Prototype filter design for weighted overlap-add filter bank based sub-band adaptive filtering applications

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***Abstract*—Weighted overlap-add (WOLA) filter bank based sub-band adaptive filtering has been widely used in various speech and audio signal processing applications. The WOLA filter bank uses a prototype analysis filter and prototype synthesis filter. The design of these prototype filters significantly impacts the performance of the adaptive filtering. In this paper we propose an optimization criterion for the prototype synthesis filter design, derived from the analysis of the oversampled uniform discrete Fourier transform (DFT) modulated filter bank for WOLA based sub-band adaptive filtering. The proposed prototype synthesis filter design involves a constrained convex optimization problem with quadratic cost function and linear equality constraints. The proposed prototype synthesis filter design method shows performance improvement for WOLA based acoustic echo cancellation (AEC) compared to the conventional prototype synthesis filters.**

***Index Terms*—acoustic echo cancellation (AEC), filter bank, sub-band adaptive filtering, system identification, weighted overlap-add (WOLA).**

I. Introduction

Adaptive filtering in the short-time Fourier transform (STFT) domain has been widely used in various speech and audio signal processing applications, and generally uses a weighted overlap-add (WOLA) filter bank [1], [2], which is essentially an oversampled uniform discrete Fourier transform (DFT) modulated filter bank.

The uniform DFT modulated filter bank uses a prototype analysis filter (or so-called analysis window) and prototype synthesis filter (or so-called synthesis window). A carefully selected prototype analysis filter reduces the side-lobe levels, restricting the cross-band dependencies [3]. The prototype synthesis filter is then designed such that it provides perfect reconstruction (PR) of the time-domain input signal (in the absence of sub-band processing).

This research work was carried out at the ESAT Laboratory of KU Leuven, in the frame of FWO Research Project nr. G0A9921N "Per-Tone Equalization and Per-Tone Precoding for Speech and Audio Signal Processing", Research Council KU Leuven: C14-21-0075 "A holistic approach to the design of integrated and distributed digital signal processing algorithms for audio and speech communication devices", and VLAIO Project nr. HBC.2022.0428 "SPARTA- Signal Processing for next-generation opticAl netwoRks and remoTe Audio experiences". The scientific responsibility is assumed by its authors.

In an oversampled uniform DFT modulated filter bank, with a given non-sparse prototype analysis filter, the PR conditions for the prototype synthesis filter design (due to the oversampling) represent an underdetermined system of linear equations. Hence, the prototype synthesis filter can be chosen from an infinite set of solutions in the affine solution space that satisfies the PR conditions. The prototype synthesis filter design then mostly relies on minimizing the norm of the prototype synthesis filter [4].

Therefore, in almost all of the WOLA filter bank based sub-band processing applications, root-raised cosine windows (e.g., root-Hann, root-Hamming, root-Blackman, etc.) are used as the prototype analysis and the prototype synthesis filter [5]–[7]. In [8], a closed-form solution for a prototype synthesis filter design is proposed which is based on minimizing the stopband attenuation. However, the proposed design is restricted to a class of filter banks where the length of the prototype analysis and prototype synthesis filter is twice the downsampling factor. In [9], closed-form solutions for different prototype synthesis filter design criteria, e.g., minimum out-of-band power, minimum quantization-error, and minimum window order, are summarised. In [10], [11] a prototype filter design is proposed which minimizes the magnitude of all aliasing components individually, in the overall transfer function (or the so-called distortion function) of the filter bank. However, the design procedures assume the desired overall transfer function as a pure delay and does not take into consideration the sub-band filter coefficients. Similarly, in [12] aliasing is minimized by designing prototype filters with large stopband attenuation. Therefore, the proposed prototype synthesis filter design criteria in the literature, ignore the sub-band filter coefficients, and are not tailored for the utilization of the oversampled uniform DFT modulated filter bank for WOLA based sub-band adaptive filtering applications.

In this paper we propose an optimization criterion for the prototype synthesis filter design, derived from the analysis of the oversampled uniform DFT modulated filter bank for WOLA based sub-band adaptive filtering applications. The proposed prototype synthesis filter design involves a constrained convex optimization problem with quadratic cost function and linear equality constraints. Simulations are used to show the performance gain achieved by the proposed prototype synthesis filter design for WOLA based acoustic echo cancellation (AEC).

The paper is organized as follows: In Section II the signal model for the WOLA based sub-band adaptive filtering is stated. In Section III, the optimization problem for the prototype synthesis filter design is proposed. In Section IV simulation results are provided for WOLA based AEC. Section V concludes the paper.

II. Signal model for WOLA based sub-band adaptive filtering

The relation between the input signal $u(n)$ and output signal $y(n)$ of a linear time-invariant (LTI) system is given as

$$y(n) = \underbrace{u(n) \star w(n)}_{v(n)} + \sigma(n) \tag{1}$$

where, $\star$ represents the convolution operation, $w(n)$ is a LTI system and $\sigma(n)$ is additive noise. Assuming a prototype analysis filter of length N ($\mathbf{h}_0 \triangleq \left[h_0(0), \cdots, h_0(N-1)\right]$) and WOLA filter bank with oversampling factor of d, the STFT domain representation of $y(n)$ and $u(n)$ can be written as

$$y(k,l) = \sum_{n=0}^{N-1} y(n + l\frac{N}{d}) h_0(n) \alpha_N^{-kn} \tag{2a}$$

$$u(k,l) = \sum_{n=0}^{N-1} u(n + l\frac{N}{d}) h_0(n) \alpha_N^{-kn} \tag{2b}$$

respectively, where k is the sub-band index, l is the time frame index and $\alpha_N \triangleq \exp^{j2\pi/N}$.

Considering $\hat{w}(k,l)$ as the complex adaptive filter coefficient for sub-band k at the time frame l, the STFT domain estimated LTI system output $\hat{v}(k,l)$ for sub-band k at the time frame l is defined as

$$\hat{v}(k,l) = u(k,l)\hat{w}(k,l) \tag{3}$$

The coefficients $\hat{w}(k,l)$, for all the sub-bands ($k = 0 \cdots N-1$), can be updated over each time frame using an adaptive algorithm. Finally, the reconstructed time-domain estimated LTI system output ($\hat{v}(n)$) is given as

$$\hat{v}(n) = \sum_{l} \sum_{k=0}^{N-1} \hat{v}(k,l) f_0(n - l\frac{N}{d}) \alpha_N^{k(n - l\frac{N}{d})} \tag{4}$$

where, $f_0(n)$ is n^{th} sample of the prototype synthesis filter of length N ($\mathbf{f}_0 \triangleq \left[f_0(0), \cdots, f_0(N-1)\right]$).

In this paper, the LTI system $w(n)$ will be assumed to be finite impulse response (FIR), with length $(d-1)\frac{N}{d}$.

III. WOLA prototype filter design

The general PR condition for a WOLA filter bank is given as

$$\sum_{m=-\infty}^{\infty} \left(h_0(i + m\frac{N}{d})\right)\left(f_0(i + m\frac{N}{d})\right) = 1$$
$$\text{for } i = 0 \cdots (\frac{N}{d} - 1) \tag{5}$$

For a FIR filter bank, with prototype analysis and prototype synthesis filter of length N, the PR condition in (5) can be alternatively written as

$$\mathbf{h}_0(i)\mathbf{f}_0(i)^\intercal = 1 \quad \text{for } i = 0 \cdots (\frac{N}{d} - 1) \tag{6}$$

where, $(.)^\intercal$ represents the transpose operation, $\mathbf{h}_0(i) \triangleq \left[h_0(i), h_0(i + \frac{N}{d}), \cdots, h_0(i + (d-1)\frac{N}{d})\right]$ is a sub-vector of the prototype analysis filter $\mathbf{h}_0$ and $\mathbf{f}_0(i) \triangleq \left[f_0(i), f_0(i + \frac{N}{d}), \cdots, f_0(i + (d-1)\frac{N}{d})\right]$ is a sub-vector of the prototype synthesis filter $\mathbf{f}_0$. Therefore, for a given prototype analysis filter, it can be shown that the solution for the prototype synthesis filter, that provides PR, is given as

$$\mathbf{f}_0(i)^\intercal = \mathbf{h}_0(i)^\ddagger + \boldsymbol{\eta}_i \quad \text{with} \quad \boldsymbol{\eta}_i \in \mathbb{N}(\mathbf{h}_0(i))$$
$$\text{for} \quad i = 0 \cdots (\frac{N}{d} - 1) \tag{7}$$

where, $(.)^\ddagger$ represents the pseudo-inverse operation and $\mathbb{N}(\mathbf{h}_0(i))$ represents the null-space of $\mathbf{h}_0(i)$. The value of $\boldsymbol{\eta}_i$ can be selected accordingly to meet the desired prototype synthesis filter characteristics.

A. Prototype analysis filter design

For WOLA based sub-band processing, a carefully selected prototype analysis filter reduces the side-lobe levels, restricting the cross-band dependencies. Therefore, frequency selectivity is the most important criterion for the prototype analysis filter design. Consequently, raised-cosine (or root-raised cosine) windows, which are characterised by reduced side-lobes and good frequency selectivity, are conventionally used as prototype analysis filter for WOLA based sub-band processing.

B. Prototype synthesis filter design

For WOLA based sub-band adaptive filtering, the goal is that the k^{th} sub-band coefficient $\hat{w}(k,l)$ converges (for $l \to \infty$) to the frequency response of the LTI system $w(n)$ evaluated at the centre frequency of the k^{th} sub-band, i.e.,

$$\hat{w}(k,\infty) = W(z = \alpha_N^k) \quad \forall k \tag{8}$$

where, $W(z)$ represents the z-transform of $w(n)$. Therefore, the N-point inverse discrete Fourier transform (IDFT) of $W(z = \alpha_N)$ is given as[1]

$$\mathbb{F}_N^* \left(W(z = \alpha_N)\right)$$
$$= \left[0, w(0), \cdots w((d-1)\frac{N}{d} - 1), 0, \cdots, 0\right] \tag{9}$$

[1]The zero is appended in the beginning of IDFT of $W(z = \alpha_N)$ to simplify the indexing in (17) - (20). Alternatively, it can be removed and the indexing in the subsequent equations can be modified accordingly.

where, $\mathbb{F}_N^*(.)$ represents the N-point IDFT operation.

For a uniform DFT modulated filter bank, (ignoring aliasing effects for the time being) the overall transfer function (or the so-called distortion function) of the filter bank with sub-band coefficients $\hat{w}(k,\infty)$ is given as [13]

$$T(z) = \sum_{k=0}^{N-1} H_k(z) F_k(z) \hat{w}(k,\infty) \tag{10}$$

where $H_k(z)$ and $F_k(z)$ are the k^{th} analysis and synthesis filter respectively. Moreover, for a uniform DFT modulated filter bank

$$H_k(z) = H_0(z\alpha_N^{-k}) \quad \text{and} \quad F_k(z) = F_0(z\alpha_N^{-k})$$
$$\text{OR}$$
$$h_k(n) = h_0(n)\alpha_N^{nk} \quad \text{and} \quad f_k(n) = f_0(n)\alpha_N^{nk} \tag{11}$$

where $h_0(n)$ and $f_0(n)$ are the prototype analysis and prototype synthesis filter, respectively.

The overall transfer function of the filter bank $T(z)$ corresponds to a time-domain filter $t(n)$ of length $2N-1$, which is given as

$$t(n) = \sum_{k=0}^{N-1} \left(h_k(n) \star f_k(n)\right) \hat{w}(k,\infty) \tag{12}$$

The time-domain filter $t(n)$ should represent the unknown time-domain LTI system such that

$$\mathbb{F}_{2N-1}^*\left(T(z=\alpha_N)\right) = \left[\mathbf{0}_N \;\; w(0)\cdots w((d-1)\tfrac{N}{d}-1) \;\; \mathbf{0}_{\frac{N}{d}-1}\right] \tag{13a}$$
$$\text{OR}$$
$$\mathbb{F}_{2N-1}^*\left(T(z=\alpha_N)\right) = \left[\mathbf{0}_{\frac{N}{d}} \;\; w(0)\cdots w((d-1)\tfrac{N}{d}-1) \;\; \mathbf{0}_{N-1}\right] \tag{13b}$$

Both of the overall transfer functions defined in (13) represent the unknown time-domain LTI system but with different delays. Their importance is discussed later in this section but we have omitted the details for their derivation here for simplicity and these will also be explained in a full report following this paper.

Using (11) the time-domain filter $t(n)$ in (12) can be written as

$$t(n) = \sum_{k=0}^{N-1} \left(h_0(n)\alpha_N^{nk} \star f_0(n)\alpha_N^{nk}\right) \hat{w}(k,\infty) \tag{14}$$

which on further expansion, can be simplified as

$$t(n) = \underbrace{\sum_m f_0(m) h_0(n-m)}_{\triangleq g_0(n)} \underbrace{\sum_{k=0}^{N-1} \hat{w}(k,\infty)\alpha_N^{nk}}_{\triangleq \hat{w}(n)} \tag{15}$$

where, $g_0(n)$ represents the convolution of the prototype analysis and the prototype synthesis filter and $\hat{w}(n)$ represents the time-domain sequence corresponding to the converged sub-band filter coefficients $\hat{w}(k,\infty)$.

It can be noticed from (15) that $\hat{w}(n)$ for $n = 0\cdots 2N-2$ is periodic with period of N, such that

$$\hat{w}(n+N) = \hat{w}(n) \quad \text{for } n = 0\cdots N-2 \tag{16}$$

Comparing (13) and (15) combined with (16) we get the desired conditions on $g_0(n)$. When (13a) is considered as the desired $t(n)$, then with

$$\hat{w}(n) = \begin{cases} w(n) & \text{for } n = 0\cdots\left((d-1)\frac{N}{d}-1\right) \\ 0 & \text{for } n = \left((d-1)\frac{N}{d}\right)\cdots N-1 \end{cases} \tag{17}$$

the desired condition on $g_0(n)$ is given as

$$g_0(n) = \begin{cases} 0 & \text{for } n = 0\cdots\left((d-1)\frac{N}{d}-1\right) \\ \frac{N}{d} & \text{for } n = N\cdots\left((2d-1)\frac{N}{d}-1\right) \end{cases} \tag{18}$$

Similarly, when (13b) is considered as the desired $t(n)$, then with

$$\hat{w}(n) = \begin{cases} w(n-\frac{N}{d}+1) & \text{for } n = \frac{N}{d}-1\cdots N-1 \\ 0 & \text{for } n = 0\cdots\frac{N}{d}-2 \end{cases} \tag{19}$$

the desired condition on $g_0(n)$ is given as

$$g_0(n) = \begin{cases} 0 & \text{for } n = \left((d+1)\frac{N}{d}-1\right)\cdots 2N-1 \\ \frac{N}{d} & \text{for } n = \frac{N}{d}-1\cdots N-1 \end{cases} \tag{20}$$

It can be noticed that the filter bank output samples when using the designed synthesis filter from (13a) depend on the input samples of the current frame only, while the filter bank output samples when using the designed synthesis filter from (13b) depend on the input samples of the previous frame also, which increases the error variance of the output samples. However, the filter bank designed from (13b) introduces less delay in the system. In the sequel, we have considered only (13a) for conciseness.

Therefore, based on (18), a convex optimization problem with quadratic cost function and linear equality constraints, is formulated in (21) ,where $\|.\|$ represents the L2 norm, ϵ is a weighting factor and $\mathbf{1}_L$ represents a vector of ones of size L. It can be shown that with the conditions on $g_0(n)$, the linear system of equations for the prototype synthesis filter design, is rank deficient. Therefore, the optimization problem for the prototype synthesis filter design in (21) contains a regularisation term for minimizing the L2 norm of the prototype synthesis filter.

IV. Simulation results

A. Acoustic echo cancellation (AEC)

For the evaluation of the proposed prototype filter design, single channel AEC is considered as a use-case scenario. For this AEC application in (1), $y(n)$ represents the microphone signal, $u(n)$ represents the far-end (loudspeaker) signal, $w(n)$ represents the room impulse response (RIR), $v(n)$ represents the echo signal and $\sigma(n)$ represents the near-end signal. Moreover, $\hat{w}(k,l)$ represents the coefficient of the estimated RIR in sub-band k at the time frame l and $\hat{v}(k,l)$ represents the STFT domain estimated echo signal in sub-band k at the time frame l.

$$\begin{aligned} \underset{f_0(n)}{\text{minimize}} \quad & \left\| g_0(0:(d-1)\frac{N}{d}-1) \right\|^2 + \left\| g_0(N:(2d-1)\frac{N}{d}-1) - \frac{N}{d}\mathbf{1}_{(d-1)\frac{N}{d}+1,1} \right\|^2 + \epsilon \left\| \mathbf{f}_0 \right\|^2 \\ \text{subject to} \quad & \mathbf{h}_0(i)\mathbf{f}_0(i)^\intercal = 1 \quad \text{for } i = 0 \cdots (\frac{N}{d}-1) \end{aligned} \tag{21}$$

B. Performance metric

The echo return loss enhancement (ERLE) is a standard performance metric for AEC, as a measure of the achieved echo suppression. It is defined by the ratio between the power of the echo signal and the power of the residual echo signal. For block processing, the ERLE for each time frame is given as

$$\text{ERLE}(l) = 10 \log_{10} \left(\frac{\sum_{n=0}^{N-1} v(n + l\frac{N}{d})^2}{\sum_{n=0}^{N-1} \xi(n + l\frac{N}{d})^2} \right) \quad \forall\, l \tag{22}$$

where, $\xi(n)$ is the residual echo signal.

C. Simulation scenario

For the simulation, a loudspeaker-enclousre-microphone (LEM) system is considered with a single microphone and a single loudspeaker. A non-stationary stochastic process with exponential decay is used to generate the RIR between the loudspeaker and the microphone, with length $L_\hbar = 512$ samples. The RIR is given as $w(n) = \sqcap_{L_\hbar}(n)\Omega(n)\exp^{-\kappa n}$, where $\sqcap_{L_\hbar}$ is a rectangular function such that $\sqcap_{L_\hbar}(n) = 1$ for $n = 0 \cdots L_\hbar - 1$, $\Omega(n)$ represents a zero-mean white Gaussian noise with unit variance and κ is decay parameter of the exponential function. Both the far-end signal and the near-end signal are zero-mean white Gaussian noise signals, such that the near-end signal is $20dB$ lower than the echo signal. The sampling frequency is 16kHz. For the WOLA filter bank a 50% overlap (i.e $d = 2$) scenario is considered with prototype analysis and prototype synthesis filter length of $N = 1024$. Finally, a recursive least squares (RLS) algorithm [14] is used for the adaptation of the sub-band coefficients $\hat{w}(k, l)$.

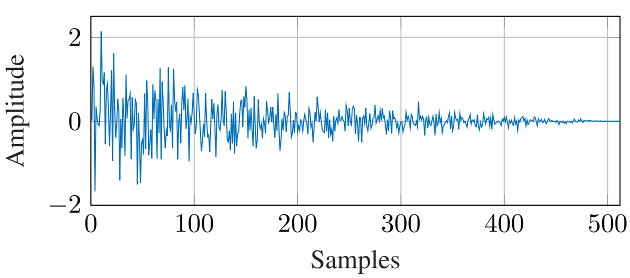

Fig. 1: Generated RIR with $L_\hbar = 512$ and $\kappa = 0.006$

D. Performance comparison

We have considered two conventional prototype analysis filters namely a rectangular window and a root-Hann window and their respective conventional prototype synthesis filters generally used in WOLA filter bank based sub-band adaptive filtering. For the rectangular window as the prototype analysis filter, Fig. 2 shows corresponding conventional prototype synthesis filter (= prototype analysis) and the proposed prototype synthesis filter based on (21). It can be observed that the

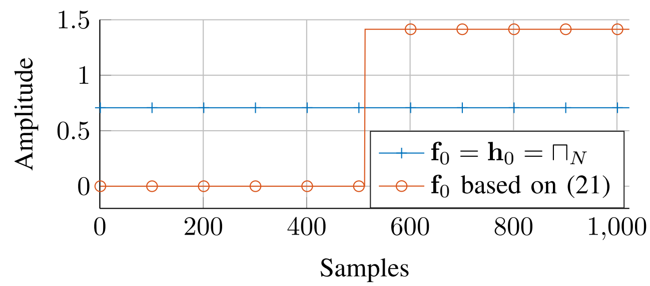

Fig. 2: Conventional ($\mathbf{f}_0 = \mathbf{h}_0$) and proposed prototype synthesis filter for a rectangular window of length N (i.e. $\sqcap_N$) as the prototype analysis filter.

proposed prototype synthesis filter in Fig. 2 resembles the prototype synthesis filter for the overlap-save (OLS) method. Similarly, Fig. 3 shows the conventional and the proposed prototype synthesis filter, for a root-Hann window as the prototype analysis filter.

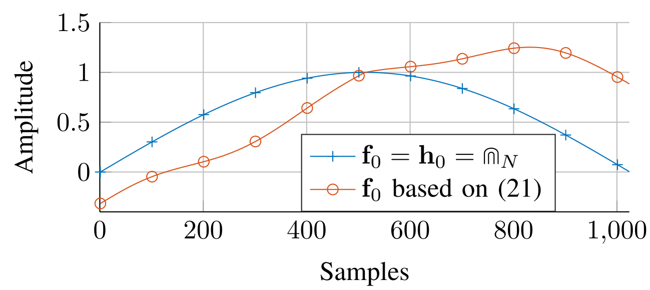

Fig. 3: Conventional ($\mathbf{f}_0 = \mathbf{h}_0$) and proposed prototype synthesis filter for a root-Hann window of length N (i.e. $\Cap_N$) as the prototype analysis filter.

Fig. 4 compares the ERLE achieved by WOLA based AEC for different prototype synthesis filters, shown in Fig. 2, when the analysis prototype is a rectangular window. Similarly, Fig. 5 compares the ERLE achieved by WOLA based AEC for different prototype synthesis filters, shown in Fig. 3, when the analysis prototype is a root-Hann window. It can be be observed from Fig. 4 and Fig. 5 that the proposed prototype synthesis filter provides a performance improvement compared to the conventional prototype synthesis filter.

It is to be noted here that with so-called per-tone-WOLA (PT-WOLA) based AEC, a more significant improvement in the achievable ERLE is observed with the proposed prototype

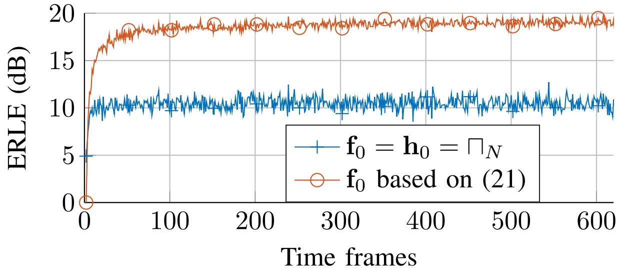

Fig. 4: ERLE achieved by WOLA based AEC for prototype filters shown in Fig. 2

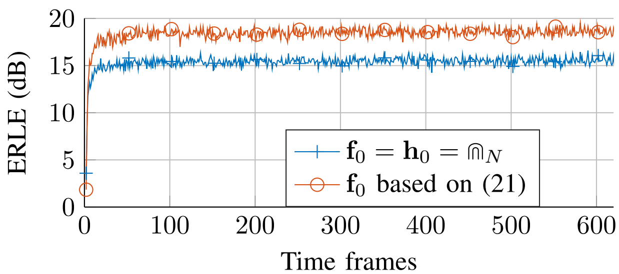

Fig. 5: ERLE achieved by WOLA based AEC for prototype filters shown in Fig. 3

synthesis filter compared to the conventional prototype synthesis filter, especially when $\mathbf{h}_0 = \Cap_N$. The achieved ERLE with PT-WOLA as shown in Fig. 6 and Fig. 7. The PT-WOLA

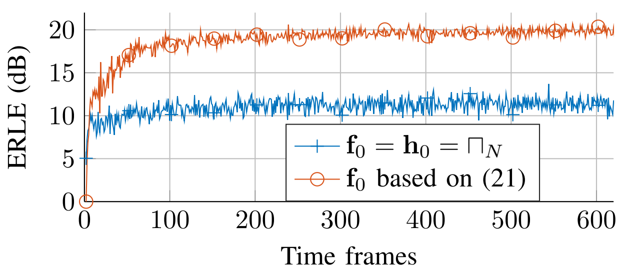

Fig. 6: ERLE achieved by PT-WOLA based AEC for prototype filters shown in Fig. 2

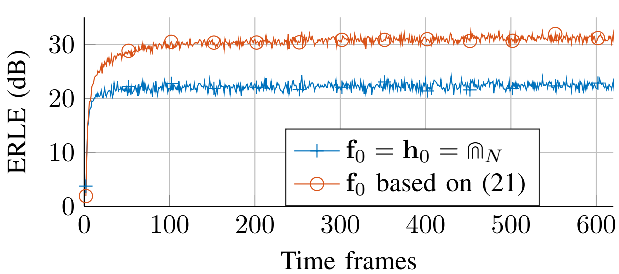

Fig. 7: ERLE achieved by PT-WOLA based AEC for prototype filters shown in Fig. 3

uses the same WOLA filter bank structure but with multi-tap STFT domain sub-band adaptive filtering. The details of the PT-WOLA based will be presented in a separate report.

V. Conclusion

A prototype synthesis filter design method has been proposed for WOLA based sub-band adaptive filtering applications. The proposed prototype synthesis filter design takes into account the overall transfer function (distortion function) of the oversampled DFT modulated filter bank along with the sub-band processing, such that it matches the unknown time-domain LTI system.

The proposed prototype synthesis filter design shows performance improvement for WOLA based AEC compared to the conventional prototype synthesis filters. Moreover, a more significant improvement in the achievable ERLE is observed with the proposed prototype synthesis filter compared to the conventional prototype synthesis filter, especially when $\mathbf{h}_0 = \Cap_N$. The details of the PT-WOLA based adaptive filtering will be presented in a separate report.

References

[1] R. Crochiere, "A weighted overlap-add method of short-time Fourier analysis/synthesis," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 28, no. 1, pp. 99–102, 1980.

[2] J. Shynk, "Frequency-domain and multirate adaptive filtering," *IEEE Signal Processing Magazine*, vol. 9, no. 1, pp. 14–37, 1992.

[3] F. Harris, "On the use of windows for harmonic analysis with the discrete Fourier transform," *Proceedings of the IEEE*, vol. 66, no. 1, pp. 51–83, 1978.

[4] Z. Shpiro and D. Malah, "Design of filters for discrete short-time Fourier transform synthesis," in *ICASSP '85. IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 10, pp. 537–540, 1985.

[5] S. Bagheri and D. Giacobello, "Robust STFT domain multi-channel acoustic echo cancellation with adaptive decorrelation of the reference signals," in *ICASSP 2021 - 2021 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pp. 131–135, 2021.

[6] M. Guo and B. Kuenzle, "Obtaining narrow transition region in STFT domain processing using subband filters," in *ICASSP 2019 - 2019 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pp. 970–974, 2019.

[7] M. L. Valero and E. A. Habets, "On the spatial coherence of residual echoes after STFT-domain multi-microphone acoustic echo cancellation," in *2018 16th International Workshop on Acoustic Signal Enhancement (IWAENC)*, pp. 141–145, 2018.

[8] T. Tanaka, "A direct design of oversampled perfect reconstruction FIR filter banks," *IEEE Transactions on Signal Processing*, vol. 54, no. 8, pp. 3011–3022, 2006.

[9] M. F. Mansour, "On the optimization of oversampled DFT filter banks," *IEEE Signal Processing Letters*, vol. 14, no. 6, pp. 389–392, 2007.

[10] J. de Haan, N. Grbic, I. Claesson, and S. Nordholm, "Filter bank design for subband adaptive microphone arrays," *IEEE Transactions on Speech and Audio Processing*, vol. 11, no. 1, pp. 14–23, 2003.

[11] J. De Haan, N. Grbic, I. Claesson, and S. Nordholm, "Design of oversampled uniform DFT filter banks with delay specification using quadratic optimization," in *2001 IEEE International Conference on Acoustics, Speech, and Signal Processing. Proceedings (Cat. No.01CH37221)*, vol. 6, pp. 3633–3636 vol.6, 2001.

[12] K. Eneman and M. Moonen, "DFT modulated filter bank design for oversampled subband systems," *Signal Processing*, vol. 81, no. 9, pp. 1947–1973, 2001.

[13] P. P. Vaidyanathan, *Multirate systems and filter banks*. Pearson Education India, 2006.

[14] S. Haykin, *Adaptive Filter Theory: International Edition*. Pearson Education, 2014.