A Recursive Least M-Estimate Adaptive Algorithm With Low Complexity for Active Control of Impulsive Noises

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***Abstract*—Adaptive active control is a crucial technology to attenuate impulsive noises. But how to design an appropriate adaptive filter to attain a flexible compromise between noise reduction and computational complexity is a challenging problem. This paper proposes a robust adaptive filtering algorithm to deal with this issue. The coefficient vector of the adaptive filter is decomposed into two sets of short sub-filters through the Kronecker product, which reduces the size of matrices and vectors in the active noise control algorithm. A robust estimator, which is insensitive to impulsive noises, is used to define a group of cost function under the recursive least-squares criterion, based on which we derive the adaptive control algorithm that is composed of two groups of alternately updating equations. The effectiveness of the proposed approach is verified by numerical simulations.**

***Index Terms*—Adaptive active control, impulsive noise, Kronecker product decomposition, computational complexity.**

I. Introduction

Active noise control (ANC) is a fundamental approach to attenuating unwanted low-frequency disturbances by introducing controllable secondary sound sources, which are employed to interfere destructively with the primary sound source [1], [2]. This technology has been extensively applied in various applications to reduce detrimental noises, such as broadband noise in flight decks, narrowband transformer noise, noise in ducts, and uncomfortable noises in living and work environments. The filtered-x least-mean square (FxLMS) algorithm is the most commonly used adaptive algorithm in ANC due to its effectiveness in most cases, low computational complexity, and ease of implementation [1], [2].

Impulsive noises exist in a wide spectrum of environments, such as the clatter of workpieces in workshops, the noise produced by pile drivers, the gunfire on the battlefield, the sound of firecrackers, door slams, objects dropping, to name but a few. This class of noises are more likely to exhibit sharp spikes or occasional bursts than one would expect from normally distributed noise that its probability density function obeys Gaussian distribution. A large number of adaptive algorithms have been developed for the active control of impulsive noises, such as the improved variants of the FxLMS algorithm [3], [4], the filtered-x least-mean p-norm algorithm (FxLMP) [5], [6], the FxlogLMS algorithm with logarithmic transformation [7], the M-estimator based FxLMS algorithm [8], the recursive least-squares (RLS) based FxlogRLS algorithm [9], the FxRLM algorithm based on the Hampel function [10], the state detector-based post-filtering algorithm [11], the filtered-x affine projection sign algorithm with a post-adaptive filter and variable step size [12], and the hybrid methods based on FxLMS- and FxRLS-types [13], [14]. Among those techniques, the FxlogRLS algorithm has attracted great attention since it uses the logarithmic function to attenuate impulsive noise and consider a commutation error to circumvent the instability of the adaptive filter. This algorithm, however, is of high computational complexity.

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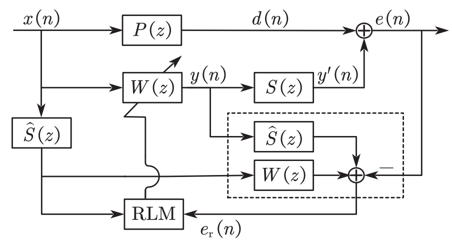

Fig. 1. Block diagram of a single channel feed-forward ANC system that takes a commutation error into account.

In this work, we extend the FxlogRLS algorithm and propose a filter-x recursive least M-estimate algorithm based on the nearest Kronecker product [15]–[18] (FxRLM-NKP) to control impulse noise. The coefficient vector of the adaptive controller is decomposed into two sets of short sub-filters through NKP, which are used to formulate the signal model. A robust estimator with an adaptive parameter is employed to define a group of cost function, from which the corresponding adaptive control algorithm is derived. The use of the NKP decomposition reduces the computational complexity of the adaptive control algorithm. The adaptive M-estimator makes the proposed FxRLM-NKP algorithm robust to impulsive noises. Simulation results show that the proposed algorithm performs well on impulsive noise control and is computationally more efficient than its original counterpart.

II. The Recursive Least M-Estimate ANC Algorithm Based on the NKP Decomposition

A. Signal Model and Optimization Criterion

A single channel feed-forward ANC system that takes a commutation error into account [9], [19] is illustrated in Fig. 1. It is composed of a reference microphone sensor for picking up the reference noise $x(n)$, an error microphone sensor for measuring residual noise $e(n)$, and a secondary sound source for generating the canceling signal $y(n)$ to attenuate the primary noise $d(n)$. $P(z)$ is the primary path and $W(z)$ is the control filter. The reference signal, $x(n)$, is filtered through $\widehat{S}(z)$, which is an estimate of the so-called secondary path $S(z)$. The commutation error $e_{\mathrm{r}}(n)$ resulting from the altered sequence between the algorithm derivation stage and the ANC applications is considered in order to improve the algorithm's stability.

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It can be seen from Fig. 1 that the error signal, $e(n)$, of the ANC system at time n is

$$e(n) = d(n) + s(n) * \left[\mathbf{w}^T(n-1)\mathbf{x}(n)\right], \tag{1}$$

where $s(n)$ is the impulse response of the secondary path, $S(z)$, at time n, $*$ denotes the linear convolution, the superscript ^{T} stands for the transpose operator,

$$\mathbf{w}(n) = [w_0(n) \;\; w_1(n) \;\; \cdots \;\; w_{L-1}(n)]^T \tag{2}$$

is the coefficient vector of the adaptive filter $W(z)$ of length L at time n, and

$$\mathbf{x}(n) = [x(n) \;\; x(n-1) \;\; \cdots \;\; x(n-L+1)]^T \tag{3}$$

is the signal vector at time n. In practical ANC applications, $S(z)$ is unknown and must be estimated. Thus, the filtered reference is generated by passing the reference signal through the estimate of the secondary path. Then, the error signal in (1) can be rewritten as

$$\begin{aligned} e(n) &= d(n) + \mathbf{w}^T(n-1)\left[\hat{s}(n) * \mathbf{x}(n)\right] \\ &= d(n) + \mathbf{w}^T(n-1)\mathbf{x}_{\mathrm{f}}(n), \end{aligned} \tag{4}$$

where

$$\begin{aligned} \mathbf{x}_{\mathrm{f}}(n) &= \hat{s}(n) * \mathbf{x}(n) \\ &= [x_{\mathrm{f}}(n) \;\; x_{\mathrm{f}}(n-1) \;\; \cdots \;\; x_{\mathrm{f}}(n-L+1)]^T, \end{aligned} \tag{5}$$

$$x_{\mathrm{f}}(n) = \hat{s}(n) * x(n), \tag{6}$$

and $\hat{s}(n)$ is the time-domain counterpart of $\widehat{S}(z)$.

In most ANC systems, control filters are usually sparse regardless of whether the reference is white Gaussian noise or impulsive noise, and so the control filters can be approximated by a low-rank model with NKP. If the reference noise is impulsive, this low-rank approximation should take a smaller order of NKP to achieve effective control performance. A benefit of using a small order of NKP is that the algorithm can be made computationally more effective. Enlightened by the NKP decomposition approach in [15]–[18], we use a low-rank model that involves the NKP between a group of short vectors to approximate the coefficient vector of the filter. Hence, the adaptive filter is decomposed through the Kronecker product as [18]

$$\mathbf{w}(n) = \sum_{p=1}^{P} \mathbf{w}_{2,p}(n) \otimes \mathbf{w}_{1,p}(n), \tag{7}$$

where $\mathbf{w}_{1,p}(n)$ and $\mathbf{w}_{2,p}(n)$ are sub-filters of length L_1 and L_2, respectively, $\otimes$ stands for the Kronecker product, P is the order of NKP, and we assume that $L = L_1 L_2$ and $P < \min\{L_1, L_2\}$. By using the following relationship [20]:

$$\begin{aligned} \mathbf{w}_{2,p}(n) \otimes \mathbf{w}_{1,p}(n) &= [\mathbf{w}_{2,p}(n) \otimes \mathbf{I}_{L_1}]\, \mathbf{w}_{1,p}(n) \\ &= [\mathbf{I}_{L_2} \otimes \mathbf{w}_{1,p}(n)]\, \mathbf{w}_{2,p}(n), \end{aligned} \tag{8}$$

where $\mathbf{I}_{L_1}$ and $\mathbf{I}_{L_2}$ are the identity matrices of size $L_1 \times L_1$ and $L_2 \times L_2$, respectively, the error signal in (4) can be expressed into two equivalent forms:

$$\begin{aligned} e_1(n) &= d(n) + \sum_{p=1}^{P} \mathbf{w}_{1,p}^T(n-1)[\mathbf{w}_{2,p}(n-1) \otimes \mathbf{I}_{L_1}]^T \mathbf{x}_{\mathrm{f}}(n) \\ &= d(n) + \sum_{p=1}^{P} \mathbf{w}_{1,p}^T(n-1)\mathbf{x}_{\mathrm{f}_{2,p}}(n) \\ &= d(n) + \mathbf{w}_1^T(n-1)\mathbf{x}_{\mathrm{f}_2}(n), \end{aligned} \tag{9}$$

$$\begin{aligned} e_2(n) &= d(n) + \sum_{p=1}^{P} \mathbf{w}_{2,p}^T(n-1)[\mathbf{I}_{L_2} \otimes \mathbf{w}_{1,p}(n-1)]^T \mathbf{x}_{\mathrm{f}}(n) \\ &= d(n) + \sum_{p=1}^{P} \mathbf{w}_{2,p}^T(n-1)\mathbf{x}_{\mathrm{f}_{1,p}}(n) \\ &= d(n) + \mathbf{w}_2^T(n-1)\mathbf{x}_{\mathrm{f}_1}(n), \end{aligned} \tag{10}$$

where

$$\mathbf{x}_{\mathrm{f}_{2,p}}(n) = [\mathbf{w}_{2,p}(n-1) \otimes \mathbf{I}_{L_1}]^T \mathbf{x}_{\mathrm{f}}(n), \tag{11}$$

$$\mathbf{x}_{\mathrm{f}_2}(n) = [\mathbf{x}_{\mathrm{f}_{2,1}}^T(n) \;\; \mathbf{x}_{\mathrm{f}_{2,2}}^T(n) \;\; \cdots \;\; \mathbf{x}_{\mathrm{f}_{2,P}}^T(n)]^T, \tag{12}$$

$$\mathbf{w}_1(n-1) = [\mathbf{w}_{1,1}^T(n-1) \;\; \mathbf{w}_{1,2}^T(n-1) \;\; \cdots \;\; \mathbf{w}_{1,P}^T(n-1)]^T, \tag{13}$$

$$\mathbf{x}_{\mathrm{f}_{1,p}}(n) = [\mathbf{I}_{L_2} \otimes \mathbf{w}_{1,p}(n-1)]^T \mathbf{x}_{\mathrm{f}}(n), \tag{14}$$

$$\mathbf{x}_{\mathrm{f}_1}(n) = [\mathbf{x}_{\mathrm{f}_{1,1}}^T(n) \;\; \mathbf{x}_{\mathrm{f}_{1,2}}^T(n) \;\; \cdots \;\; \mathbf{x}_{\mathrm{f}_{1,P}}^T(n)]^T, \tag{15}$$

$$\mathbf{w}_2(n-1) = [\mathbf{w}_{2,1}^T(n-1) \;\; \mathbf{w}_{2,2}^T(n-1) \;\; \cdots \;\; \mathbf{w}_{2,P}^T(n-1)]^T. \tag{16}$$

To smooth out the momentary fluctuations due to large bursts in impulsive noise and mitigate the adverse effect of the impulsive characteristics on the adaptive filter, a robust M-estimator is employed to define a set of cost functions as follows:

$$\begin{cases} \mathcal{J}_\rho\left[\mathbf{w}_1(n)\right] = \sum_{i=1}^{n} \lambda_1^{n-i} \rho[\varepsilon_1(i)], & \text{(17a)} \\ \mathcal{J}_\rho\left[\mathbf{w}_2(n)\right] = \sum_{i=1}^{n} \lambda_2^{n-i} \rho[\varepsilon_2(i)], & \text{(17b)} \end{cases}$$

where

$$\varepsilon_1(i) = d(i) + \mathbf{w}_1^T(n)\mathbf{x}_{\mathrm{f}_2}(i), \tag{18}$$

$$\varepsilon_2(i) = d(i) + \mathbf{w}_2^T(n)\mathbf{x}_{\mathrm{f}_1}(i), \tag{19}$$

are the *a posteriori* errors at time index i, $0 < \lambda_1, \lambda_2 < 1$ are two forgetting factors, which are set to different values since possibly unequal error signals $\varepsilon_1(i)$ and $\varepsilon_2(i)$ cause the optimal forgetting factors in (17a) and (17b) to be different, and $\rho(\cdot)$ is a Cauchy estimator, which is defined as [21]

$$\rho_C[\varepsilon_l(i)] = \frac{\xi_l^2}{2} \log\left[1 + \left(\frac{\varepsilon_l(i)}{\xi_l}\right)^2\right], \; l = 1, 2, \tag{20}$$

the parameter ξ_l is adaptively estimated by using the variance estimate of $\varepsilon_l(n)$ and median operation [22].

Note that the algorithm presented in this work is largely different from the logarithmic transformation method developed in [9]. The cost function of the algorithm in this work is established by using a Cauchy estimator, whose parameter ξ_l is adaptively estimated to better track impulsive noises. More importantly, the proposed algorithm uses the NKP to decompose the coefficient vector of the adaptive filter into two sets of shorter sub-filters, which can largely reduce the computational complexity of the adaptive algorithm.

B. Adaptive Algorithm

According to (17a), the derivative of $\mathcal{J}_\rho[\mathbf{w}_1(n)]$ with respect to $\mathbf{w}_1(n)$ is deduced as

$$\frac{\partial \mathcal{J}_\rho[\mathbf{w}_1(n)]}{\partial \mathbf{w}_1(n)} = \sum_{i=1}^{n} \lambda_1^{n-i} \frac{\rho'[\varepsilon_1(i)]}{\varepsilon_1(i)} \mathbf{x}_{\mathrm{f}_2}(i)[d(i) + \mathbf{w}_1^T(n)\mathbf{x}_{\mathrm{f}_2}(i)], \tag{21}$$

where $\rho'(\cdot)$ is the first-order derivative of $\rho(\cdot)$. Letting

$$\gamma_1(i) = \frac{\rho'[\varepsilon_1(i)]}{\varepsilon_1(i)} \tag{22}$$

and the derivative of (21) be equal to zero, we obtain one normal equation as follows:

$$\mathbf{R}_2(n)\mathbf{w}_1(n) = \mathbf{p}_2(n), \tag{23}$$

where

$$\begin{aligned}\mathbf{R}_2(n) &= \sum_{i=1}^{n} \lambda_1^{n-i}\gamma_1(i)\mathbf{x}_{\mathrm{f}_2}(i)\mathbf{x}_{\mathrm{f}_2}^T(i) \\ &= \lambda_1 \mathbf{R}_2(n-1) + \gamma_1(n)\mathbf{x}_{\mathrm{f}_2}(n)\mathbf{x}_{\mathrm{f}_2}^T(n), \end{aligned} \tag{24}$$

$$\begin{aligned}\mathbf{p}_2(n) &= -\sum_{i=1}^{n} \lambda_1^{n-i}\gamma_1(i)d(i)\mathbf{x}_{\mathrm{f}_2}(i) \\ &= \lambda_1 \mathbf{p}_2(n-1) - \gamma_1(n)d(n)\mathbf{x}_{\mathrm{f}_2}(n). \end{aligned} \tag{25}$$

In a similar fashion, one can obtain from (17b) another normal equation as follows:

$$\mathbf{R}_1(n)\mathbf{w}_2(n) = \mathbf{p}_1(n), \tag{26}$$

where

$$\begin{aligned}\mathbf{R}_1(n) &= \sum_{i=1}^{n} \lambda_2^{n-i}\gamma_2(i)\mathbf{x}_{\mathrm{f}_1}(i)\mathbf{x}_{\mathrm{f}_1}^T(i) \\ &= \lambda_2 \mathbf{R}_1(n-1) + \gamma_2(n)\mathbf{x}_{\mathrm{f}_1}(n)\mathbf{x}_{\mathrm{f}_1}^T(n), \end{aligned} \tag{27}$$

$$\gamma_2(i) = \frac{\rho'[\varepsilon_2(i)]}{\varepsilon_2(i)}, \tag{28}$$

$$\begin{aligned}\mathbf{p}_1(n) &= -\sum_{i=1}^{n} \lambda_2^{n-i}\gamma_2(i)d(i)\mathbf{x}_{\mathrm{f}_1}(i) \\ &= \lambda_2 \mathbf{p}_1(n-1) - \gamma_2(n)d(n)\mathbf{x}_{\mathrm{f}_1}(n). \end{aligned} \tag{29}$$

It is easy to check, according to (20), that $\gamma_1(i)$ and $\gamma_2(i)$ can be expressed as

$$\gamma_l(i) = \frac{1}{1+\left(\frac{\varepsilon_l(i)}{\xi_l}\right)^2}, \; l = 1, 2. \tag{30}$$

Based on (23)–(25), (26), (27), and (29), we derive the FxRLM-NKP algorithm, which is summarized in Table I. Notice that in the algorithm implementation, we employ the *a priori* errors with $\mathbf{w}_1(n-1)$ and $\mathbf{w}_2(n-1)$ instead of $\mathbf{w}_1(n)$ and $\mathbf{w}_2(n)$ to compute $\gamma_1(n)$ and $\gamma_2(n)$, respectively.

As pointed out in [9], the signal flow path of the ANC system is different from the typical derivation stage. This difference brings out the commutation error in the error signal [9], [19]. In the active control of impulsive noises, such a commutation error cannot be ignored due to its fast abrupt change. So, the commutation error has to be incorporated into the update equation of the proposed algorithm to deal with its adverse effect. To do this, we express, according to the error formulation in (1), (9), and (10), the estimates of the primary noise $d(n)$ and the secondary path into the following two equivalent forms:

$$\begin{aligned}\hat{d}(n) &= e_1(n) - \hat{s}(n) * \sum_{p=1}^{P} \mathbf{w}_{1,p}^T(n-1)[\mathbf{w}_{2,p}(n-1)\otimes \mathbf{I}_{L_1}]^T\mathbf{x}(n) \\ &= e_1(n) - \hat{s}(n) * \sum_{p=1}^{P} \mathbf{w}_{1,p}^T(n-1)\mathbf{x}_{2,p}(n) \\ &= e_1(n) - \hat{s}(n) * \left[\mathbf{w}_1^T(n-1)\mathbf{x}_2(n)\right] \end{aligned} \tag{31}$$

TABLE I

THE FXRLM-NKP ALGORITHM.

Initialization
$\mathbf{w}_{2,p}(0) = [\eta \quad 0 \quad \cdots \quad 0]^T,\ p = 1, 2, \ldots, P \ (0 < \eta \le 1)$
$\mathbf{w}_{1,p}(0) = [\eta \quad 0 \quad \cdots \quad 0]^T,\ p = 1, 2, \ldots, P \ (0 < \eta \le 1)$
$\mathbf{Q}_2 = \delta_2 \mathbf{I}_{PL_1 \times PL_1},\ \mathbf{Q}_1 = \delta_1 \mathbf{I}_{PL_2 \times PL_2},\ \delta_1 > 0,\ \delta_2 > 0$
For $n = 1, 2, \ldots$, compute
$\mathbf{x}_{\mathrm{f}}(n) = \hat{s}(n) * \mathbf{x}(n)$
$\mathbf{x}_{\mathrm{f}_{2,p}}(n) = [\mathbf{w}_{2,p}(n-1) \otimes \mathbf{I}_{L_1}]^T \mathbf{x}_{\mathrm{f}}(n)$
$\mathbf{x}_{\mathrm{f}_2}(n) = \left[\mathbf{x}_{\mathrm{f}_{2,1}}^T(n) \ \ \mathbf{x}_{\mathrm{f}_{2,2}}^T(n) \ \ \cdots \ \ \mathbf{x}_{\mathrm{f}_{2,P}}^T(n)\right]^T$
$\mathbf{x}_{2,p}(n) = \left[\mathbf{w}_{2,p}(n-1) \otimes \mathbf{I}_{L_1}\right]^T \mathbf{x}(n)$
$\mathbf{x}_2(n) = \left[\mathbf{x}_{2,1}^T(n) \ \ \mathbf{x}_{2,2}^T(n) \ \ \cdots \ \ \mathbf{x}_{2,P}^T(n)\right]^T$
$e_1(n) = d(n) + \mathbf{w}_1^T(n-1)\mathbf{x}_{\mathrm{f}_2}(n)$
$\gamma_1(n) = \frac{1}{1+\left(\frac{e_1(n)}{\xi_1}\right)^2}$
$\mathbf{k}_2(n) = \frac{\lambda_1^{-1}\mathbf{Q}_2(n-1)\mathbf{x}_{\mathrm{f}_2}(n)}{\frac{1}{\gamma_1(n)} + \lambda_1^{-1}\mathbf{x}_{\mathrm{f}_2}^T(n)\mathbf{Q}_2(n-1)\mathbf{x}_{\mathrm{f}_2}(n)}$
$\mathbf{Q}_2(n) = \lambda_1^{-1}\mathbf{Q}_2(n-1) - \lambda_1^{-1}\mathbf{k}_2(n)\mathbf{x}_{\mathrm{f}_2}^T(n)\mathbf{Q}_2(n-1)$
$e_{\mathrm{r}_1}(n) = e_1(n) - \hat{s}(n) * \left[\mathbf{w}_1^T(n-1)\mathbf{x}_2(n)\right] + \mathbf{w}_1^T(n-1)\mathbf{x}_{\mathrm{f}_2}(n)$
$\mathbf{w}_1(n) = \mathbf{w}_1(n-1) - \mathbf{k}_2(n)e_{\mathrm{r}_1}(n)$
$\quad = \left[\mathbf{w}_{1,1}^T(n) \ \ \mathbf{w}_{1,2}^T(n) \ \ \cdots \ \ \mathbf{w}_{1,P}^T(n)\right]^T$
$\mathbf{x}_{\mathrm{f}_{1,p}}(n) = \left[\mathbf{I}_{L_2} \otimes \mathbf{w}_{1,p}(n-1)\right]^T \mathbf{x}_{\mathrm{f}}(n)$
$\mathbf{x}_{\mathrm{f}_1}(n) = \left[\mathbf{x}_{\mathrm{f}_{1,1}}^T(n) \ \ \mathbf{x}_{\mathrm{f}_{1,2}}^T(n) \ \ \cdots \ \ \mathbf{x}_{\mathrm{f}_{1,P}}^T(n)\right]^T$
$\mathbf{x}_{1,p}(n) = [\mathbf{I}_{L_2} \otimes \mathbf{w}_{1,p}(n-1)]^T \mathbf{x}(n)$
$\mathbf{x}_1(n) = \left[\mathbf{x}_{1,1}^T(n) \ \ \mathbf{x}_{1,2}^T(n) \ \ \cdots \ \ \mathbf{x}_{1,P}^T(n)\right]^T$
$e_2(n) = d(n) + \mathbf{w}_2^T(n-1)\mathbf{x}_{\mathrm{f}_1}(n)$
$\gamma_2(n) = \frac{1}{1+\left(\frac{e_2(n)}{\xi_2}\right)^2}$
$\mathbf{k}_1(n) = \frac{\lambda_2^{-1}\mathbf{Q}_1(n-1)\mathbf{x}_{\mathrm{f}_1}(n)}{\frac{1}{\gamma_2(n)} + \lambda_2^{-1}\mathbf{x}_{\mathrm{f}_1}^T(n)\mathbf{Q}_1(n-1)\mathbf{x}_{\mathrm{f}_1}(n)}$
$\mathbf{Q}_1(n) = \lambda_2^{-1}\mathbf{Q}_1(n-1) - \lambda_2^{-1}\mathbf{k}_1(n)\mathbf{x}_{\mathrm{f}_1}^T(n)\mathbf{Q}_1(n-1)$
$e_{\mathrm{r}_2}(n) = e_2(n) - \hat{s}(n) * \left[\mathbf{w}_2^T(n-1)\mathbf{x}_1(n)\right] + \mathbf{w}_2^T(n-1)\mathbf{x}_{\mathrm{f}_1}(n)$
$\mathbf{w}_2(n) = \mathbf{w}_2(n-1) - \mathbf{k}_1(n)e_{\mathrm{r}_2}(n)$
$\quad = \left[\mathbf{w}_{2,1}^T(n) \ \ \mathbf{w}_{2,2}^T(n) \ \ \cdots \ \ \mathbf{w}_{2,P}^T(n)\right]^T$
$\mathbf{w}(n) = \sum_{p=1}^{P} \mathbf{w}_{2,p}(n) \otimes \mathbf{w}_{1,p}(n)$
$e(n) = d(n) + \mathbf{w}^T(n)\mathbf{x}_{\mathrm{f}}(n)$

and

$$\begin{aligned}\hat{d}(n) &= e_2(n) - \hat{s}(n) * \sum_{p=1}^{P} \mathbf{w}_{2,p}^T(n-1)[\mathbf{I}_{L_2}\otimes \mathbf{w}_{1,p}(n-1)]^T\mathbf{x}(n) \\ &= e_2(n) - \hat{s}(n) * \sum_{p=1}^{P} \mathbf{w}_{2,p}^T(n-1)\mathbf{x}_{1,p}(n) \\ &= e_2(n) - \hat{s}(n) * \left[\mathbf{w}_2^T(n-1)\mathbf{x}_1(n)\right], \end{aligned} \tag{32}$$

where

$$\mathbf{x}_{2,p}(n) = [\mathbf{w}_{2,p}(n-1)\otimes \mathbf{I}_{L_1}]^T\mathbf{x}(n), \tag{33}$$

$$\mathbf{x}_2(n) = [\mathbf{x}_{2,1}^T(n) \ \ \mathbf{x}_{2,2}^T(n) \ \ \cdots \ \ \mathbf{x}_{2,P}^T(n)]^T, \tag{34}$$

$$\mathbf{x}_{1,p}(n) = [\mathbf{I}_{L_2}\otimes \mathbf{w}_{1,p}(n-1)]^T\mathbf{x}(n), \tag{35}$$

$$\mathbf{x}_1(n) = [\mathbf{x}_{1,1}^T(n) \ \ \mathbf{x}_{1,2}^T(n) \ \ \cdots \ \ \mathbf{x}_{1,P}^T(n)]^T. \tag{36}$$

Substituting the estimates in (31) and (32) into (9) and (10), respectively, and rewriting the original $e_1(n)$ and $e_2(n)$ as $e_{\mathrm{r}_1}(n)$ and $e_{\mathrm{r}_2}(n)$, respectively, give the error signals:

$$e_{\mathrm{r}_1}(n) = e_1(n) - \hat{s}(n) * [\mathbf{w}_1^T(n-1)\mathbf{x}_2(n)] + \mathbf{w}_1^T(n-1)\mathbf{x}_{\mathrm{f}_2}(n), \tag{37}$$

$$e_{\mathrm{r}_2}(n) = e_2(n) - \hat{s}(n) * [\mathbf{w}_2^T(n-1)\mathbf{x}_1(n)] + \mathbf{w}_2^T(n-1)\mathbf{x}_{\mathrm{f}_1}(n). \tag{38}$$

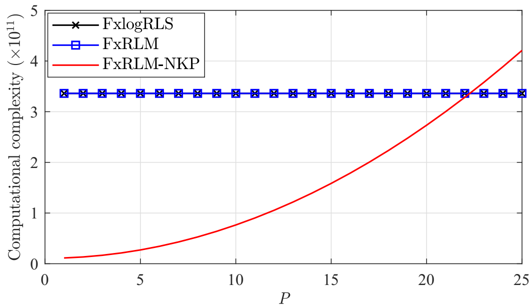

Fig. 2. Computational complexity of the FxlogRLS, FxRLM, and FxRLM-NKP algorithms versus the parameter P, where the length of the adaptive filter is set to $L = 1024$.

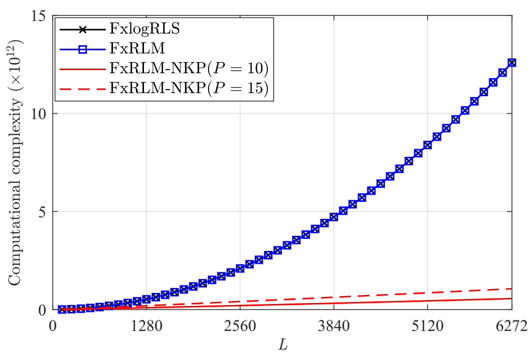

Fig. 3. Computational complexity of the FxlogRLS, FxRLM, and FxRLM-NKP algorithms versus the length L of the adaptive filter, where the parameter P is set to 10 and 15, respectively.

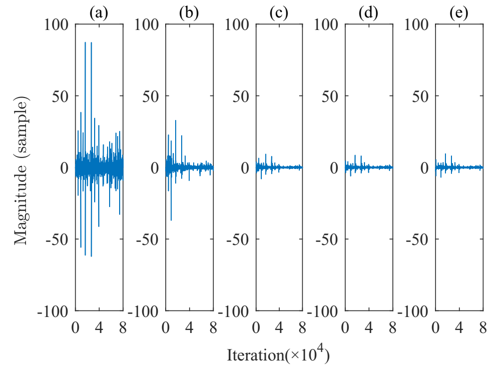

Fig. 4. Simulation results for the case of $\alpha = 1.5$ with $P = 15$: (a) original primary noise, (b) residual noise of the FxLMS algorithm; (c) residual noise of the FxlogRLS algorithm, (d) residual noise of the FxRLM algorithm, and (e) residual noise of the FxRLM-NKP algorithm.

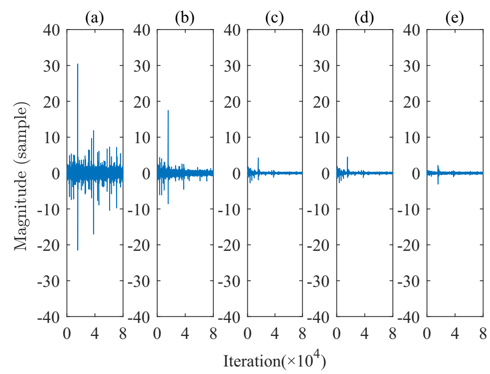

Fig. 5. Simulation results for the case of $\alpha = 1.8$ with $P = 10$: (a) original primary noise, (b) residual noise of the FxLMS algorithm; (c) residual noise of the FxlogRLS algorithm, (d) residual noise of the FxRLM algorithm, and (e) residual noise of the FxRLM-NKP algorithm.

III. Simulations

In this section, numerical simulations are conducted to validate the effectiveness of the proposed FxRLM-NKP algorithm, and compare it with the FxLMS, FxlogRLS, FxRLM algorithms. The primary path $P(z)$ and secondary path $S(z)$ are modeled as FIR filters of length 1600 and 500, respectively, and we assume that the secondary path $S(z)$ with the minimum phase is known *a priori*, i.e., $\widehat{S}(z) = S(z)$. The length of the global adaptive control filter $\mathbf{w}(n)$ is set to $L = 1024$, the length of the sub-filters $\mathbf{w}_1(n)$ and $\mathbf{w}_2(n)$ is set to $L_1 = L_2 = 32$. Two reference noise signals are modeled by the symmetric α-stable (SαS) distribution [23]. One is the highly impulsive with $\alpha = 1.5$, and the other is less impulsive with $\alpha = 1.8$.

The computational complexity is evaluated in terms of the number of multiplications/divisions required for the implementation of an ANC algorithm. The number of additions/subtractions are neglected because they are much quicker to calculate in most generic hardware platforms. Figure 2 illustrates the computational complexity of the FxlogRLS, FxRLM, and FxRLM-NKP algorithms versus the parameter P, where the length of the adaptive filter is set to $L = 1024$. As seen, as the value of P increases, the computational complexity of the proposed FxRLM-NKP algorithm increases. If $P \leq 22$, its computational cost is less than that of the other two algorithms. Figure 3 plots the computational complexity of the FxlogRLS, FxRLM, and FxRLM-NKP algorithms versus the length L of the adaptive filter, where the parameter P is set to 10 and 15, respectively. It can be seen that as the length L of the adaptive filter increases, the computational complexities of the studied algorithms increase. In comparison, the computational cost of the proposed FxRLM-NKP algorithm is much less than those of the other two algorithms, especially for large L. This indicates that in terms of the computational efficiency, the proposed algorithm is the most effective among the three compared algorithms.

Figures 4 and 5 present the noise reduction performance of the studied algorithms. As seen, for the two types of impulsive noises, the FxLMS algorithm performs worst due to its lack of robustness. The FxlogRLS algorithm obtains good control performance as it uses a logarithmic function to prevent from large fluctuation in the filter coefficients. The FxRLM algorithm achieves comparable noise reduction performance due to the immunity of the adaptive Cauchy estimator over impulsive noises. The proposed FxRLM-NKP algorithm attains a comparable control performance as FxRLM but with a much lower computational cost. The FxRLM-NKP algorithm has the lowest complexity among the three studied robust control algorithms.

IV. Conclusions

In this paper, an FxRLM-NKP algorithm was developed for the active control of impulsive noises. With NKP, the control filter is decomposed into two sets of short sub-filters, which significantly reduces the computational complexity of the algorithm. A Cauchy estimator was employed to define the robust cost function, from which the corresponding control algorithm was deduced. The proposed algorithm exploits the property of the Cauchy estimator and an adaptive parameter to achieve active control of impulse noises. The complexity analysis showed that the proposed FxRLM-NKP algorithm is computationally much more efficient than FxRLM and FxlogRLS and the simulation results showed that they achieve comparable performances.

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