Statistical prior knowledge for robust medical image segmentation by level set

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***Abstract*—Region-based active contours give interesting results when applied for objects detection with poor contrasting edges and noise, but for medical image segmentation, where objects of interest are often present with inhomogeneity, these models fail. In [16], we presented an original approach that integrate prior knowledge on the target shape into active contours with free registration based on a complete and stable set of invariant descriptors. In this work, we intend to present a novel method that incorporate statistical prior knowledge into level set model for robust medical image segmentation resulting from machine learning algorithms. In fact, for medical imaging, in many situations like tumors segmentation, prior knowledge on shape is insufficient or unavailable because it vary from a patient to another. The adopted approach is demonstrated with two unsupervised learning methods. The first one use the Fuzzy C-Means (FCM) and the second one is based on the Hidden Markov Random Field Expectation-Maximization algorithm (HMRF-EM). Both of the two are used to classify the pixels of the target image into two classes: the object and the background to construct our reference shape to be used as prior knowledge. A weighting schema is then considered to guide the active contour model under both forces: the traditional and the proposed one. We validate the robustness and the accuracy of our approach using ground truths given from experts segmentation and standard data bases. Experiments and comparative studies with several recent works are highlighted and show that the integrated method achieves good performance results.**

***Index Terms*—Prior knowledge, Segmentation, EM, FCM**

I. Introduction

Image segmentation was always considered as a complex and critical task in image processing and computer vision. Its aim is to make the representation of an image more significant and easier to be analyzed. i.e. it is used to subdivide a particular image into several zones with homogeneous intensities. Until now, a large variety of algorithms are encountered in literature and researchers have made great efforts to increase the efficiency of image segmentation processes. Front propagation methods based on an implicit representation of the evolving front proved to lead to convincing results for basic segmentation purposes [2], [5], [6], [19], [20]. Among the region-based methods, Chan and Vese [7] is a popular and representative model which is based on a simplified Mumford–Shah functional for segmentation [8]. The Chan-Vese model is successfully used for images containing two regions that have distinct average pixel intensities. But unfortunately, it remains sensitive to the initial contour, it also suffers from the problems of intensity in-homogeneity, occlusions and clutter. In order to solve the above problems, many solutions proposed to incorporate prior knowledge into the active contour model. The prior constraint can be related to shape, pixel intensity or both of them. In 2011, Melih and al. [22] used a statistical level set method which employs the Expectation Maximization (EM) algorithm to initialize and estimate the parameters. In 2013, Mezghich and al. [15] proposed the introduction of the shape prior after the registration of binary images associated with level set functions of the active contour and a reference shape. In 2016, Vakili and al. [18] used the level set method as a numerical technique for tracking interfaces which is able to determine the curve to shrink or expand by using the Bayesian rule and Mixture of Gaussian distribution to involve regional feature of images. In [9], Agus and al. proposed a framework which integrates two machine learning algorithms, the k-nearest neighbours (KNN) and the support vector machine (SVM), with a region-based active contour model to classify each pixel of the image according to the learning data. In 2019, Eltanboly and al. [10] introduced a statistical approach based on both shape and intensity constraints into the Osher and Sethian model [6]. Gong et al. [21] proposed a deep learning based active contour framework for pancreas segmentation. In [11], Khosravanian and al. presented a level set method for automatic brain tumor segmentation. This approach is based on superpixel fuzzy clustering and lattice Boltzmann method. In this paper, we propose a new method that incorporates an intensity-based prior constraint into Chan and Vese model [7] using either HMRF-EM or FCM algorithm to help the classic model achieve promising segmentation results. This approach helps to construct prior knowledge even in absence of prior learning (references or templates) on target shape which are frequently encountered in medical imaging like tumors segmentation. The rest of the paper is organised as follows : in section 2, we briefly recall the principle of active contours, then we present in section 3 the two studied machine learning algorithms for pixels classification. In section 4, we develop our approach for constructing prior knowledge to be added to level set model. In section 5, we highlight experimental results to prove the efficiency and robustness of the proposed method. Finally, we conclude the work and present future perspectives.

II. Level Set based active contours

Chan and Vese [7] have introduced an energy function that is defined by:

$$\begin{aligned} E_{CV}\left(c_1, c_2, C\right) = & \mu \cdot \operatorname{Length}(C) + v \operatorname{Area}(\operatorname{inside}(C)) \\ & + \lambda_1 \cdot \int_{\text{inside } (C)} \left|f(x, y) - c_1\right|^2 dx dy \\ & + \lambda_2 \cdot \int_{\text{outside } (C)} \left|f(x, y) - c_2\right|^2 dx dy \end{aligned} \tag{1}$$

Where C represents the curve in evolution, c_1 and c_2 are the mean values of f inside and outside C, respectively, f is the image intensity, $\mu, v \geq 0$ and $\lambda_1, \lambda_2 > 0$ fixed parameters. The goal is to search for c_1, c_2 and C such that $E_{CV}\left(c_1, c_2, C\right)$ is minimal.

We can solve this problem by using the level set technique introduced by Osher and Sethian where the curve C is represented implicitly using the zero-level set function $\phi(x, y)$. Hence $E_{CV}\left(c_1, c_2, C\right)$ can be reformulated as follows:

$$\begin{aligned} & E_{CV}\left(\phi, c_1, c_2\right) = \mu \int_{\Omega} \delta(\phi)|\nabla \phi(x, y)| dx dy \\ & \quad + v \int_{\Omega} H(-\phi(x, y)) dx dy \\ & \quad + \lambda_1 \int_{\Omega}\left|f(x, y) - c_1\right|^2 H(-\phi(x, y)) dx dy \\ & \quad + \lambda_2 \int_{\Omega}\left|f(x, y) - c_2\right|^2 H(\phi(x, y)) dx dy \end{aligned} \tag{2}$$

Where Ω is the domain of the image, H and δ are, respectively, the Heaviside function and the Dirac measure, as follows:

$$H(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{if } z < 0 \end{cases} \quad \delta(z) = \frac{d}{dz} H(z) \tag{3}$$

This minimization problem is solved by using the Euler-Lagrange equations and updating the level set function $\phi(x, y)$ by the gradient descent method:

$$\frac{\partial \phi}{\partial t} = \delta(\phi)\left[\mu \operatorname{div}\left(\frac{\nabla \phi}{|\nabla \phi|}\right) - \lambda_1\left(f - c_1\right)^2 + \lambda_2\left(f - c_2\right)^2\right] \tag{4}$$

Where c_1 and c_2 can be updated at each iteration respectively by:

$$\begin{aligned} & c_1(\phi) = \frac{\int_{\Omega} f(x, y)(1 - H(\phi(x, y))) dx dy}{\int_{\Omega}(1 - H(\phi(x, y))) dx dy} \\ & c_2(\phi) = \frac{\int_{\Omega} f(x, y) H(\phi(x, y)) dx dy}{\int_{\Omega} H(\phi(x, y)) dx dy} \end{aligned} \tag{5}$$

This model is very efficient in image segmentation as it is able to achieve a wider range of convergence in presence of poor edges, low contrast and noise. However, this model has some limitations as well, in fact, it is only suitable for the bi-phase image. If the intensities inside and outside the curve are not homogeneous, the constants c_1 and c_2 cannot be accurate. As a result, the model will not be able to achieve good segmentation results.

III. Statistical image segmentation

We will present in this section, two unsupervised machine learning algorithms that we will use in order to construct our prior knowledge. The prior constraint will be then integrated into level set active contour.

A. HMRF-EM algorithm

In this work, we will use the HMRF-EM algorithm to classify the pixels. This algorithm combines the Expectation Maximization (EM) algorithm and the Hidden Markov Random Field (HMRF) modeling [12]. First we will recall briefly some required fundamental concepts in the context of image processing and analysis.

We try to estimate a classification $\widehat{x}$ of the image according to the criterion of maximum a posteriori (MAP) :

$$\widehat{x} = \arg \max_x P(y \mid x) P(x) = \arg \min_x U(x \mid y) + U(x) \tag{6}$$

where the prior energy function $U(x)$ is defined by Eq. 6 and $U(y \mid x)$ is given by:

$$U(x \mid y) = \sum_{i \in S} \frac{\left(y_i - \mu_{x_i}\right)^2}{2 \sigma_{x_i}^2} + \log\left(\sigma_{x_i}\right) \tag{7}$$

To estimate the parameters of the Gaussian mixture (μ, σ), we use the EM algorithm. Then, the HMRF-EM can be described as follows: for a given iteration t

- Estimation of $\widehat{x}$ by the ICM.
- Estimation of $\widehat{\theta} = (\widehat{\mu}, \widehat{\sigma})$ by the EM algorithm :

$$\mu_{\ell}^{(t)} = \frac{\sum_{i \in S} P^{(t)}\left(\ell \mid y_i\right) y_i}{\sum_{i \in S} P^{(t)}\left(\ell \mid y_i\right)} \tag{8}$$

$$\left(\sigma_{\ell}^{(t)}\right)^2 = \frac{\sum_{i \in S} P^{(t)}\left(\ell \mid y_i\right)\left(y_i - \mu_{\ell}^{(t-1)}\right)^2}{\sum_{i \in S} P^{(t)}\left(\ell \mid y_i\right)} \tag{9}$$

These expressions are similar to the classical EM formula used in the context of finite Gaussian mixture except that

$$P^{(t)}(\ell \mid y_i) = \frac{f^{(t)}\left(y_i ; \theta_{\ell}^{(t-1)}\right) \cdot P^{(t)}\left(\ell \mid x_{\aleph_i}\right)}{P^{(t)}\left(y_i\right)} \tag{10}$$

As we can note here the spatial information is encoded in the prior distribution $P\left(\ell \mid \hat{x}_{N_i}\right)$ which is expressed by

$$P\left(\ell \mid \hat{x}_{N_i}\right) = \frac{\exp\left(-U\left(\ell \mid x_{N_i}\right)\right)}{\sum_{\xi \in L} \exp\left(-U\left(\xi \mid x_{N_i}\right)\right)} \tag{11}$$

As an iterative procedure, the HMRF-EM algorithm requires an initialization step. In fact, the ICM uses a prior classification in the first phase for the energy minimization. In this work, the EM classification results are considered as a starting point.

B. FCM algorithm

Fuzzy C-Means (FCM) is a fuzzy unsupervised classification algorithm. Resulting from the k-means algorithm, it introduces the notion of a fuzzy set in defining classes.

The FCM algorithm [17] attributes pixels to each category using fuzzy memberships. Suppose $X(x_1, x_2, ., x_N)$ an image of N pixels to be partitioned into c clusters.

The values of the membership degrees are grouped in a matrix

$U=[u_{ik}]$ for: $1\leq$ i $\leq$n, $1\leq$ k $\leq$c ; Where u_{ik} denotes the membership degree of pixel i to class k. In order to have a better partition, the following constraints are imposed on the elements of U:

$$\begin{aligned} &u_{ik}\in[0,1]; \\ &\Sigma_k u_{ik}=1; \text{for } \forall i. \end{aligned} \tag{12}$$

The algorithm evolves the partition (U-Matrix) by minimizing the following objective function:

$$J=\sum_{i=1}^{N}\sum_{k=1}^{c}u_{ik}^{m}\left\|x_i-c_k\right\|^2 \tag{13}$$

Where c_k is the k^{th} cluster center, $||.||$ is a normalized metric, and $m>1$ is a parameter controlling the fuzziness degree (usually $m=2$); The principal steps of the Fuzzy C-means algorithm are:

1) Initialization of the membership matrix and the c_k centers (random initialization).
2) Evolve the U partition matrix and the centers according to these two equations.
(1) $u_{ik}=1/\left(\sum_{j=1,c}\left(d_{ik}/d_{ij}\right)^{(2/(m-1))}\right)$
Where : $d_{ij}=||x_i-c_j||$,
(2) $c_k=\left(\Sigma_i\left(u_{ik}\right)^m\cdot x_i\right)/\left(\Sigma_i\left(u_{ik}\right)^m\right)$
3) Convergence can be found by measuring changes in the membership function or the cluster center during two successive iteration steps.

IV. Proposed approach

In this section, we present our original formulation of the intensity-based prior knowledge. Manual image segmentation by experts can take long time and requires a learning base in the supervised case. The FCM and HMRF-EM can solve this problem by segmenting an image without any prior information, therefore it is able to guide the Chan and Vese [7] algorithm to converge easily toward the target object. The basic idea is to take the image segmented by FCM or HMRF-EM as a prior information. Consider $I(x,y)$ a medical image to be segmented. The FCM and HMRF-EM algorithm classifies each pixel according to its degree of membership to Class 1 (object) or Class 2 (background). So for each given pixel $P(x,y)$, we construct a reference level set function based on pixel intensity defined as follows : $\phi_{ref_{int}}$

$$\phi_{ref\ _{int}}(x,y)=\begin{cases}-1, P(x,y)\in Class1\\ \ \ 1, P(x,y)\in Class2\end{cases} \tag{14}$$

Where the pixels labelled as object take the value -1 and those which represent the background take the value 1. And ϕ is the evolving level function using the Chan and Vese [7] algorithm. Note that the level set function (also called image distance map) is positive on the outside of the curve (zero level) and negative on the inside. Thus we propose to minimize the following energy:

$$E_{\text{intensity}}=\int_{\Omega}H(g(x,y))dxdy \tag{15}$$

Where

$$g(x,y)=-\phi(x,y)\cdot\operatorname{sign}\left(\phi_{ref_{int}}(x,y)\right)$$

As we can see, this energy corresponds to the area of variability between the image segmented by FCM or HMRF-EM and the image segmented by Chan-Vese model. [7].

The new energy's functional to be minimized is:

$$E_{tot}=w\cdot E_{CV}+(1-w)\cdot E_{\text{intensity}} \tag{16}$$

Where w is a weighting factor between the two energies having values between [0,1]. More discussion about this scalar will be detailed in the next section. To minimise this energy, the gradient descent method is used for the level set function ϕ, $\frac{\partial E}{\partial\phi}=-\frac{\partial\phi}{\partial t}$. Then, the total discrete evolution equation is:

$$\begin{aligned}\frac{\phi_{ij}^{n+1}-\phi_{ij}^{n}}{\Delta t}&=w\delta_{\varepsilon}\left(\phi_{ij}^{n}\right)\Big[-\mu K_{i,j}+v+\lambda_1\left(f_{i,j}-c_1\right)^2\\ &-\lambda_2\left(f_{i,j}-c_2\right)^2\Big]+(1-w)\operatorname{sign}\left(\phi_{ref_{int_{i,j}}}\right)\delta_z\left(g_{ij}^{n}\right)\end{aligned} \tag{17}$$

An overview of the proposed method is shown in Fig. 1.

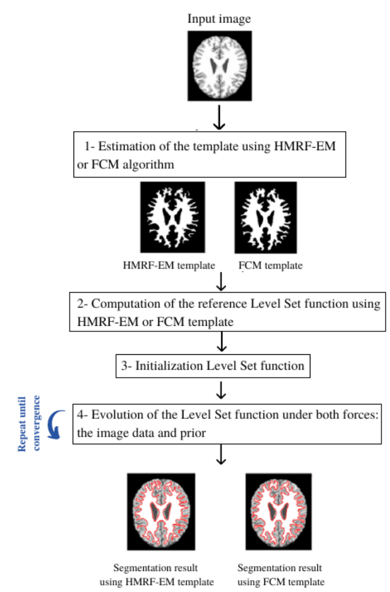

Fig. 1. Brief schematic overview of the proposed method

V. Experimental results

In order to further verify the superiority of the proposed method, a variety of medical images from different data bases are used including Brain Web, Liver tumor and Kidney images. All data are obtained from different patients and their associated ground truths are given by experts. The proposed method is implemented on a computer with 1.60 GHz Intel(R) Core(TM) i5, 8 GB RAM and a Windows 10 operating system. We had set the weighting factor $w=0.5$. To assess the accuracy of segmentation, two popular metrics are used; i.e, the Jaccard index (JI) and the Dice coefficient, also known as the similarity index (SI). Let A and B be the segmentation result and the ground truth respectively. The metrics are defined by:

$$\mathrm{JI} = \frac{|A \cap B|}{|A \cup B|} \tag{18}$$

$$\mathrm{SI} = \frac{2|A \cap B|}{|A| + |B|} \tag{19}$$

Where $|.|$ designates the number of elements. The reference templates are constructed using the FCM and HMRF-EM algorithms, as shown by Fig. 2. The number of classes for each image is carefully chosen. A number of qualitative

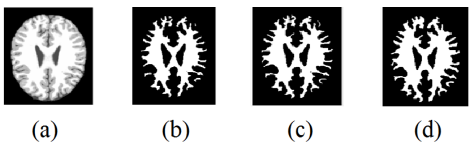

Fig. 2. Segmentation results for four images : (a) A given medical image, (b) It's ground truth, (c) HMRF-EM result, (d) FCM result.

segmentation results are shown in Fig. 3 while the quantitative results are presented in Fig. 4. In Fig. 3, we perform on images obtained from Brain Web data base. We shows that Chan and Vese's model has difficulties in segmenting this type of images (Brain web data base) because it contains several gray levels. Thanks to the prior information, we can see that the results obtained by our model far exceed those of the classical model.

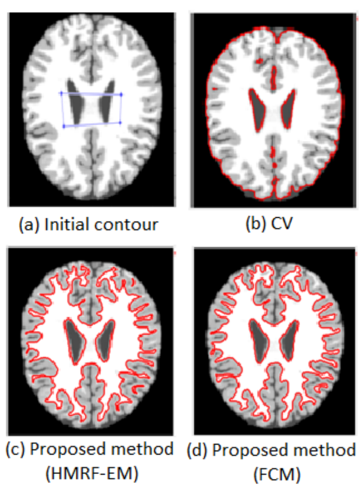

Fig. 3. Segmentation results of brain white matter.

For comparison purposes, our model is compared to [9]'s model for liver tumor segmentation. This model used supervised algorithms (KNN and SVM) to construct and integrate prior knowledge to level set. Accuracy of our two models are shown in Table 1. As it can be seen in Fig. 5, the proposed methods generally generate higher accuracy for the segmentation of liver tumor images. We end this part of experiments

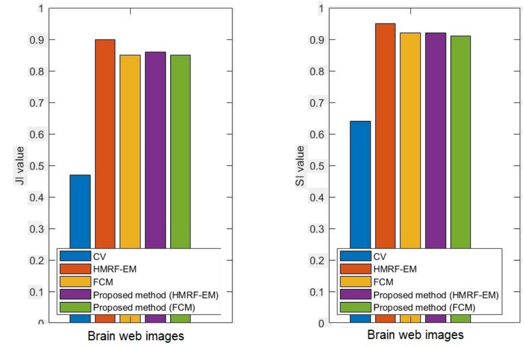

Fig. 4. Illustration of the JI and SI values for images shown in Fig. 3

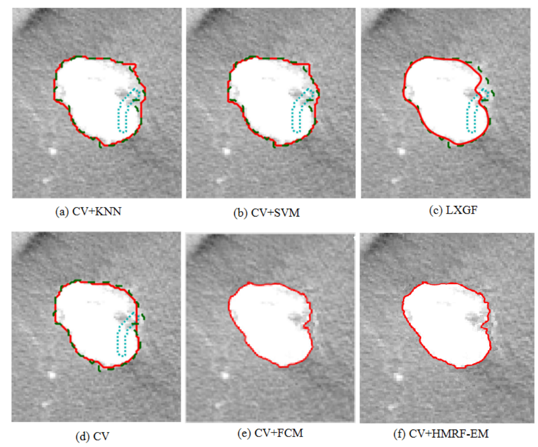

Fig. 5. Segmentation results for the liver tumor using various methods. Red solid lines denote the final segmentation, blue dotted lines denote the initialization contour, and green dashed lines are for the ground truth.

TABLE I
COMPARISON OF SEGMENTATION RESULTS FOR TUMOR LIVER.

	CV-HMRF-EM	CV-FCM	CV	CV+KNN	CV+SVM	LXGF
JI	0.95	0.94	0.66	0.93	0.93	0.68
SI	0.96	0.98	0.70	0.97	0.97	0.77

by comparing our model with the well known deep learning Mask R-CNN [13]. The method, called Mask R-CNN [13], extends Faster R-CNN by adding a branch for predicting an object mask in parallel with the existing branch for bounding box recognition. Fig. 6 and Fig. 7 demonstrate sample results comparing our method versus the method proposed in [13]. Table 2 summarizes the results of the CT Kidney images.

VI. Conclusion

We have proposed a framework to integrate machine learning algorithms with region-based active contour models. The framework utilizes unsupervised algorithms. So no need to several references for an offline learning step. The proposed intensity prior energy helps the evolving curve to converge

TABLE II
COMPARISON OF SEGMENTATION FOR CT KIDNEY IMAGES FIG. 9 .

	FCM		HMRF-EM		CV-FCM		CV-HMRF-EM		MASK R-CNN	
	SI	JI	SI	JI	SI	JI	SI	JI	SI	JI
Image 1	0.79	0.64	0.77	0.62	0.91	0.84	0.91	0.84	0.92	0.85
Image 2	0.87	0.77	0.84	0.73	0.93	0.87	0.91	0.84	0.86	0.75
Image 3	0.83	0.72	0.82	0.7	0.94	0.88	0.92	0.85	0.84	0.73
Image 4	0.87	0.78	0.85	0.74	0.93	0.88	0.92	0.85	0.86	0.76

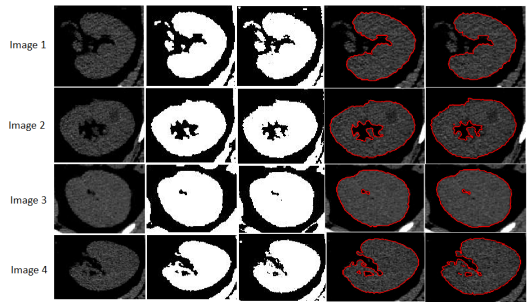

Fig. 6. Segmentation results of different methods for the CT Kidney images. First column: original image. Second to last columns are FCM, HMRF-EM, proposed method(FCM), proposed method (HMRF-EM) respectively.

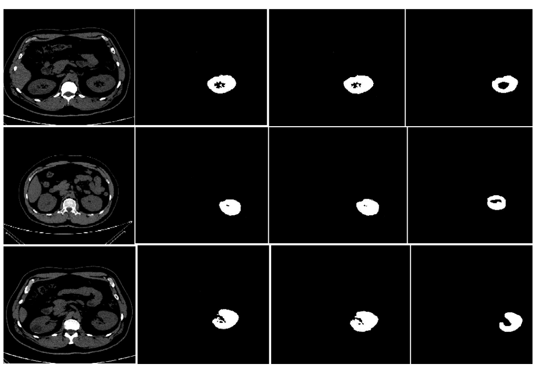

Fig. 7. Segmentation results of different methods for the CT Kidney images. First column: original image. Second to last columns are proposed method(FCM), proposed method (HMRF-EM), Mask R-CNN respectively

to the desired object. The improved model with FCM and HMRF-EM incorporated into the Chan-Vese [7] confirmed the efficiency of our proposed approach compared to the traditional Chan-Vese and also many recent models in terms of precision and computation time, in fact it is able to improve the segmentation results of active contours in the presence of occlusion, noise and low contrast. As future perspectives, we are working to propose a hybrid model based on both shape and intensity priors.

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