ON THE DESIGN OF ROBUST DIFFERENTIAL BEAMFORMERS WITH UNIFORM CIRCULAR MICROPHONE ARRAYS

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ABSTRACT

While circular microphone arrays (CMAs) have been used in a wide range of applications, e.g., smart speakers and conferencing phones, how to design robust beamformers that can obtain the highest possible directivity with such arrays remains a challenging issue. In this paper, we study this problem and present a method to design robust differential beamformers based on uniform CMAs (UCMAs). The approach casts the differential beamforming problem into one of optimization, which attempts to maximize the directivity factor (DF) subject to some constraints on symmetric nulls in the beampattern and the white noise gain (WNG). The robust beamformers are obtained by transforming the optimization problem into a quadratic eigenvalue problem (QEP). Simulation results show that the deduced beamformers can obtain high DFs and consistent beampatterns, regardless of the steering angle, and the WNG is effectively controlled to be larger than a pre-specified threshold.

***Index Terms*—**Uniform circular arrays, differential beamforming, directivity factor, white noise gain, quadratic eigenvalue problem.

1. INTRODUCTION

Microphone arrays with signal processing techniques have been utilized in a wide spectrum of applications [1–4]. One of the fundamental array signal processing techniques is beamforming, which is a spatial filtering process that attempts to recover the speech signal of interest (often called the target or desired speech signal) incident to the array from the look direction while attenuating the unwanted sound signals incident from other directions. Since speech signals are broadband, the beamformers are desired to form the same spatial response over a broad frequency band. This has led to the development of differential beamformers [5–7], which can achieve high directivity with consistent broadband beampatterns [8–10]. Microphone arrays that are equipped with differential beamformers are called differential microphone arrays (DMAs).

DMAs can be implemented with different array topologies, e.g., one-dimensional (1D), two-dimensional (2D), and three-dimensional (3D) ones. The choice of a topology depends on many factors including but not limited to the array performance, the steering flexibility, and the size of the device in which the array is embedded. For thin devices such as television panels and tablets, linear differential microphone arrays (LDMAs) are generally preferred. Consequently, many different array design and differential beamforming methods have been developed for the linear topology [5–7, 11–14]. While they have been widely used, LDMAs have some prominent drawbacks. The major one is that the performance changes significantly with the steering direction and in some directions other than the endfire ones, the beamformer may even break down, leading to noise amplification instead of attenuation. In many applications such as teleconferencing and smart home systems, steering has to be a priority as the target signal may be incident to the array from any azimuth angle between 0° and 360°. In such scenarios, 2D or 3D geometries need to be used. The most popular 2D array is the circular one, which can achieve consistent performance over different azimuth angles. As a result, many beamforming methods with circular microphone arrays (CMAs) have been developed [15–18], among which the differential one, based on either null constraints [8, 19] or series expansion [20, 21], has attracted much interest. However, how to design differential beamformers with CMAs that can obtain the highest possible directivity and is robust enough to implement with commercially available sensors, remains a challenging problem, which requires further efforts.

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In this paper, we present a method to design robust differential beamformers based on uniform CMAs (UCMAs). We formulate the differential beamforming problem into one of optimization, which attempts to maximize the directivity factor (DF) under some constraints on beampattern nulls as well as the white noise gain (WNG). This optimization problem is subsequently transformed into a quadratic eigenvalue problem (QEP). Note that the QEP was investigated in [12] to design robust differential beamformers with linear arrays. Therefore, the work in this paper can be regarded as an extension of the ideas in [12] from linear microphone arrays to circular microphone arrays. Simulations demonstrate that the deduced robust differential beamformers can obtain high DFs and consistent beampatterns, regardless of the steering angle. Moreover, the WNG can be effectively controlled to be larger than a pre-specified threshold and, therefore, the degree of robustness of the beamformer can be adjusted by changing the value of the lower WNG threshold.

2. SIGNAL MODEL AND EVALUATION METRICS

Consider a UCMA that consists of M omnidirectional sensors with radius r as illustrated in Fig. 1. If the azimuth angle is denoted as θ and the speed of sound is $c = 340$ m/s, the phase-delay vector (also known as the steering vector) along the direction θ is written as

$$\mathbf{d}_\theta(f) = \begin{bmatrix} e^{\jmath \frac{2\pi f r}{c}\cos(\theta-\psi_1)} & \ldots & e^{\jmath \frac{2\pi f r}{c}\cos(\theta-\psi_M)} \end{bmatrix}^T, \quad (1)$$

where the superscript ^{T} denotes the transpose operator, $\jmath$ denotes the imaginary unit, ψ_m is the azimuth angle of the mth microphone, and f is the frequency. For simpler notation, we will drop the dependence on f in the remainder of this paper.

Assume that the target speech is incident from the direction θ_{s}. Then, the received signal of the UCMA in frequency-domain is writ-

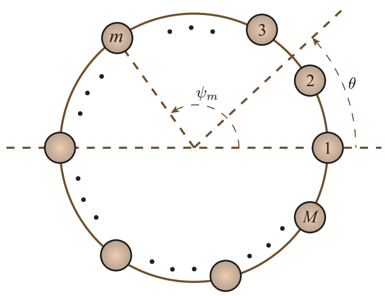

Fig. 1. Illustration of a UCMA that has M sensors, where ψ_{m} denotes the azimuth angle of the mth microphone and θ denotes the azimuth angle.

ten as

$$\begin{aligned}\mathbf{y} &= \begin{bmatrix} Y_1 & Y_2 & \cdots & Y_M \end{bmatrix}^T \\ &= \mathbf{d}_{\theta_{\mathrm{s}}} X + \mathbf{v},\end{aligned} \tag{2}$$

where Y_m is the received signal of the mth ($m = 1, 2, \ldots, M$) sensor, X denotes the desired signal, and $\mathbf{v}$ denotes the noise vector defined analogously to $\mathbf{y}$.

For beamforming, a spatial filter:

$$\mathbf{h} = \begin{bmatrix} H_1 & H_2 & \cdots & H_M \end{bmatrix}^T, \tag{3}$$

is applied to the signals observed at the array to recover the desired signal, i.e.,

$$Z = \mathbf{h}^H \mathbf{y}, \tag{4}$$

where Z is an estimate of the desired signal and the superscript ^{H} is for the conjugate-transpose operator. Generally, the beamformer needs to ensure the desired source signal from θ_{s} to be passed through without distortion. So, the beamforming filter, $\mathbf{h}$, should satisfy the following distortionless constraint:

$$\mathbf{h}^H \mathbf{d}_{\theta_{\mathrm{s}}} = 1. \tag{5}$$

This constraint should always be considered in the design of beamformers. But before discussing how to design differential beamformers, we first present three metrics, which will be used to analyze and evaluate the studied beamformers. The definitions of these metrics can be easily found in many books and papers. So, we simply give their mathematical expressions that will be used in this work. They are

- the beampattern:

$$\mathcal{B}_\theta(\mathbf{h}) = \mathbf{h}^H \mathbf{d}_\theta, \tag{6}$$

- WNG [22]:

$$\mathcal{W}(\mathbf{h}) = \frac{\left|\mathbf{h}^H \mathbf{d}_{\theta_{\mathrm{s}}}\right|^2}{\mathbf{h}^H \mathbf{h}}, \tag{7}$$

- and DF [8]:

$$\mathcal{D}(\mathbf{h}) = \frac{\left|\mathbf{h}^H \mathbf{d}_{\theta_{\mathrm{s}}}\right|^2}{\mathbf{h}^H \mathbf{\Gamma}_{\mathrm{d}} \mathbf{h}}, \tag{8}$$

where $\mathbf{\Gamma}_{\mathrm{d}}$ is a matrix of size $M \times M$, whose (ij)th element is $[\mathbf{\Gamma}_{\mathrm{d}}]_{ij} = \mathrm{sinc}(2\pi f \delta_{ij}/c)$, with δ_{ij} being the spacing between the ith and jth array elements.

3. ROBUST DIFFERENTIAL BEAMFORMERS WITH UCMAS

Theoretically, the directivity pattern of an Nth-order DMA is determined by the look direction as well as the direction information of its nulls. Such information was exploited to design differential beamformers, leading to the so-called null constrained method [11, 14]. For UCMAs, however, the null constrained method developed for linear microphone array cannot be directly used due to the symmetry property of the circular topology [8,21]. But there is a way to deal with this issue, i.e., adding symmetric null information [21]. Specifically, assume that there are N nulls at $\theta_{\mathrm{s}} + \theta_n$, with $0 < \theta_n \le \pi$, $n = 1, 2, \ldots, N$, we can construct the following linear system of equations to design an Nth-order DMA with a UCMA:

$$\mathbf{A}^H \mathbf{h} = \mathbf{i}, \tag{9}$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{d}_{\theta_{\mathrm{s}}} & \mathbf{d}_{\theta_{\mathrm{s}}-\theta_1} & \mathbf{d}_{\theta_{\mathrm{s}}+\theta_1} & \cdots & \mathbf{d}_{\theta_{\mathrm{s}}-\theta_N} & \mathbf{d}_{\theta_{\mathrm{s}}+\theta_N} \end{bmatrix} \tag{10}$$

is of size $M \times (2N+1)$, $\mathbf{i} = \begin{bmatrix} 1 & 0 & \ldots & 0 \end{bmatrix}^T$ is a vector of length $2N+1$. The above constraints require the use of at least $2N+1$ microphones. Now, suppose that the array consists of M microphones with $M \ge 2N+1$. We can identify the beamforming filter by optimizing the WNG under the constraints given in (9), i.e.,

$$\min_{\mathbf{h}} \mathbf{h}^H \mathbf{h} \quad \mathrm{s.\,t.} \quad \mathbf{A}^H \mathbf{h} = \mathbf{i}. \tag{11}$$

The solution of (11) leads to the maximum WNG (MWNG) beamformer:

$$\mathbf{h}_{\mathrm{MWNG}} = \mathbf{A}\left(\mathbf{A}^H \mathbf{A}\right)^{-1} \mathbf{i}. \tag{12}$$

Another way to identify the beamforming filter is through maximizing the DF subject to the constraints in (9), i.e.,

$$\min_{\mathbf{h}} \mathbf{h}^H \mathbf{\Gamma}_{\mathrm{d}} \mathbf{h} \quad \mathrm{s.\,t.} \quad \mathbf{A}^H \mathbf{h} = \mathbf{i}. \tag{13}$$

The solution leads to the maximum DF (MDF) beamformer, i.e.,

$$\mathbf{h}_{\mathrm{MDF}} = \mathbf{\Gamma}_{\mathrm{d}}^{-1} \mathbf{A}\left(\mathbf{A}^H \mathbf{\Gamma}_{\mathrm{d}}^{-1} \mathbf{A}\right)^{-1} \mathbf{i}. \tag{14}$$

In comparison with the MWNG beamformer, the MDF one is able to achieve a much higher DF, but it is not robust and sensitive to sensor and array imperfections. One solution is to maximize the DF while ensuring the WNG to be equal or larger than a minimum threshold value $\mathcal{W}_0$. This WNG constrained problem is mathematically formulated as

$$\min_{\mathbf{h}} \mathbf{h}^H \mathbf{\Gamma}_{\mathrm{d}} \mathbf{h} \qquad \mathrm{s.\,t.} \quad \begin{cases} \mathbf{h}^H \mathbf{h} = \mathcal{W}_0^{-1} \\ \mathbf{A}^H \mathbf{h} = \mathbf{i} \end{cases}. \tag{15}$$

Note that an optimization problem similar to (15) was formulated in [12] to design differential beamformers with uniform linear arrays. The difference lies in not only the geometry but also the linear system constraints.

According to the analysis given in [12], any fixed beamformer satisfying the constraints in (9) can be written as

$$\mathbf{h} = \mathbf{h}_{\mathrm{MWNG}} + \widetilde{\mathbf{U}} \mathbf{g}, \tag{16}$$

where $\mathbf{g}$ is a filter of length $K = M - 2N - 1$, the matrix $\widetilde{\mathbf{U}}$ of size $M \times K$ is the null space of $\mathbf{A}\mathbf{A}^H$, and $\mathbf{h}_{\mathrm{MWNG}}^H \widetilde{\mathbf{U}} \mathbf{g} = 0$.

With (16), one can check that

$$\mathbf{h}^H\mathbf{\Gamma}_{\rm d}\mathbf{h} = \mathbf{h}_{\rm MWNG}^H\mathbf{\Gamma}_{\rm d}\mathbf{h}_{\rm MWNG} - \mathbf{g}^H\mathbf{t} - \mathbf{t}^H\mathbf{g} + \mathbf{g}^H\mathbf{\Psi}\mathbf{g}, \quad (17)$$

$$\mathbf{h}^H\mathbf{h} = \mathbf{h}_{\rm MWNG}^H\mathbf{h}_{\rm MWNG} + \mathbf{g}^H\mathbf{g} = \mathcal{W}_0^{-1}, \quad (18)$$

where

$$\mathbf{t} = -\widetilde{\mathbf{U}}^H\mathbf{\Gamma}_{\rm d}\mathbf{h}_{\rm MWNG}, \quad (19)$$

$$\mathbf{\Psi} = \widetilde{\mathbf{U}}^H\mathbf{\Gamma}_{\rm d}\widetilde{\mathbf{U}}. \quad (20)$$

With (17) and (18), the optimization problem in (15) can be transformed into the following form:

$$\min_{\mathbf{g}} \left(\mathbf{g}^H\mathbf{\Psi}\mathbf{g} - \mathbf{g}^H\mathbf{t} - \mathbf{t}^H\mathbf{g}\right) \quad \text{s. t.} \quad \mathbf{g}^H\mathbf{g} = \eta, \quad (21)$$

where

$$\eta = \mathcal{W}_0^{-1} - \mathbf{h}_{\rm MWNG}^H\mathbf{h}_{\rm MWNG}. \quad (22)$$

To solve (21), we use the Lagrange multiplier method to obtain the following cost function:

$$\mathcal{J}(\mathbf{g}, \lambda) = \mathbf{g}^H\mathbf{\Psi}\mathbf{g} - \mathbf{t}^H\mathbf{g} - \mathbf{g}^H\mathbf{t} - \lambda\left(\mathbf{g}^H\mathbf{g} - \eta\right), \quad (23)$$

where λ is a Lagrange multiplier. Identifying the derivative of $\mathcal{J}(\mathbf{g}, \lambda)$ with respect to $\mathbf{g}$ and setting the results to zero gives, respectively,

$$(\mathbf{\Psi} - \lambda\mathbf{I}_K)\,\mathbf{g} = \mathbf{t}, \quad (24)$$

$$\mathbf{g}^H\mathbf{g} = \eta, \quad (25)$$

where $\mathbf{I}_K$ is the identity matrix of $K \times K$. To obtain the solution for (21), one needs to determine the smallest value of λ that satisfies (24) and (25) [12, 13, 23], which can be derived as follows.

Let us set

$$\widetilde{\mathbf{g}} = (\mathbf{\Psi} - \lambda\mathbf{I}_K)^{-2}\,\mathbf{t} = (\mathbf{\Psi} - \lambda\mathbf{I}_K)^{-1}\,\mathbf{g}. \quad (26)$$

Then, (24) and (25) can be transformed into the QEP [24] (detailed derivation can be obtained by following the work in [12, 13]):

$$\left(\lambda^2\mathbf{P}_2 + \lambda\mathbf{P}_1 + \mathbf{P}_0\right)\widetilde{\mathbf{g}} = \mathbf{Q}(\lambda)\,\widetilde{\mathbf{g}} = \mathbf{0}, \quad (27)$$

where

$$\mathbf{P}_2 = \mathbf{I}_K, \quad (28)$$

$$\mathbf{P}_1 = -2\mathbf{\Psi}, \quad (29)$$

$$\mathbf{P}_0 = \mathbf{\Psi}^2 - \frac{\mathbf{t}\mathbf{t}^H}{\eta}. \quad (30)$$

Now, the solution of (21) is obtained as [12, 13]

$$\mathbf{g} = \left[\mathbf{\Psi} - \Re(\lambda_0)\,\mathbf{I}_K\right]^{-1}\mathbf{t}, \quad (31)$$

where λ_0 is the smallest eigenvalue of $\mathbf{Q}(\lambda)$, which can be found by solving the QEP as a generalized eigenvalue problem (GEP) [13]. Finally, the robust differential beamformer filter is

$$\mathbf{h}_{\rm R} = \mathbf{h}_{\rm MWNG} + \widetilde{\mathbf{U}}\left[\mathbf{\Psi} - \Re(\lambda_0)\,\mathbf{I}_K\right]^{-1}\mathbf{t}. \quad (32)$$

4. SIMULATIONS

We evaluate the performance of the proposed method through simulations in this section.

In the first set of simulations, we consider to design robust differential beamformers using a UCMA with $M = 8$ and $r = 1$ cm, which has a null at $\theta_{\rm s} + 120^\circ$, i.e., $\theta_1 = 120^\circ$, and the value of $\mathcal{W}_0$ is set to three different levels, i.e., -30 dB, -25 dB, and -20 dB. The differential beamformer is designed according to (32).

Figure 2 plots the DFs as well as the WNGs of the proposed method as a function of frequency for the three different values of $\mathcal{W}_0$. One can see that the robust differential beamformer designed

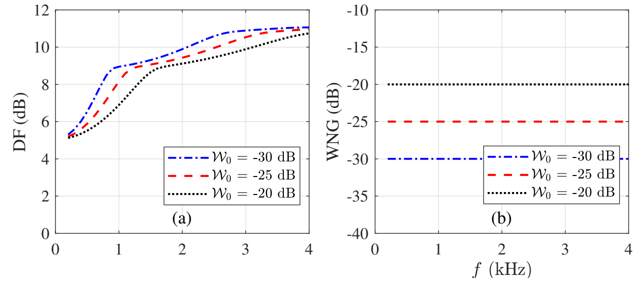

Fig. 2. DFs and WNGs of the robust differential beamformer designed with a UCMA: (a) DFs and (b) WNGs. Conditions: $M = 8$, $r = 1$ cm, $\theta_{\rm s} = 30^\circ$, and $\theta_1 = 120^\circ$.

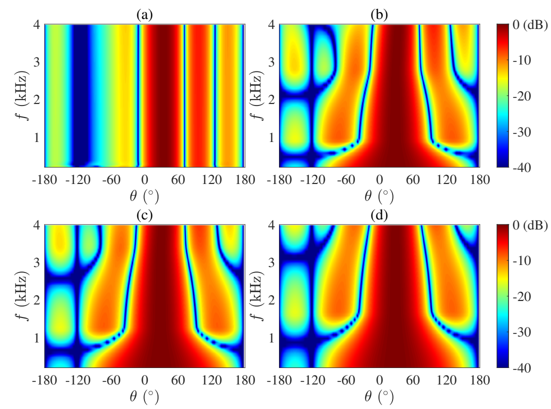

Fig. 3. Broadband beampatterns corresponding to the MDF and the proposed robust differential beamformers with a UCMA for different values of $\mathcal{W}_0$: (a) the MDF beamformer, (b) the proposed, $\mathcal{W}_0 = -30$ dB, (c) the proposed, $\mathcal{W}_0 = -25$ dB, and (d) the proposed, $\mathcal{W}_0 = -20$ dB, Conditions: $M = 8$, $r = 1$ cm, $\theta_{\rm s} = 30^\circ$, and $\theta_1 = 120^\circ$.

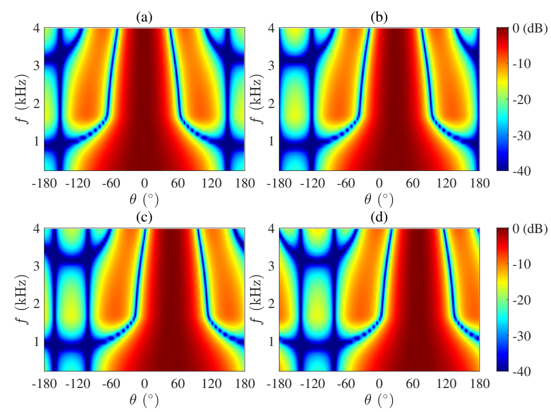

Fig. 4. Broadband beampatterns corresponding to the proposed robust differential beamformers based on a UCMA with different steering angles where: (a) $\theta_{\rm s} = 0^\circ$, (b) $\theta_{\rm s} = 30^\circ$, (c) $\theta_{\rm s} = 50^\circ$, and (d) $\theta_{\rm s} = 70^\circ$, Conditions: $M = 8$, $r = 1$ cm, $\mathcal{W}_0 = -20$ dB, and $\theta_1 = 120^\circ$.

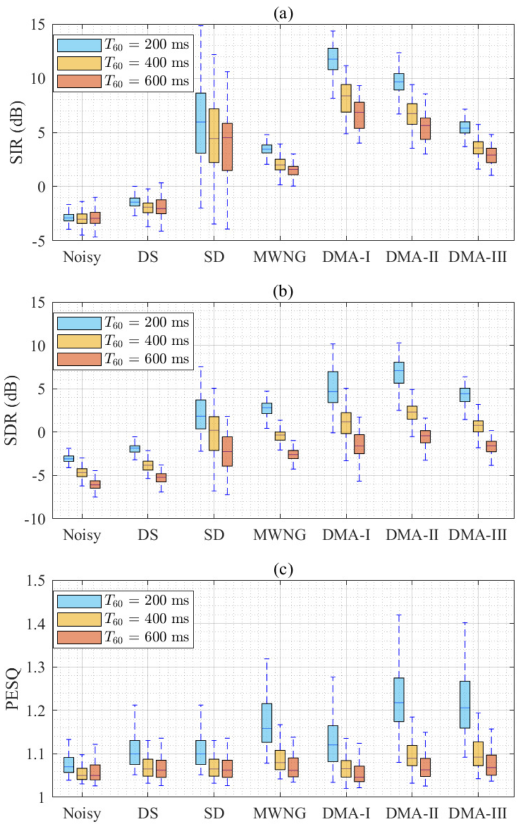

Fig. 5. Box plots of the input SIR, SDR, and PESQ of the noisy signal received by the reference microphone and the output SIR, SDR, and PESQ of the DS, robust SD, MWNG, proposed DMA-I, DMA-II, and DMA-III beamformers, designed with UCMAs, in three different reverberated environments. (a) SIR, (b) SDR, and (c) PESQ. Conditions: $M = 8$, $r = 2$ cm, the reverberation time of the three reverberated environments are $T_{60} = 200$ ms, 400 ms, and 600 ms, respectively (what shown is the averaged results over 100 simulations conducted in each environments).

with the proposed method is successful in controlling the WNG to the specified thresholds. We then design the MDF beamformer according to (14) with the same array. Figure 3 plots the broadband beampatterns corresponding to the MDF beamformer and the proposed method, again, with three different values of $\mathcal{W}_0$. It is observed that the MDF beamformer has consistent beampatterns over the studied frequency band. The proposed method is more robust than MDF, but it achieves the robustness by sacrificing the beampattern consistency, i.e., the beampattern changes over frequencies. We also set $\mathcal{W}_0 = -20$ dB and designed robust differential beamformers with different steering direction for $\theta_s \in [0^\circ, 30^\circ, 50^\circ, 70^\circ]$. The broadband beampatterns corresponding to the designed beamformer are plotted in Fig. 4, which shows that the beampatterns designed with the proposed method do not change with the steering direction.

In the second set of simulations, we evaluate the performance of the proposed method in reverberant environments. We consider a room of size: $6 \text{ m} \times 8 \text{ m} \times 3 \text{ m}$ with three reverberation conditions: 1) room-I (the reflection coefficients for the six walls are all equal to 0.59, and the reverberation time, T_{60}, is approximately 200 ms); 2) room-II (the reflection coefficients for the six walls are all equal to 0.82 and T_{60} is approximately 400 ms); and 3) room-III (the reflection coefficients for the six walls are all equal to 0.885 and T_{60} is approximately 600 ms). For ease of exposition, we denote the positions in the room with a 3D Cartesian coordinate system where one corner in the floor of the room is chosen as origin. A UCMA with $M = 8$ and $r = 2$ cm is used and the center of the array is at $(3, 3, 1)$. We conduct 100 simulations in every reverberation condition. In every simulation, a desired point sound source is 2 m away from the array center (the source and the microphone sensors are in the same horizontal plane), but its direction is randomly generated with a uniform distribution with $\theta_s \in [0, \pi]$. Two equal-power interference sources are located at the directions $\theta_s + 70^\circ$ and $\theta_s + 160^\circ$, respectively, with a distance of 2 m from the array center. The background noise is white Gaussian noise and the signal-to-noise ratio (SNR) is randomly set in a range from 20 dB to 30 dB (uniformly distributed). We generate the room impulse responses from the source to array elements with the image model method [25]. The clean source signals are arbitrarily selected from the TIMIT database [26]. The array observations are generated by convolving the clean speech signals with the impulse responses followed by adding background noise. The sampling frequency is 16 kHz.

We assume that the source direction is given as the *a priori* information and set one null at $\theta_s + \theta_1 = 250^\circ$, and designed three robust differential beamformers with the proposed method: 1) DMA-I ($\mathcal{W}_0 = -20$ dB), 2) DMA-II ($\mathcal{W}_0 = -10$ dB), and 3) DMA-III ($\mathcal{W}_0 = 0$ dB). Besides, the MWNG, delay-and-sum (DS), and the diagonal-loading based superdirective (SD) (with a loading parameter of 0.01) beamformers are designed for comparison [8]. We implement all those beamformers in the STFT domain while setting a frame size of 512 (Kaiser window with a shape factor of 1.9π and 75% overlap) and a 512-point FFT. The first microphone is selected as the reference microphone.

The signal-to-distortion ratio (SDR), signal-to-interference ratio (SIR) [27], and PESQ [28] are used as the performance measures. The results are shown as box plots in Fig. 5. One can see that all the studied beamformers bring improvement in SIR, SDR, and PESQ. But their performance decreases as the reverberation increases, which corroborates what has been observed in the literature. In comparison, the three beamformers designed with the proposed method, i.e., DMA-I, DMA-II, and DMA-III are all superior to the DS and MWNG beamformers thanks to their high directivity. They also produced higher SDR and PESQ than the diagonal loading based SD beamformer in the low reverberant environment. In comparison, the DMA-I obtains the highest SIR, which shows its effectiveness in terms of interference suppression. The DMA-II has a slightly lower SIR than the DMA-I but achieves the highest SDR and PESQ among the designed beamformers. The underlying reason is that the DMA-II was designed with a more proper level of robustness than DMA-I but with a slightly lower DF. In addition, DMA-III has lower SIR, SDR, and PESQ than DMA-I and DMA-II since DMA-III sacrifices too much DF for a higher value of WNG, which is indeed not needed.

5. CONCLUSIONS

This paper studied the problem of designing robust differential beamformers with UCMAs. We first discussed how to design differential beamformers with UCMAs using symmetric null constraints and then expressed the resulting beamformer as the sum of two orthogonal beamformers: a MWNG and a reduced-rank beamformer. Benefiting from this useful decomposition, we presented a method to design robust differential beamformers, which formulates the beamforming problem into one of optimization that maximizes the DF subject to some constraints on symmetric nulls in the beampattern and WNG. The robust beamformers are obtained by transforming the optimization problem into one of QEP. Design examples and Monte Carlo simulations were conducted to evaluate the presented method, and the results demonstrate that the presented method is able to obtain the specified value of WNG with the highest possible DF.

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