

Multi-Target Doppler Ambiguity Identification for a PMCW Automotive Radar System

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Abstract—For the detection of weak and strong targets a high dynamic range in the range correlation processing of a phase-modulated continuous wave (PMCW) radar system is desired. However, the Doppler frequency shift can impair the received signal, resulting in a decreased peak-to-sidelobe ratio (PSR). The Doppler impact can be counteracted by utilizing the velocity information obtained in the current measurement cycle. However, this velocity information can be ambiguous. This paper presents a novel algorithm for addressing this velocity ambiguity issue in a multi-target scenario for a PMCW radar system. The main lobe power is used as a feature to resolve the velocity ambiguity, allowing for correct compensation of the phase progression in the range profile. The feasibility of the proposed algorithm is demonstrated in a simulated environment.

Index Terms—Automotive radar, phase-modulated continuous wave, Doppler ambiguity, Doppler shift, digital modulation

I. INTRODUCTION

Automotive radar is a key technology for modern advanced driver assistance systems (ADAS) and fully-automated driving. Today's radar systems mostly employ analog modulation schemes, such as frequency-modulated continuous wave (FMCW). Time division multiplexing (TDM) and Doppler division multiplexing (DDM) are typically used as multiplexing strategies in multiple input multiple output (MIMO) configurations. Digital modulation schemes such as PMCW have gained more attention in recent years, enabling multiplexing in the code domain with code division multiplexing (CDM), which is very beneficial for MIMO.

In PMCW radar systems, the phase of a carrier wave is modulated according to a pseudo-random sequence. In [1] it has been shown that the Doppler shift due to the relative motion between the radar systems and the targets negatively affects the correlation processing along the fast-time dimension in PMCW radar systems. The Doppler shift manifests as an additional linearly progressing phase along the received signal. When this received signal is correlated with the reference signal, its additional linearly progressing phase leads to a decrease in the main lobe level and an increase in the sidelobe levels in the correlation result. As a result, the PSR is decreased, which is a decisive factor in describing the detection sensitivity of a radar system. In addition, in the presence of a Doppler shift, weak targets can be masked by the increased sidelobe levels.

Several types of code families suitable for PMCW radar with different characteristics exist. Multiple studies have investigated the influence of the Doppler shift on these code families, e.g., [2]–[4]. It has been shown that some codes are more Doppler-tolerant than others. However, regardless of the code family selected, the Doppler shift should be compensated for to maintain optimum performance. For Doppler-tolerant codes, such as the Gold code, compensation further decreases the influence of the Doppler shift. Doppler compensation however also allows the use of codes with lower Doppler tolerance but preferred properties, e.g., for auto- and cross-correlation. This in particular applies to the almost perfect auto-correlation sequence (APAS) considered in this paper.

Approaches to Doppler mitigation have been presented in [1] and [5]. However, these approaches are limited to targets with unambiguous velocities and do not consider targets with velocity ambiguity. Velocity ambiguity results from the fact that radar systems can only measure velocities correctly up to a certain value, typically dependent on the slow-time sampling period utilized by the radar system. This velocity is referred to as the maximum ambiguous velocity. If the relative velocity of the target is greater than this value, aliasing occurs in slow-time, and the measured relative velocity differs from the real velocity. In FMCW radar systems, approaches such as the Chinese remainder theorem (CRT) and density-based spatial clustering of applications with noise (DBSCAN) exist, but these approaches require multiple measurement cycles to resolve the Doppler ambiguity [6].

The contribution of this paper includes the presentation of a novel method for the identification of ambiguous velocities. The main lobe level after the range processing is used as a feature for the identification of velocity ambiguity. While in previous works, the identification has to consider multiple measurement cycles jointly, the proposed method can identify targets with ambiguous velocities based on a single measurement cycle. Furthermore, it is not only possible to identify targets with ambiguous velocities but also to determine the real relative velocities of the targets.

II. PMCW SIGNAL MODEL

PMCW is a digital modulation scheme. Depending on a pseudo-random periodic sequence, the phase of a carrier wave

is modulated. A single symbol of the sequence is called a chip, and it is defined as $x[n] = \exp(j\phi[n])$. For simplicity, it is assumed that binary symbols are used, so that $\phi[n] \in \{0, \pi\}$ and consequently $x[n] \in \{-1, 1\}$. A complete sequence consists of N_c chips, and in its equivalent complex baseband (ECB) representation it can be defined as

$$x_{\text{ECB}}(t) = \sum_{n_c=0}^{N_c-1} x[n_c] \text{rect}\left(\frac{t - n_c T_c}{T_c}\right), \quad (1)$$

where n_c is the chip index, N_c is the sequence length, T_c is the chip duration, and $\text{rect}(\cdot)$ denotes the rectangular function.

The signal $x_{\text{ECB}}(t)$ is used to modulate a carrier wave with a frequency f_c . A single transmitted sequence in its representation as an analytic signal is given as

$$x_+(t) = \exp(j2\pi f_c t) x_{\text{ECB}}(t). \quad (2)$$

Several consecutive transmitted sequences are used to form the coherent processing interval (CPI) transmit waveform of the radar, which is given as

$$x_{+, \text{CPI}}(t) = \sum_{n_{\text{slow}}=0}^{N_{\text{slow}}-1} x_+(t - n_{\text{slow}} T_{s2s}), \quad (3)$$

where n_{slow} is the sequence index, N_{slow} is the total number of consecutive transmitted sequences, and T_{s2s} is the sequence-to-sequence duration. At each $t = n_{\text{slow}} T_{s2s}$ a sequence is transmitted. The sequence-to-sequence duration is at least as long as the sequence duration $T_{\text{seq}} = N_c T_c$, so that $T_{s2s} \geq T_{\text{seq}}$ holds, i.e. the individual sequences do not overlap. In practice, it can be assumed that the sequence-to-sequence duration is greater than the sequence duration.

The received signal $y_+(t)$, in the interval $(n_{\text{slow}} - 1)T_{s2s} \leq t < n_{\text{slow}} T_{s2s}$, is a sum of attenuated and time-delayed replicas of the n_{slow} -th transmitted sequence. If targets are moving, $y_+(t)$ is additionally, time-scaled. In the following, the signal after the down-conversion of $y_+(t)$ is called $y_{\text{ECB}}(t)$. This signal is sampled and digitized. The CPI received data can then be represented as a two-dimensional matrix by

$$y[n_{\text{fast}}, n_{\text{slow}}] = y_{\text{ECB}}(n_{\text{fast}} T_c + n_{\text{slow}} T_{s2s}), \quad (4)$$

where n_{fast} is the sample index along fast-time and n_{slow} along the slow-time. The sampling rate of the analog-to-digital converter (ADC) needs to be at least as great as the Nyquist rate. The bandwidth is inversely proportional to the chip duration, $B \propto 1/T_c$. The shorter the chip duration, the higher the required sampling rate of the ADC. Using a quadrature demodulator, the sampling period along fast-time is set to the chip duration T_c and the sampling period along slow-time is equal to the sequence-to-sequence duration T_{s2s} .

The cross-correlation along fast time between the discrete signals $x[n]$ and $y[n_{\text{fast}}, n_{\text{slow}}]$ is defined as

$$r_{xy}[k, n_{\text{slow}}] = \sum_{n_{\text{fast}}=0}^{N_c-1} x^*[\text{mod}(n_{\text{fast}} - k, N_c)] y[n_{\text{fast}}, n_{\text{slow}}], \quad (5)$$

where $k \in \{0, 1, \dots, N_c - 1\}$ denotes the lag of the correlation, $\text{mod}(\cdot)$ is the modulo-operator, and $(\cdot)^*$ is the complex conjugate. The perfect correlation is given when for each lag the correlation value is equal to zero except for the lag at the round-trip time delay τ as

$$r_{xy}[k, n_{\text{slow}}] = \begin{cases} 0 & \text{for } kT_c \neq \tau \\ N_c & \text{for } kT_c = \tau. \end{cases} \quad (6)$$

The round-trip delay is proportional to the range between the radar system and the target. As derived in [7], perfect correlation properties are only possible over a limited range when binary symbols are used. The correlation result in (5) can be interpreted as the range profile, and it is usually a superposition of reflections from multiple targets.

The Doppler shift due to relative motion between the radar system and the target affects the burst of consecutive transmitted sequences along fast-time and slow-time. As shown in [1], the phase shift along fast-time is considered parasitic since it decreases the PSR in the range profile, and it should be compensated. The phase change along slow-time in contrast is desired and shall be estimated since it carries the relevant information for velocity estimation. Therefore, the range profile $r_{xy}[k, n_{\text{slow}}]$ is transformed into a range-Doppler map, providing information about the ranges and relative velocities of targets by peaks located at the respective bins, by applying a discrete Fourier transform (DFT) along slow-time [8].

The Doppler shift along fast-time and slow-time is a linearly increasing phase modeled by

$$\chi[n_{\text{fast}}, n_{\text{slow}}] = \exp(-j2\pi f_D (n_{\text{fast}} T_c + n_{\text{slow}} T_{s2s})), \quad (7)$$

where f_D is the Doppler frequency. The Doppler frequency is defined as

$$f_D = \frac{2v_r}{\lambda}, \quad (8)$$

where v_r is the relative velocity between the radar system and the target, and λ is the wavelength of the carrier wave. The wavelength is defined as $\lambda = c/f_c$, where c is the velocity of propagation. The relative velocity can be either positive or negative. While negative velocities indicate an approaching target, positive velocities indicate a target that is moving away from the radar system. As can be seen in (7), the term $2\pi f_D n_{\text{fast}} T_c$ denotes the phase change along fast-time, which is intended to be compensated in the range profile processing.

Fig. 1 shows the influence of the Doppler shift on the main lobe level after correlation. An APAS of the length of 1044 is used for the simulation. According to (6) the correlation value for a static scenario is equal to the sequence length, $r_{xy}[\tau/T_c, n_{\text{slow}}] = N_c$. The higher the relative velocity v_r , the greater the decrease in the main lobe level.

In the following, this finding is used as a feature to identify targets with ambiguous velocities. Furthermore, the real relative velocities are estimated based on the main lobe level.

III. DOPPLER AMBIGUITY

When a target with an absolute relative velocity larger than the maximum ambiguous velocity, i.e., $|v_r| > v_{\text{max}}$, is

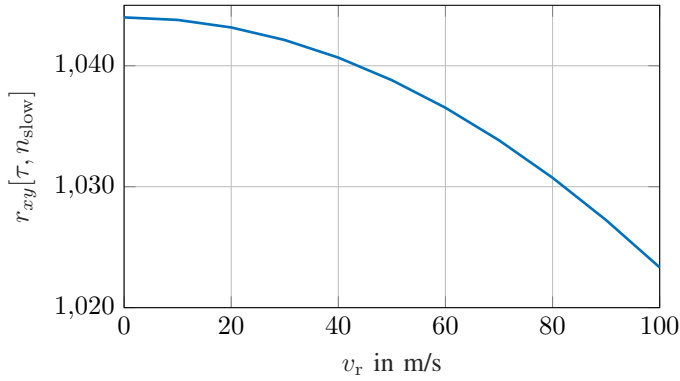


Fig. 1. Main lobe level after correlation for APAS (1044) as a function of relative velocity. An increase in the relative velocity and hence in the Doppler shift leads to a decrease in the level of the main lobe.

detected, its measured relative velocity $|v_{\text{meas}}| \leq v_{\text{max}}$ differs from the real velocity v_r . According to [9], the maximum relative velocity is limited by the sampling along the slow-time as

$$v_{\text{max}} = \frac{\lambda}{4T_{s2s}}. \quad (9)$$

The velocity resolution depends on the number of consecutive transmitted sequences and is calculated as

$$\Delta v = \frac{\lambda}{2N_{\text{slow}}T_{s2s}}. \quad (10)$$

The greater the number of consecutive transmitted sequences, the higher the velocity resolution. The relationship between v_{meas} and v_r is described as

$$v_{\text{meas}} = \begin{cases} v_r & \text{for } |v_r| \leq v_{\text{max}} \\ v_r + \kappa 2v_{\text{max}} & \text{for } |v_r| > v_{\text{max}} \end{cases} \quad (11)$$

where $\kappa \in \mathbb{Z}$ is a positive or negative integer value.

An example scenario is shown in Fig. 2. The scenario consists of one target with an unambiguous relative velocity and four targets with ambiguous relative velocities. The measured relative velocity of the four targets differs from the real relative velocity. Further processing is required to estimate the real relative velocity between the radar system and these targets.

IV. DOPPLER IDENTIFICATION

When a target is moving relative to the radar system, the Doppler shift adds a progressive phase shift to the received signal. The sidelobe level increases and is not equal to zero, while the main lobe level decreases. It follows,

$$r_{xy}[k, n_{\text{slow}}] = \begin{cases} \neq 0 & \text{for } kT_c \neq \tau \\ < N_c & \text{for } kT_c = \tau. \end{cases} \quad (12)$$

In the following, the main lobe level is used as a feature for Doppler ambiguity identification and the estimation of the real relative velocity of each target. Compared to the approach from [1], this also allows identifying and simultaneously compensating the Doppler shift for multiple targets with ambiguous velocities. A multi-velocity hypothesis approach is used to

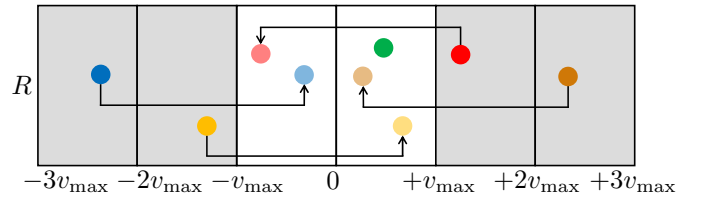


Fig. 2. Doppler ambiguity: Scenario with multiple targets with ambiguous velocities. The measured relative velocities differ from the real relative velocities of the targets. Only the green target has an unambiguous velocity, and its measured relative velocity is equal to the real relative velocity.

identify targets with ambiguous velocities. For hypothetical velocities, a Doppler compensation is applied to the range correlation. According to (11), hypothetical relative velocities are calculated as

$$v_{\text{hypo}} = v_{\text{meas}} + \kappa 2v_{\text{max}}. \quad (13)$$

Equivalent to (7) the Doppler shift compensation factor along fast-time can be defined as

$$\chi_{\text{comp}}[n_{\text{fast}}] = \exp(j2\pi f_D n_{\text{fast}} T_c), \quad (14)$$

where the Doppler frequency $f_D = 2v_{\text{hypo}}/\lambda$ depends on the hypothetical relative velocity.

In the following, an algorithm for the main lobe level-based Doppler identification (MLLDI) method is presented. The algorithm in a compact form is described in Algorithm 1.

A. Step 1: Identification of Potential Targets

The local maxima for each column of the range-Doppler map are extracted. A threshold, e.g., based on the constant false alarm rate (CFAR) principle, is used to distinguish between noise and targets. The 2D-CFAR threshold can be applied to the range-Doppler map. Any local maximum above this threshold is assumed to be a potential target.

B. Step 2: Inverse Range Correlation

An inverse correlation is applied to each column of the range-Doppler map. This step transforms the range axis back to a time-domain signal but keeps the Doppler axis. I.e., the data is represented in the fast-time / Doppler dimension. It is required for the following Doppler compensation step. The resulting matrix is referred to as the Doppler map.

C. Step 3: Multi-Velocity Hypothesis Doppler Compensation

Each column of the Doppler map is compensated with a multi-velocity hypothesis. The Doppler compensation factor χ_{comp} depends on the factor κ , which according to (13) determines the hypothetical relative velocity v_{hypo} . The number of hypothetical relative velocities considered is a preset parameter of the algorithm. In an automotive context, it is usually sufficient to consider $-2 \leq \kappa \leq 2$. The compensation with the hypothetical velocities results in several Doppler maps.

Algorithm 1 Main Lobe Level-based Doppler Identification

Step 1: For each column in the range-Doppler map, extract potential targets above a defined threshold

Step 2: Apply inverse correlation to each column of range-Doppler map to obtain Doppler-map

Step 3: For each κ compute a compensated Doppler map based on v_{hypo}

Step 4: Apply correlation to each column of the Doppler map to generate compensated range-Doppler map

Step 5: For each potential target, find κ which maximizes the main lobe level in the compensated range-Doppler map

Step 6: Compensate the range-Doppler map using the identified relative velocities

D. Step 4: Range Correlation

To obtain the compensated range-Doppler maps similar to (5), again correlations are performed between the columns (fast-time) of the compensated Doppler maps and the transmitted sequence $x[n]$. Each range-Doppler map corresponds to the Doppler compensation with one value of κ .

E. Step 5: Main Lobe Level Evaluation for Hypothetical Relative Velocities

For each κ , the main lobe levels of the potential targets are evaluated. The relative velocity that leads to the greatest main lobe level is identified as the real relative velocity for the specific target. If the greatest main lobe is achieved with $\kappa = 0$, then the respective target has an unambiguous velocity, while a factor $\kappa \neq 0$ indicates a target with an ambiguous velocity. For multiple targets, at the same Doppler frequency, a specific κ and therefore an individual relative velocity is estimated.

F. Step 6: Doppler Compensation

After identifying the real relative velocities for the targets, the Doppler shift can be compensated for each column of the range-Doppler map. Individual targets at the same Doppler frequency bin can have different relative velocities and therefore, different Doppler compensation factors. Each column is a superposition of the reflected signals from all targets in the Doppler frequency bin. Therefore, it is necessary to compensate for each reflected signal individually. This results in a range-Doppler map with decreased sidelobes and increased main lobe levels.

V. SIMULATION RESULTS

The simulated PMCW radar system is parametrized similarly to an automotive-verified FMCW radar system to achieve realistic and comparable performance. The simulation parameters can be found in Table I. For binary phase shift keying (BPSK) an APAS with a length of 516 chips is used. The chip duration is equal to 4ns. The generation of APAS is derived in [10] and detailed information on its properties is given in [11]. According to (9) and (10), the maximum relative velocity is about 28.79 m/s with a resolution of 0.22 m/s.

TABLE I
PARAMETERS OF 79 GHz PMCW RADAR SYSTEM

Parameter		Symbol	Value	Unit
Chip	bandwidth	B	250	MHz
	sampling rate	f_s	250	MHz
	duration	T_c	4.0	ns
Sequence	code		APAS	
	length	N_c	516	chips
	usable length	$N_{c,\text{use}}$	258	chips
	duration	T_{seq}	2.06	μs
	interval	T_{s2s}	32.95	μs
	repetitions	N_{slow}	256	

TABLE II
TARGET LIST

Index	Range		Rel. velocity		κ
	in m	bin	in m/s	bin	
1	23.98	40	19.57	215	0
2	113.92	190	19.57	215	0
3	29.98	50	64.33	160	1
4	59.96	100	64.33	160	1
5	107.93	180	105.72	90	2
6	95.93	160	-78.05	35	-1

This maximum velocity does not cover the range of relative velocities occurring in automotive scenarios and therefore the measured relative velocity can be ambiguous. The maximum range is about 309.39 m with a resolution of 0.60 m, where the maximum range is limited by the number of chips in the sequence [9]. To avoid ghost targets, the usable length of APAS is limited to half of the actual sequence length. As a consequence, the usable range is also limited to half of the actual measurable range. Therefore, the usable range is limited to 154.59 m. Due to their perfect auto-correlation properties for half the range, APAS have a high PSR due to zero sidelobes.

According to (11), the maximum factor $\kappa \in \{-2, -1, 0, 1, 2\}$ is chosen to cover possible relative velocities in automotive scenarios. For $\kappa = 2$ and $v_{\text{max}} = 30$ m/s, the maximum relative velocity that can be estimated is about 150 m/s. Theoretically, greater values of κ can be used to estimate greater relative velocities. The Doppler compensation with the real relative velocity leads to the greatest increase in the main lobe level and is identified as the correct relative velocity. This relative velocity can be used for the Doppler compensation of the range profile. The correct relative velocities can be calculated according to (13).

Fig. 3a shows the uncompensated range-Doppler map for a multi-target scenario. The normalized amplitude of the range-Doppler map is called \mathcal{V} . Some targets have ambiguous relative velocities, while others have unambiguous relative velocities. The target list is given in Table II. The MLLDI method is applied to the range profiles to identify the real relative velocities of the targets. The Doppler compensation is then applied based on the estimated relative velocities. The resulting range-Doppler map is shown in Fig. 3b. The normal-

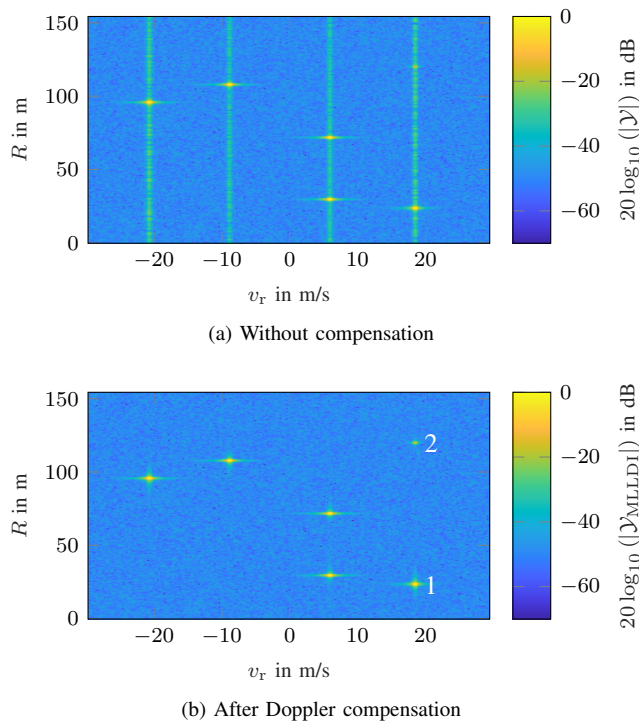


Fig. 3. Range-Doppler maps for a multi-target scenario. After Doppler compensation, the main lobe level is increased and the sidelobe level is decreased. A previously obscured weak target is visible after the identification of the correct relative velocities and application of Doppler compensation.

ized amplitude is denoted as $\mathcal{Y}_{\text{MLLDI}}$. Doppler compensation increases the PSR and suppresses the sidelobes below the noise floor. The noise floor is set to an artificially low level. A weak target, indicated by 2, previously obscured by the sidelobes of a strong target, indicated by 1, is detectable after the Doppler compensation.

It should be noted that this paper focuses on the identification of velocity ambiguity. For simplicity, only equal real relative velocities are assumed for targets at the same Doppler frequency bin. The Doppler compensation for multiple targets with different relative velocities at the same Doppler frequency bin is beyond the scope of this paper.

VI. CONCLUSION AND OUTLOOK

Identification and resolution of Doppler ambiguity in radar systems is necessary to provide valid information about detected targets. Estimation of relative velocities is required to compensate for the negative effect of Doppler shift on range correlation. Without compensation, target masking can occur, and weak targets can be obscured by strong sidelobes from other targets. We have presented a novel method for identifying velocity ambiguities and estimating the real relative velocities of multiple targets in a single measurement cycle. The main lobe level of the correlation signal is used as a feature for the identification. The proposed method uses a multi-velocity hypothesis Doppler compensation for the received signal and evaluates the main lobe level of each target

after range correlation. The Doppler compensation with the real relative velocity leads to the largest increase in the main lobe level for each target and results in an increased PSR.

While only point targets are considered in this work, in automotive scenarios objects may appear as extended targets and span multiple adjacent range and velocity bins. Under these conditions, the performance of the proposed method can be further investigated and compared with other methods in terms of processing requirements and performance. Furthermore, in real-world scenarios, a range profile can consist of multiple targets with different relative velocities at the same Doppler frequency. To compensate for the Doppler shift of different relative velocities it is necessary to compensate for the influence of each signal individually. The Doppler compensation for multiple targets at the same Doppler frequency bin is planned for future work.

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