

Adaptive Feedback Active Noise Control by Robust Controller Interpolation

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Abstract—Feedback active noise control (ANC) offers an economical means to attenuate noise and is thus used in a variety of problems, e.g., in automotive or hearables. In practice, stability needs to be guaranteed also in the face of variations in the controlled system. Typically, a constrained optimization problem is solved to integrate this so-called robust stability criterion into the system design. However, the tracking of constraints at runtime remains challenging in resource-constrained environments such as hearables. In this contribution, we propose an efficient algorithm which is suitable for such environments. This algorithm uses internal model control (IMC) to steer a parallel connection of time-invariant filters via uncertainty-aware least-mean-square (LMS) adaptation, thus facilitating the adaptation at high sampling rates. The novel algorithm is evaluated through simulation and measurements with human listeners using ANC headphones as an example use case. The results show good ANC performance despite significant savings.

Index Terms—Active noise control, adaptive filters, constrained optimization, feedback control, robust control

I. INTRODUCTION

The suppression of acoustic noise using methods of active noise control (ANC) has attracted steady interest from researchers and engineers. Advances in digital signal processor (DSP) technology and algorithms have enabled wide application of ANC, e.g., in automotive and hearables. Typically, ANC systems use a time-invariant or adaptive digital filter with a finite impulse response (FIR) or an infinite impulse response (IIR). A low processing latency is crucial in generating phase-inverted cancellation signals, which naturally leads to high sampling rates [1]–[4].

One suitable strategy to implement ANC is robust feedback. This involves a microphone that is placed at the desired location of cancellation. An advantage is that noise is cancelled directly at the microphone position. However, so-called robust stability, i.e., stability of the feedback loop in all practical situations, must be considered in the design. This usually entails a set of hard constraints on the filter as implied by a model of the uncertainty that is inherent to the system [5]–[9].

Often, the correlation properties of noise are time-variant, which suggests the use of adaptive algorithms. Existing adaptive algorithms that maintain robust stability are based on a constrained least-mean-square (LMS) algorithm that requires a frequency discretization to maintain a significant number of constraints at runtime [4], [8] or expensive linear matrix inequalities (LMIs) [10]. Such strategies are not feasible in resource-constrained environments if long impulse responses

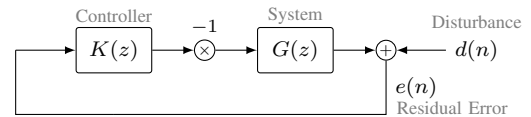


Fig. 1. Block diagram of a discretized system model of a feedback system.

are needed to model the physical system. Commercial ANC headphones thus often use time-invariant approaches instead.

In this contribution, we propose an efficient adaptive feedback ANC algorithm with guaranteed robust stability based on linear parameter-varying (LPV) control. Typically, LPV control is applied to known harmonic or tonal disturbances and combines several time-invariant controllers using an observer-based switching or interpolation logic [11], [12]. The approach followed in this work is to combine LPV control and LMS adaptation in a suitable way. For this, we use internal model control (IMC) [13], [14] and derive an update equation which minimizes the residual error using few adaptive weights.

The distinctive novelty in this work is an efficient mechanism to ensure robust stability. This mechanism is based on sufficient robust stability conditions that we integrate into the adaptation at low computational expense. Thus, our algorithm guarantees robust stability using significantly fewer resources than existing algorithms, so that LMS adaptation without subsampling and at high sampling rates becomes practically feasible.

In a case-study using over-ear ANC headphones, we compare the novel algorithm to an existing one that is computationally more expensive. We further implement an ANC prototype that is suitable for state-of-art consumer electronics using a low-power DSP and low-order IIR filters and assess the performance using measurements with human listeners.

II. PROBLEM STATEMENT

We consider a feedback ANC system which aims to reduce acoustic noise at a defined microphone position using a digital controller $K(z)$. Fig. 1 shows a system theoretic diagram of the system at hand using a standard digital model [2] that comprises the disturbance $d(n)$ to be reduced, the residual error $e(n)$ obtained from a microphone, and the controlled system $G(z)$, also termed plant or secondary path. The controlled system is modeled as linear time-invariant (LTI) filter $G(z)$ which captures the acoustic sound propagation and properties of the hardware such as microphones, loudspeakers and converters. Accurate knowledge on $G(z)$ is vital for ANC [4].

A. Design Specifications

To formalize the goal of reducing the residual error, one typically considers the sensitivity $S(z)$ and the complementary sensitivity $T(z)$, defined as [15]

$$S(z) = \frac{1}{1 + K(z)G(z)}, \quad (1a)$$

$$T(z) = \frac{K(z)G(z)}{1 + K(z)G(z)}. \quad (1b)$$

The sensitivity relates the residual error $e(n)$ to the disturbance $d(n)$. For a high attenuation, $S(z)$ assumes a low magnitude response. In contrast, a low magnitude of $T(z)$ is related to a high robustness against instability of the feedback loop. As $S(z) + T(z) = 1 \forall z$, an inevitable trade-off between robustness and performance is implied. Further, attenuation at one frequency implies amplification at another when $G(z)$ is not minimum-phase due to the waterbed effect [15].

A further difficulty is that $G(z)$ is generally unknown in practice [4]. This bears the hazard of feedback loop instability, which needs to be addressed in the design. A common criterion is nominal stability, which is given when the closed loop is stable for a nominal model $G_0(z)$ of the secondary path. Often, the more rigorous specification of robust stability based on a multiplicative uncertainty model is adopted [5]–[8]. In this case, variations in $G(e^{j\Omega})$ are treated by a Fourier domain set

$$\Pi : \{G(e^{j\Omega}) = G_0(e^{j\Omega}) \cdot (1 + \Delta(e^{j\Omega})W_2(e^{j\Omega}))\}, \quad (2)$$

which intuitively forms a disk in the complex plane. Stability can then be guaranteed for any $G(e^{j\Omega}) \in \Pi$ using the condition

$$|W_2(e^{j\Omega})T_0(e^{j\Omega})| < 1. \quad (3)$$

Here, $\Delta(e^{j\Omega})$ is any transfer function satisfying $|\Delta(e^{j\Omega})| \leq 1$, $W_2(e^{j\Omega})$ is an experimentally determined uncertainty bound that quantifies the uncertainty of the system under control, $T_0(z)$ is obtained from (1b) using $G_0(z)$ and $\Omega \in [0, 2\pi)$ is the normalized angular frequency [8], [15].

B. Internal Model Control

The internal model control (IMC) configuration provides a parametrization of all controllers that stabilize a given plant [13] and is thus suitable to realize adaptive feedback ANC. An IMC controller is obtained by substituting the controller $K(z)$ as

$$K(z) = \frac{Q(z)}{1 - G_0(z)Q(z)}, \quad (4)$$

with $G_0(z)$ a time-invariant plant model and $Q(z)$ a feedforward filter which serves as a new optimization variable. In the nominal case $G_0(z) = G(z)$, the feedback loops cancel out so that $S(z)$ and $T(z)$ become affine functions of $Q(z)$

$$S(z) = 1 - G(z)Q(z) \quad (5a)$$

$$T(z) = G(z)Q(z) \quad (5b)$$

and (3) becomes equivalent to the convex condition

$$|W_2(e^{j\Omega})G_0(e^{j\Omega})Q(e^{j\Omega})| < 1. \quad (6)$$

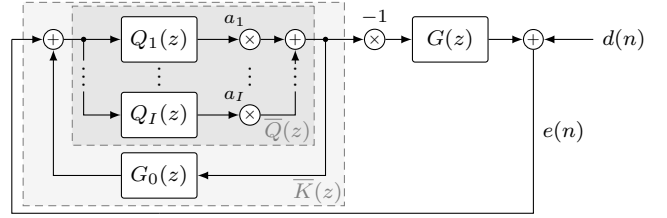


Fig. 2. Model of adaptive feedback controller $\bar{K}(z)$ under study.

This permits the use of feedforward methods in feedback ANC, as used in time-invariant and adaptive control systems [9], [14].

In general, the model assumption $G_0(z) = G(z)$ is violated in practice, since $G_0(z)$ is a time-invariant model for a time-varying system $G(z)$. Inserting (4) into (1a) reveals that $S(z)$ is then given by

$$S(z) = \frac{1 - G_0(z)Q(z)}{1 + Q(z)[G(z) - G_0(z)]}. \quad (7)$$

These variations are taken into account by the uncertainty model, and the influence of the residual term $G(z) - G_0(z)$ can be minimized by selecting a suitable nominal model $G_0(z)$, e.g., based on experimental measurement data [4]. As an alternative, one could use an adaptive model $G_0(z)$ to identify $G(z)$ in the loop [14], however, this is beyond the scope of this work.

III. ROBUST CONTROLLER INTERPOLATION

In the following, we propose a novel strategy for implementing robust adaptive feedback ANC. The main benefit of this strategy will be that fewer weights than usual are adapted so that robust stability is attained using few constraints. This leads to a simple adaptation scheme that is compatible with the high sampling rates that ANC systems are typically operated at.

Fig. 2 shows a block diagram of the proposed system. The adaptive IMC controller $\bar{K}(z)$ features a time-invariant plant estimate $G_0(z)$ and I time-invariant filters $Q_i(z)$, $i \in \{1, \dots, I\}$ that are connected in parallel. We propose to steer the contribution of the $Q_i(z)$ at runtime using I weights a_i as in [16]. The interpolated filter $\bar{Q}(z)$ is given by

$$\bar{Q}(z) = \sum_{i=1}^I a_i Q_i(z) \quad (8)$$

and $\bar{K}(z)$ is obtained from (4) using $\bar{Q}(z)$ instead of $Q(z)$. We define the weight vector $\mathbf{a} = [a_1 \ \dots \ a_I]^T \in \mathbb{R}^I$ as the optimization variable and impose $I+1$ constraints on its entries,

$$a_i \geq 0 \ \forall i \quad \text{and} \quad \sum_{i=1}^I a_i = \mathbf{1}^T \mathbf{a} \leq 1, \quad (9)$$

with $\mathbf{1} \in \mathbb{R}^I$ a vector of ones. This particular set of constraints is a sufficient condition for robust stability under certain circumstances that are elaborated by the following theorem:

Theorem 1. *Let $\bar{K}(z)$ be given by (4) using $\bar{Q}(z)$ with each $Q_i(z)$ robustly stable in terms of (3) for a certain uncertainty bound $W_2(e^{j\Omega})$. Then, the interpolation $\bar{Q}(z)$ according to (8) is also robustly stable for weights a_i that satisfy (9).*

Proof. Using (6) and (8), we note that robust stability of $\overline{K}(z)$ is guaranteed if and only if the following inequality holds:

$$\left| W_2(e^{j\Omega}) G_0(e^{j\Omega}) \sum_{i=1}^I a_i Q_i(e^{j\Omega}) \right| < 1 \quad (10)$$

We show that (10) holds for all a_i satisfying (9) by finding an upper bound for the left-hand side of (10) using the triangle inequality $|\sum_i a_i Q_i(z)| \leq \sum_i |a_i Q_i(z)|$. This leads to

$$\sum_{i=1}^I a_i \cdot |W_2(e^{j\Omega}) G_0(e^{j\Omega}) Q_i(e^{j\Omega})| < 1, \quad (11)$$

where (11) implies (10). As per assumption that all individual terms $|W_2(e^{j\Omega}) G_0(e^{j\Omega}) Q_i(e^{j\Omega})|$ satisfy (6), it follows that

$$\sum_{i=1}^I a_i \cdot |W_2(e^{j\Omega}) G_0(e^{j\Omega}) Q_i(e^{j\Omega})| < \sum_{i=1}^I a_i \leq 1, \quad (12)$$

with $\mathbf{1}^T \mathbf{a} \leq 1$ given by (9). Thus, (12) implies (11) which, in turn, implies (10). \square

Theorem. 1 states that robust stability in terms of the widely used unstructured uncertainty model persists for combinations of controllers if IMC is employed and (9) is met. Thus, the adaptation is robust against variations in $G(z)$. This does not cover the stability of the adaptation itself, which is treated in the literature and which is essentially given when algorithm parameters such as learning rates are chosen appropriately [4].

The proposed algorithm requires a suitable design of the time-invariant part, i.e., $G_0(z)$ and $Q_i(z)$, as well as an update rule to find optimal coefficients a_i within the bounds of (9) at runtime. Both of these aspects are discussed below.

A. Time-Invariant Part

First, we consider the time-invariant part of the system. The underlying question is how a-priori knowledge is exploited in the design. In general, the required number and type of filters is application-dependent. For example, in a car headrest system, properties of the disturbance often can be estimated in advance since potential noise sources such as engines are known. For headphones, a fair estimate of the passive insulation is usually available [17], which suggests the mild assumption that the disturbance is predominantly of low frequency. We note that both, $G_0(z)$ and $Q_i(z)$ can be found using standard methods.

The selection of a nominal model $G_0(z)$ is a common task in control applications. This aspect is treated in Sec. II-B and in the literature [15]. From now on, we thus assume that a suitable model $G_0(z)$ is available.

To design the controllers, standard methods that consider robust stability can be used. This includes methods based on IMC [7], [8] that determine the filters $Q_i(z)$ directly. However, \mathcal{H}_∞ synthesis [5] or non-convex optimization [6] can also be used to first design a controller $K_i(z)$ and then solve (4) for $Q_i(z)$. Joint design procedures [16] are also promising but exceed the scope of this work. A common benefit is that specifications are divided among multiple controllers to mitigate adverse effects such as overshoot due to the waterbed effect.

B. Time-Variant Part

In the following, we assume that $Q_i(z)$ that correspond to robustly stable controllers $K_i(z)$ are available and focus on the adaptation of the weights a_i at runtime. We derive a variant of the LMS algorithm to obtain an update rule which aims to minimize the expected error signal power and guarantees robust stability using the devised set of constraints (9). The objective function considers the error $e(n)$, which is given by

$$e(n) = d(n) - d(n) * g_0(n) * \bar{q}(n) \quad (13)$$

in the nominal case [4]. Here, $g_0(n)$ and $\bar{q}(n)$ denote the impulse responses of $G_0(z)$ and $\overline{Q}(z)$, respectively. We follow a stochastic approach to obtain the optimum coefficient vector \mathbf{a} in each time instant n based on the problem

$$\begin{aligned} \mathbf{a} = \arg \min_{\mathbf{a}} \quad & J(n) := e^2(n) \\ \text{subject to } & a_i \geq 0 \quad \forall i \quad \text{and} \quad \mathbf{1}^T \mathbf{a} \leq 1. \end{aligned} \quad (14)$$

Equation (14) casts the design objective as a convex problem with a guaranteed global optimum. However, an iterative solver is needed to solve (14) [18], which collides with requirements on complexity and latency that most ANC systems face. A penalty method [9] was proposed for these cases, which we apply to the problem at hand with some modifications. By introducing penalty terms, we obtain a modified cost function

$$\tilde{J}(n) = J(n) + \sigma \max\{\mathbf{1}^T \mathbf{a}, 1\}^2 + \sigma \sum_{i=1}^I \max\{-a_i, 0\}^2. \quad (15)$$

The scaling factor $\sigma \in \mathbb{R}^+$ is used to control the strictness that constraints are enforced with. If all constraints are satisfied, $J(n)$ and $\tilde{J}(n)$ have the same global minimum. Otherwise at least one penalty term becomes active, thus reducing the constraint violation successively until it becomes zero.

We use the method of steepest descent with learning rate $\gamma \in \mathbb{R}^+$ to obtain the coefficient update rule

$$a_i(n+1) = a_i(n) - \frac{\gamma}{2} \cdot \frac{\partial \tilde{J}(n)}{\partial a_i(n)}, \quad 1 \leq i \leq I. \quad (16)$$

The index n indicates that the weights are now time-variant. The derivative of $\tilde{J}(n)$ with respect to $a_i(n)$ is given by

$$\begin{aligned} \frac{\partial \tilde{J}(n)}{\partial a_i(n)} = & -2e(n) \cdot [d(n) * g_0(n) * q_i(n)] \\ & + \sigma \cdot (\mathbf{1}^T \mathbf{a}(n) - 1) \cdot [\text{sign}(\mathbf{1}^T \mathbf{a}(n) - 1) + 1] \\ & + \sigma \cdot a_i(n) \cdot [\text{sign}(-a_i(n)) + 1], \end{aligned} \quad (17)$$

where the function $\text{sign}(\cdot)$ returns the sign of its argument.

IV. EVALUATION

We study the proposed algorithm using Bose QC45 over-ear headphones as an example. We removed the manufacturers ANC electronics and instead established a direct connection to a DSP which runs the ANC algorithm. For clarity, we consider only the left ear channel in the following.

To account for variations in $G(z)$ we use the uncertainty model from [7]. Fig. 3 shows the 56 measurements that this

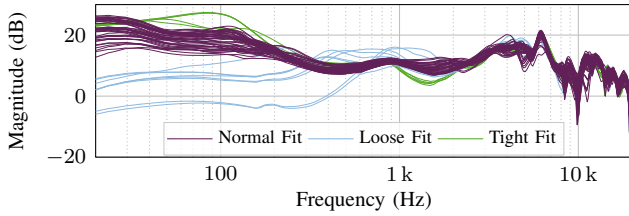


Fig. 3. 56 magnitude responses measured for $|G(z)|$.

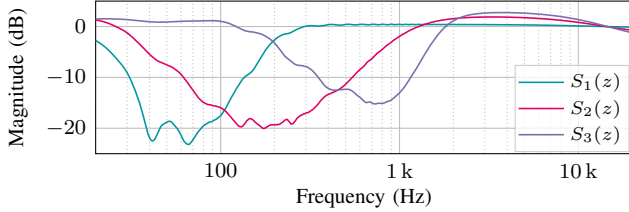


Fig. 4. Nominal sensitivities $S_i(z)$ achieved by the controllers $K_i(z)$.

model is based on. It reflects the wearing situations of 16 different persons and also covers corner cases, e.g., loose fits and tight fits, which are relevant to ensure stability when handling the device.

We used six different types of ambient noise from the ETSI EG 202 396-1 database [19] that the ANC system is intended to suppress. These signals are examples of real-world noise with different correlation and stationarity properties.

A. Simulation

In the following, we compare the proposed algorithm to the adaptive system from [9] using an FIR filter $Q(z)$ with 512 adjustable tap weights. This algorithm is a suitable baseline due to its conceptual similarity, i.e., it is also based on IMC and imposes the robust stability constraints using penalty functions. Due to the higher number of weights, the approach is more versatile than ours, but a slow convergence rate is a potential drawback as noted in [9].

For our algorithm, we designed $I = 3$ time-invariant FIR filters $Q_i(z)$ with 512 taps at a sampling rate $f_s = 48$ kHz using [7] and [8] and used (16) to adapt the weights $a_i(n)$. The performance goals were chosen to increase the control bandwidth over time-invariant schemes, resulting in an arrangement with spectral separation similar to a filterbank. Fig. 4 shows the nominal sensitivities $S_i(z)$ that the designed controllers $K_i(z)$ achieve. Note that the number of adaptive parameters is about two orders of magnitude smaller for our algorithm.

The comparison is made under equivalent conditions, i.e., both adaptive filters can influence 512 taps, use the same uncertainty model and use the average normal fit as nominal model $G_0(z)$. The penalty scaling and the learning rates for both algorithms were chosen to maximize the attenuation with a comparable and low constraint violation.

Fig. 5 shows the disturbance $d(n)$ and the achieved residual errors $e(n)$ for the nominal case $G_0(z) = G(z)$. For reference, the noise type is indicated and the evolution of the weights

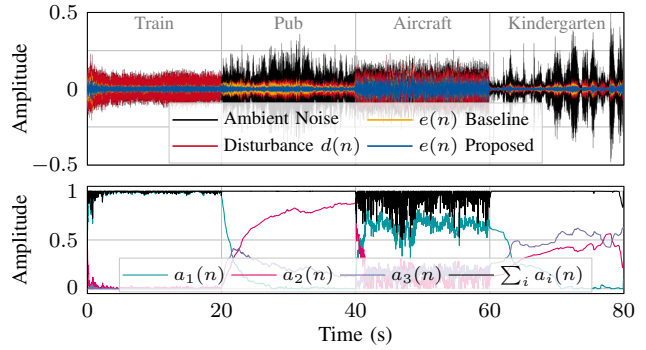


Fig. 5. Simulation results: ambient noise, remaining disturbance $d(n)$ after passive attenuation and error signals $e(n)$ achieved by the baseline [9] and the proposed algorithm as well as the evolution of the weights $a_i(n)$.

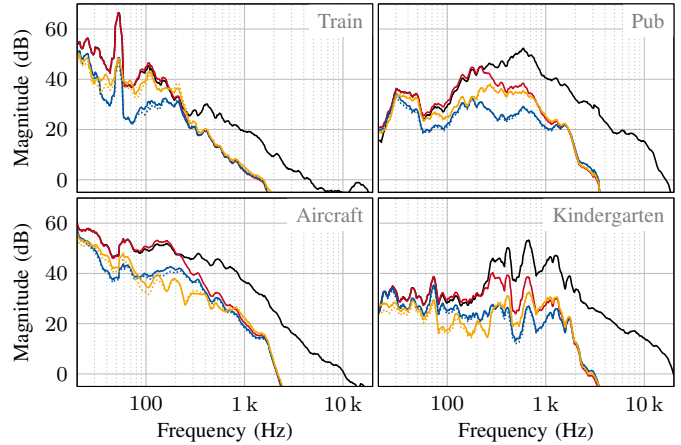


Fig. 6. Simulated signal PSDs: ambient noise (—), disturbance (—), and errors using baseline algorithm [9] (—) and proposed method (—).

$a_i(n)$ over time is also shown. The figure reveals that the constraints are maintained tightly.

We extend the analysis by considering Fig. 6, which depicts the long term signal power spectral densities (PSDs) of this experiment. For clarity, the magnitude responses were smoothed using a $1/6$ octave average filter. To consider the case of an imperfect plant model, i.e., $G_0(z) \neq G(z)$, we further evaluated the performance for all 16 listeners. The decibel-averaged error signals are marked in Fig. 6 as dotted lines. We observe that the performance for *train* and *aircraft* is similar for both methods. However, the baseline is outperformed for the instationary noises *kindergarten* and *pub*, especially for frequencies above 250 Hz that result from abrupt speech onsets. Here, our algorithm profits from the low number of weights which imply a faster convergence rate. This advantage was also reported in [20], albeit for a feedforward ANC system.

B. Real-Time ANC System

We study the ANC performance in a real-time application. For this, the proposed adaptive controller was implemented on an Analog Devices ADAU1787 audio codec operating at a sampling rate $f_s = 192$ kHz. We designed $I = 3$ new filters $Q_i(z)$

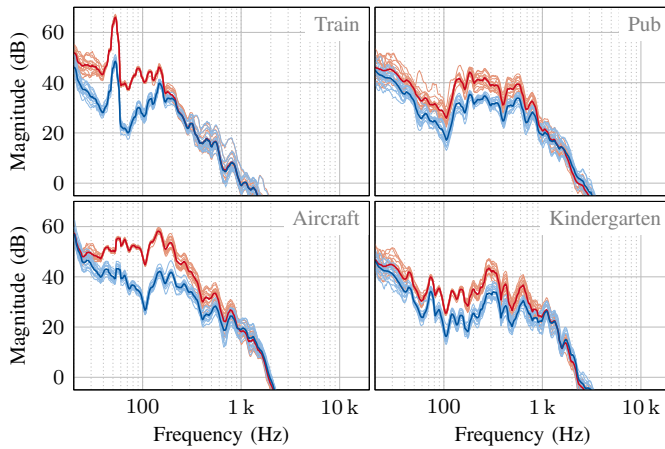


Fig. 7. Measured error signal PSDs with ANC off (—) or on (—) of 15 individual subjects (thin bright lines) and decibel-averaged case (thick lines).

for this rate and applied an order reduction algorithm [15]. The resulting 16th order IIR filters are compatible with the resource-constrained environment and achieve a performance very similar to that shown in Fig. 4. The algorithm [9] was not considered due to insufficient resources on this hardware.

Using the headphone microphones, we measured the error signal $e(n)$ for 15 different subjects in a measurement room that complies with the recommendation ITU-R BS.1116-2 [21]. We played back the ambient signal shown in Fig. 5 over one subwoofer and 8 studio speakers with a circular distribution around the subject. Two measurements were made for each subject, one without and one with the ANC system activated. This allows to assess the active attenuation in an isolated manner. The overall setup was very similar to Sec. IV-A, however, signal levels differed slightly due to the playback setup. The results are cross-validated because 5 of the 15 subjects were not considered in the design of the filters $Q_i(z)$.

To analyze the measurements, we divided them by noise type. Fig. 7 shows the error signal magnitudes for all subjects with ANC off and on, respectively. For clarity, the curves were smoothed using a $1/6$ octave band filter. The decibel-averaged case is also indicated for reference. We observe a spread of about 5 dB around the mean active attenuation, which is partially due to the measurement setup.

The low frequency components in *pub* and *kindergarten*, e.g., chair movement sounds, are more significant compared to Fig. 6 due to the playback setup. The algorithm thus emphasizes low frequencies more, which results in a slightly lower average attenuation of about 10 dB. For the noises *train* and *aircraft*, we achieved a performance similar to Fig. 6 with a peak attenuation of about 20 dB. Overall, the algorithm successfully adapts to the intricate time-varying correlation properties of the signals in a way that is robust to the person-induced variance.

V. CONCLUSION

We propose a novel algorithm for adaptive feedback ANC which guarantees robust stability. The algorithm interpolates between robust time-invariant controllers without subsampling

using LMS adaptation and IMC. The main difference to existing methods is a simplified mechanism to guarantee robust stability using few constraints on the adaptive weights. The low number of adaptive weights offers a reduction of computational complexity and a faster convergence rate at the expense of versatility. The resulting algorithm was evaluated using simulations as well as measurements with human listeners and a real-time prototype headphone using low-power DSP hardware. For resource-constrained ANC applications such as hearables, the algorithm offers a considerable performance increase at moderate computational cost.

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