radar Conference*, May 2014, pp. 0050–0055.

[16] V. B. Gantovnik, Z. Gurdal, L. T. Watson, and C. M. Anderson-Cook, “Genetic algorithm for mixed integer nonlinear programming problems using separate constraint approximations,” *AIAA Journal*, vol. 43, no. 8, pp. 1844–1849, Aug. 2005.

[17] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*, Prentice Hall PTR, 1993.

[18] R. L. Rardin, *Optimization in Operations Research*, vol. 166, Prentice Hall Upper Saddle River, NJ, 1998.

[19] M. Gen and R. Cheng, *Genetic Algorithms and Engineering Optimization*, vol. 7, John Wiley & Sons, 1999.

[20] MATLAB cooperation, “Mixed integer ga optimization [Online],” Avaiable: https://www.mathworks.com/help/gads/mixed-integer-optimization.html.