Sensor node calibration in presence of a dominant reflective plane

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***Abstract*—Recent advances in simultaneous estimation of both receiver and sender positions in ad-hoc sensor networks have made it possible to automatically calibrate node positions – a prerequisite for many applications. In man-made environments there are often large planar reflective surfaces that give significant reverberations. In this paper, we study geometric problems of receiver-sender node calibration in the presence of such reflective planes. We establish a rank-1 factorization problem that can be used to simplify the estimation. We also show how to estimate offsets, in the Time difference of arrival case, using only the rank constraint. Finally, we present a new solver for the minimal cases of sender-receiver position estimation. These contributions result in a powerful stratified approach for the node calibration problem, given a reflective plane. The methods are verified with both synthetic and real data.**

***Index Terms*—TDOA, TOA, reverberations, minimal problems, self-calibration**

I. Introduction

Accurate receiver-sender node positions are a key prerequisite for many applications such as microphone array calibration, radio antenna array calibration, mapping and positioning [1]. If all senders and receivers are synchronized, it is possible to obtain absolute distance measurements between senders and receivers. These measurements can be used for self-calibration and such problems (time of arrival problems, TOA) have been studied in a large body of work [2]–[10]. A variant of the TOA-problem is time difference of arrival (TDOA), where the receivers are synchronized and the senders are unsynchronized [11]–[13].

Large planar surfaces that act as acoustic or radio mirrors exist in both natural and man-made environments. In such cases, the received signal contains both the part from the direct path as well as parts that have been reflected against surfaces. This has been utilized for GNSS altimetry [14], estimating the shape of a room [15], [16], and has the potential to be used in receiver-sender node position calibration [17], [18]. In this paper, we study how such reverberations can be exploited. In particular, we study the case of a dominant unknown plane, e.g., the floor plane. In this case, for each receiver there are two detections, the direct and the indirect one reflected in the floor. We assume that these detections are correctly identified,

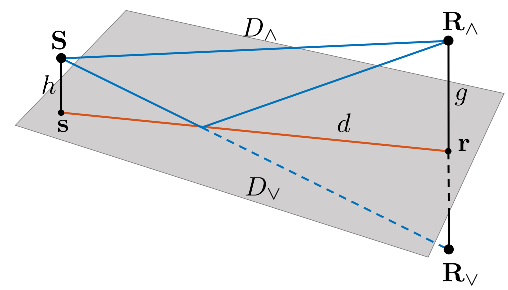

Fig. 1: Schematic of the geometry for one sender $\mathbf{S}$ and a mirror pair of receivers $(\mathbf{R}_\wedge, \mathbf{R}_\vee)$. Also shown is the direct distance $D_\wedge$ and the reflected distance $D_\vee$. The sender and the receivers project to $\mathbf{s}$ and $\mathbf{r}$ respectively in the unknown mirror plane, with corresponding distance d.

although, in general, finding which surfaces a particular echo has bounced of is a problem in itself known as echo labeling [16], [19]. We study how the geometry of this situation can be used, study minimal cases of reconstruction and use the new solvers for robust structure from motion estimation[1] . This leads to a powerful stratified formulation that separates the problem into TDOA offset estimation, height estimation and planar position estimation.

II. System Overview and Contributions

The general problem we address involves m receiver positions $\mathbf{R}_i \in \mathbb{R}^3$, $i = 1, \dots, m$ and n sender positions $\mathbf{S}_j \in \mathbb{R}^3$, $j = 1, \dots, n$. These could for example represent the microphone positions and locations of sound emissions, respectively. The arrival time of a signal sent from sender j to receiver i is denoted t_{ij}, and the time that it is emitted is denoted τ_j. Multiplying the travel time $t_{ij} - \tau_j$ with the speed v of the signal, we obtain the distance between sender and receiver

$$D_{ij} = Z_{ij} - o_j = \|\mathbf{R}_i - \mathbf{S}_j\|, \tag{1}$$

where $Z_{ij} = v t_{ij}$, $o_j = v\tau_j$ and $\|.\|$ denotes the ℓ^2-norm. The speed v is throughout the paper assumed to be known and constant. Let Z_{ij} be noisy measurements that typically suffer from small approximately Gaussian noise, outliers with

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[1]Code: https://github.com/Etomer/Reflective-Self-Calibration

substantially larger errors and missing data. Estimating $\mathbf{R}_i$, $\mathbf{S}_j$ and o_j from Z_{ij} is known as the TDOA node calibration problem. If the offsets o_j are assumed to be known, we have the corresponding TOA node calibration problem. When building systems to solve such problems robustly, often a number of key system components need to be developed. Some of these are standard components, but some need to be specifically designed if we have special setups of the geometry. In this paper, we address such a specific case, namely when we know that there is a dominant reflective plane present in the scene. This gives a number of system benefits, but also puts a number of constraints on the system. In this case, we assume that we measure both a direct distance $D_\wedge$ and a reflected distance $D_\vee$ (or $Z_\wedge$ and $Z_\vee$ for the TDOA case). We can model the reflections using the true receivers $\mathbf{R}_\wedge$, and mirrored receivers $\mathbf{R}_\vee$ (see Fig. 1), so that

$$D_{\wedge ij} = Z_{\wedge ij} - o_j = \|\mathbf{R}_{\wedge i} - \mathbf{S}_j\|, \quad (2)$$
$$D_{\vee ij} = Z_{\vee ij} - o_j = \|\mathbf{R}_{\vee i} - \mathbf{S}_j\|, \quad (3)$$

defines our sensor node calibration problem.

In Algorithm 1, an overview of our proposed stratified approach for solving this node calibration problem is shown. The specific components that we have developed, and that also make up the main contribution of our paper, are shown in bold face.

Algorithm 1 Proposed System (main contributions in bold)

Require: TOA or TDOA meaurements between senders and receivers with unknown positions in 3D

1: **Using the assumption of an (unknown) reflective plane, separate the problem into a rank-1 problem and a planar estimation problem (Section III)**
2: **If we have a TDOA problem, use the rank-1 constraint to solve for the unknown time-offsets (Section IV)**
3: Solve the rank-1 problem in a robust way (allowing for outliers and missing data). We use a RANSAC [20] approach, giving the unknown heights of senders and receivers up to a global unknown parameter. (Section V)
4: **Solve for the unknown global parameter and the unknown planar positions of the receivers and senders in a robust way using novel minimal solvers. (Section VI)**
5: Use non-linear refinement of all unknowns, e.g., by using gradient descent or Levenberg-Marquardt.

III. Mirror Geometry

The first thing we must consider is that we have a Euclidean ambiguity in our solution. This means that for a given solution, i.e., the position of the dominant reflective plane, senders and receivers, all Euclidean transformations of the solution is also a valid solution to the problem. In order remove this ambiguity, we fix the six degrees of freedoms by specifying our coordinate system. We do this by choosing the reflective plane as the z-plane, placing the first receiver on the z-axis and the second receiver in the yz-plane with positive x-coordinate.

Denoting the z-coordinates (heights) of the receivers g_i and senders h_j, and denoting the horizontal distance between $\mathbf{R}_i$ and $\mathbf{S}_j$ as d_{ij} (see Fig. 1), we get

$$D^2_{\wedge ij} = d^2_{ij} + (g_i - h_j)^2 = d^2_{ij} + g^2_i + h^2_j - 2g_ih_j, \quad (4)$$
$$D^2_{\vee ij} = d^2_{ij} + (g_i + h_j)^2 = d^2_{ij} + g^2_i + h^2_j + 2g_ih_j. \quad (5)$$

From these equations we can derive

$$D_{\Delta ij} \equiv \frac{D^2_{\vee ij} - D^2_{\wedge ij}}{4} = g_ih_j, \quad (6)$$
$$D_{\Sigma ij} \equiv \frac{D^2_{\vee ij} + D^2_{\wedge ij}}{2} = d^2_{ij} + g^2_i + h^2_j. \quad (7)$$

The first type of equation, (6), only involves the heights, g_i and h_j, and not the horizontal distance d_{ij}. Grouping together measurements from several sender-receiver pairs we get

$$D_\Delta = \begin{pmatrix} g_1h_1 & \dots & g_1h_n \\ \vdots & \ddots & \vdots \\ g_mh_1 & \dots & g_mh_n \end{pmatrix} = \begin{pmatrix} g_1 \\ \vdots \\ g_m \end{pmatrix} \begin{pmatrix} h_1 & \cdots & h_n \end{pmatrix}. \quad (8)$$

Estimating the heights g_i and h_j has now turned into a rank-1 matrix factorization problem. We will discuss solution strategies for this problem in Section V.

The second type of equation (7) can be used to calculate the horisontal distances d_{ij} between the projections on the mirror plane of the receivers and senders,

$$d^2_{ij} = D_{\Sigma ij} - g^2_i - h^2_j. \quad (9)$$

This almost leads to an ordinary TOA-problem in one dimension less, i.e., for the unknown projected receiver and senders positions $\mathbf{r}_i$ and $\mathbf{s}_j$ in the plane (see Fig. 1), we have

$$d^2_{ij} = \|\mathbf{r}_i - \mathbf{s}_j\|^2, \quad (10)$$

where d^2_{ij} depends on the estimates of the heights. This dependance leads to a slightly modified TOA-problem, which is discussed in Section VI.

The following sections are concentrated on finding robust initial solutions to the calibration problem. This is typically followed by nonlinear optimization over all inlier data and parameters in a least-squares sense, i.e., we minimize a cost such as

$$\sum_{ij} L(D_{\wedge ij} - ||\mathbf{R}_{\wedge i} - \mathbf{S}_j||_2) + L(D_{\vee ij} - ||\mathbf{R}_{\vee i} - \mathbf{S}_j||_2), \quad (11)$$

for a robust loss function L, using some gradient descent method, e.g., Levenberg–Marquardt.

IV. Offset Estimation

In this section, we will show how the rank constraint on D_Δ can be used to solve for the offsets o_j present when considering the TDOA-problem. As a reminder, the measurements are given by $Z_{\wedge ij}$ and $Z_{\vee ij}$ and relate to the distances according to

$$D_{\wedge ij} = Z_{\wedge ij} - o_j, \quad D_{\vee ij} = Z_{\vee ij} - o_j. \quad (12)$$

By insertion in (6), we observe that $D_\triangle$ is linear in o_j.

$$D_{\triangle ij} = \frac{Z_{\vee ij}^2 - Z_{\wedge ij}^2}{4} - \frac{Z_{\vee ij} - Z_{\wedge ij}}{2} o_j \tag{13}$$

The rank-1 constraint on $D_\triangle$ implies that all 2×2-minors vanish. Each minor will be a quadratic polynomial containing the monomials $\{o_{j_1}o_{j_2}, o_{j_1}, o_{j_2}, 1\}$ for some indices $j_1 \neq j_2$, $j_1, j_2 \in \{1, \dots, n\}$. The polynomial system formed in this way can be written $A\boldsymbol{v} = \boldsymbol{b}$, where A and $\boldsymbol{b}$ only depend on the data $(Z_{\wedge ij}, Z_{\vee ij})$ and $\boldsymbol{v}$ collects all non-constant monomials of the minors. Provided $m \geq 3$ and $n \geq 2$, the linear system is well-defined and $\boldsymbol{v}$ can be solved for. The offsets o_j are then easily extracted as the linear monomials in $\boldsymbol{v}$. Note that this method only utilizes the rank constraint on $D_\triangle$, works independently of the dimension of the space and turns the TDOA problem into a TOA problem. How to solve the TOA problem is the topic of the next two sections.

V. Height Estimation

In Section III, we saw that the full TOA self-calibration problem, with a mirror plane, decomposes into two separate problems. The problem of estimating the unknown heights turns into a low rank matrix factorization problem (8). Given a solution to this problem, it is clear that the rank-1 factorization of our data will only be determined up to an unknown parameter $\lambda \neq 0$, i.e., $D_\triangle = \hat{\mathbf{g}}\hat{\mathbf{h}}^T$, where $\mathbf{g} = \lambda\hat{\mathbf{g}}$ and $\mathbf{h} = \frac{1}{\lambda}\hat{\mathbf{h}}$.

If we have no missing data and no noise in our measurement, it is easy to find a solution to the factorization problem, simply by choosing $\hat{\mathbf{g}}$ as the first column of $D_\triangle$ and $\hat{\mathbf{h}}$ as the first row of $D_\triangle$ divided by $\hat{g}_1$. Consequently, with m receivers and n senders we only use $m + n - 1$ of the mn available equations. There are hence $(m-1)(n-1)$ constraints (invariants) that the noiseless realization should fulfill. In general, we will have noise, gross outliers and missing data in our measurement matrix. It is well known that the least-squares estimate is given by truncating the singular value decomposition of $D_\triangle$ to rank one [21]. However, if we have gross outliers this is not the best estimate, and if we have missing data we cannot even compute the singular value decomposition. There has been much previous work on low-rank matrix factorization [22]–[25].

In this case, we can solve the factorization in an easier way since $D_\triangle$ only has rank one and for most of these problems $n \gg m$. We find the solution by fixing $\hat{g}_1 = 1$ and then solve for $\hat{g}_i$ using a RANSAC-voting scheme with vote j computed as $D_{\triangle ij}/D_{\triangle 1j}$. We then solve for each $\hat{h}_j$ by using a RANSAC-voting scheme, where vote i is given by $D_{\triangle ij}/\hat{g}_i$. We then decide which entries in $D_\triangle$ are inliers by checking which entries in $|\hat{\mathbf{g}}\hat{\mathbf{h}}^T - D_\triangle|$ are less than some chosen tolerance.

VI. Planar Position Estimation

We will now turn our attention to the problem of estimating projected planar positions of the receivers and senders in the mirror plane, given that we have estimates of the heights. In the previous section, we saw that there were $(m-1)(n-1)$ invariants in the data, that are always fulfilled for noiseless data. This means that the number of excess constraints $\mathcal{E}$ is

$$\begin{aligned}\mathcal{E} &= 2mn - (3m + 3n - 3) - (m-1)(n-1) &(14)\\ &= mn - 2n - 2m + 2. &(15)\end{aligned}$$

Setting $\mathcal{E} = 0$ gives the two minimal cases $(m, n) = (3, 4)$ and $(m, n) = (4, 3)$, which are the minimal amount of data that is required to solve the full TOA-problem. Note that, in these cases, the heights are slightly overdetermined when there is noise in the measurements. From (9) and (10) we get

$$\|\mathbf{r}_i - \mathbf{s}_j\|^2 = D_{\Sigma ij} - \lambda^2\hat{g}_i^2 - \frac{1}{\lambda^2}\hat{h}_j^2, \tag{16}$$

where $\mathbf{r}_i$, $\mathbf{s}_j$ and λ are the unknown parameters. The scale λ is what makes (16) different from a standard planar TOA self-calibration problem, for which the minimal case is $(m, n) = (3, 3))$ [6], [7]. Using algebraic tools, it could be possible to eliminate the receiver and sender positions from (16), resulting in equations in only λ. This would enable a complete separation of the height estimation in the previous section and the planar position estimation treated here. However, we have found this elimination to be intractable[2]. Instead, we will eliminate only the senders and produce a solver for the receivers in conjunction with λ.

Our approach for solving the planar TOA-problem together with λ is to formulate the problem as a polynomial equation system and then use an existing automatic solver generator [26]. The generated solver consists of a linear system (the so-called elimination template), and an eigendecomposition of the same size as the number of solutions to the problem.

To start, let $(m, n) = (3, 4)$, and fix the coordinate system as described in Section III, i.e., let $\mathbf{r}_1 = \mathbf{0}$. We can then construct the linear systems $A\mathbf{s}_j = \boldsymbol{b}_j$, where

$$A = \begin{bmatrix} -2\mathbf{r}_2^T \\ -2\mathbf{r}_3^T \end{bmatrix} \quad \text{and} \quad \boldsymbol{b}_j = \begin{bmatrix} d_{2j}^2 - d_{1j}^2 - \mathbf{r}_2^T\mathbf{r}_2 \\ d_{3j}^2 - d_{1j}^2 - \mathbf{r}_3^T\mathbf{r}_3 \end{bmatrix}. \tag{17}$$

Since $d_{1j}^2 = \mathbf{s}_j^T\mathbf{s}_j$, we can eliminate the senders and form the equation system

$$d_{1j}^2 = \boldsymbol{b}_j^T(AA^T)^{-1}\boldsymbol{b}_j \quad \text{for} \quad j = 1, \dots, 4, \tag{18}$$

provided that A is invertible. Here, we will perform a change of variables, and instead of parameterizing the receivers in the coordinates $\mathbf{r}_i$, we use the squared inter-receiver distances c_{12}^2, c_{13}^2 and c_{23}^2, where $c_{ik} = \|\mathbf{r}_i - \mathbf{r}_k\|$, as we have observed this to produce more stable solvers.

For (18) to become polynomial it has to be multiplied with $\lambda^2\det(AA^T)$, resulting in

$$\det(AA^T)(\lambda^2 D_{\Sigma 1j} - \lambda^4\hat{g}_1^2 - \hat{h}_j^2) = \lambda^2\boldsymbol{b}_j^T\operatorname{adj}(AA^T)\boldsymbol{b}_j \tag{19}$$

[2]The approach is nevertheless possible for the 2D equivalent of the 3D mirroring problem considered here. Then $(m, n) = (2, 2)$ and the constraint becomes a single quartic polynomial in λ^2.

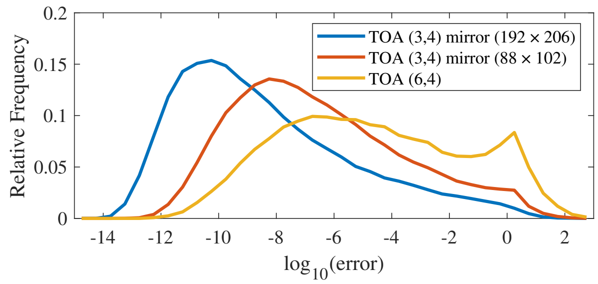

Fig. 2: Distribution of distance errors from multiple trials when providing solvers with noiseless data.

for $j = 1, \dots, 4$, where

$$AA^T = 2\begin{bmatrix} 2c_{12}^2 & c_{12}^2 + c_{13}^2 - c_{23}^2 \\ c_{12}^2 + c_{13}^2 - c_{23}^2 & 2c_{13}^2 \end{bmatrix}, \quad (20)$$

$$\boldsymbol{b}_j = \begin{bmatrix} D_{\Sigma 2j} - \lambda^2 \hat{g}_2^2 - D_{\Sigma 1j} + \lambda^2 \hat{g}_1^2 - c_{12}^2 \\ D_{\Sigma 3j} - \lambda^2 \hat{g}_3^2 - D_{\Sigma 1j} + \lambda^2 \hat{g}_1^2 - c_{13}^2 \end{bmatrix}. \quad (21)$$

However, this introduces spurious solutions causing the ideal generated by the polynomial system to not be zero-dimensional. For example, if $\lambda = 0$ the system reduces to the single equation $\det(AA^T) = 0$ which has infinitely many solutions. These spurious solutions can be removed by saturating with the unknowns $\{c_{12}^2, c_{13}^2, c_{23}^2, \lambda^2\}$ when generating the solver [9]. The produced solver has 14 solutions and an elimination template (see [26]) of size 192×206. The template can be reduced to 88×102 by saturating with $\{c_{12}^2, c_{13}^2, c_{23}^2\}$ algebraically before generating the solver. From the solutions, $\mathbf{r}_i$ and d_{ij} are easily found, after which $\mathbf{s}_j$ can be found by solving the linear systems in (17).

Observe that by relaxing the mirroring constraints on $\mathbf{R}_\wedge$ and $\mathbf{R}_\vee$, we get the minimal TOA problem $(m, n) = (6, 4)$ for which solvers already exist [7]. However, those solvers are slower and unnecessarily big in the sense that they have 38 solutions and a template size of 493×531 [27]. Furthermore, together with the offset estimation in Section IV, we have constructed minimal solvers also for the TDOA case. Without the presence of a reflective plane, these problems are significantly more difficult [28].

VII. Experiments

To evaluate the stability of our solvers, we generated synthetic TOA data consisting of receiver and sender coordinates drawn from $\mathcal{N}(0, 1)$. $D_\wedge$ and $D_\vee$ were calculated accordingly without added noise, and the heights were estimated as in Section V up to the scaling factor λ. Fig. 2 shows the norm of the distance errors resulting from the estimated node positions. As can be seen, the proposed $(3, 4)$ solvers produce smaller errors than the existing $(6, 4)$ TOA solver. They are also significantly faster with execution times of 1.6 ms (192×206) and 0.7 ms (88×102), compared to the 18 ms of the $(6, 4)$ TOA solver.

In order to test our methods in a real setting, we constructed a controlled TOA-experiment. We used a number of

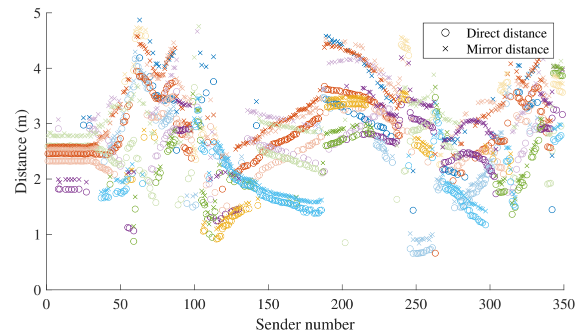

Fig. 3: Distance measurements for the real sound experiment, with different colors for different microphones. Both the estimated direct path and the estimated mirror path are shown. Note that there is a very large amount of missing data.

synchronized microphones and a moving loudspeaker playing a musical piece. The experiment was done in an environment which also featured an independent motion capture system, in order to evaluate the results. Distance estimates were found using GCC-PHAT [29] between the microphones, and in order to have a controlled experiment we used the ground truth to estimate the time offset between the speaker and the microphones. The resulting measurements for 11 microphones and 349 speaker positions are shown in Fig. 3. The dataset contains very little outliers, but very large amounts of missing data and noise in the measurements. We then proceeded to estimate both sender and receiver positions, using our stratified approach. The heights were found as described in Section V, after which the minimal solver (192×206) described in Section VI was used to estimate initial solutions for $\mathbf{r}_i$, $\mathbf{s}_j$ and λ. We used the solver in a RANSAC-voting scheme by letting the solutions vote for the correct height scaling factor. This gives an initial solution for the planar positions for three receiver and four sender positions, as well as an estimate of the global height scale. This solution was then extended using trilateration, with subsequent non-linear refinement. The results for the heights and the planar reconstruction are shown in Fig. 4, where also the ground truth is shown. The resulting mean errors in 3D-positions were in this case 8.7 cm for the receivers and 13 cm for the senders. Note that for a majority sender positions we have only three distance measurements.

VIII. Conclusion

In this paper, we have described how dominant reflective planes can be used to give powerful constraints on TOA and TDOA node calibration problems. We have developed tractable methods, that in a stratified way, solves for time offsets, node heights and planar positions of nodes, using minimal solvers that can be efficiently applied in bootstrapping algorithms. We have further applied these methods to both synthetic and real data, with promising result.

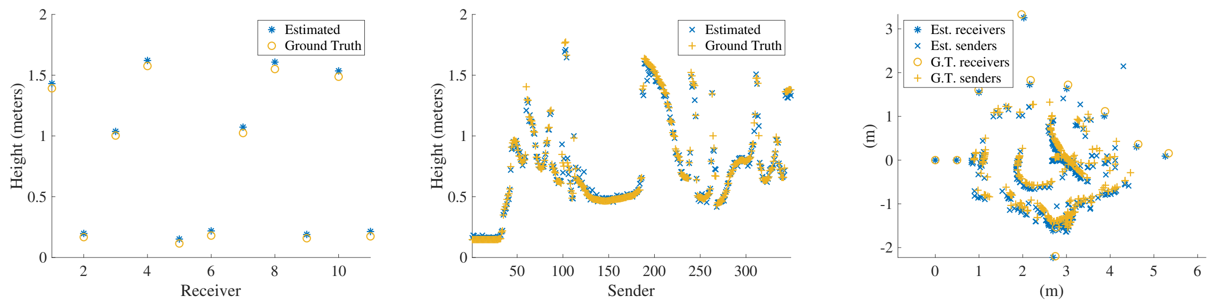

Fig. 4: Results on the real sound experiment. Left and middle show the estimated heights (in blue) for the receivers and senders, respectively, compared to the ground truth (in yellow). To the right, the estimated planar positions (in blue) for the receivers and senders are shown. The reconstruction has been rigidly registered to the ground truth (in yellow).

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