Channel Parameter Estimation Using a Wideband LFM Preamble: Comparison of the Fractional Fourier Transform and Matched Filtering

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***Abstract*—In this paper, we present an improvement of an existing method to estimate the channel parameters of an underwater acoustic channel in a wideband scenario. For underwater acoustic communications, the channel is characterized by paths having both time delay and Doppler scale. In this paper we focus on using a linear frequency modulated (LFM) preamble to estimate the channel parameters. We propose an improvement to the state-of-the-art method, which utilizes the fractional Fourier transform (FrFT), and show that matched filtering and the FrFT method can both be viewed as integral transforms having the same kernel. We argue that, contrary to the prevailing belief in the literature, matched filtering is superior compared to the FrFT method for channel parameter estimation. We support our findings using numerical experiments.**

***Index Terms*—Delay-Doppler channel, wideband, fractional Fourier transform, matched filtering.**

I. Introduction

The main difficulty of underwater acoustic communications is the fact that the channel severely distorts the transmitted signal. Since the carrier frequency is comparable to the signal bandwidth, one has to deal with a wideband scenario. In particular, the Doppler effect cannot be approximated by a frequency shift, but manifests itself as a Doppler scale[1]. In order to be able to communicate through such a channel, one first has to estimate the channel parameters, such as attenuation, time delay and Doppler scale.

Estimating channel parameters has long been a topic of interest in (underwater acoustic) communications research. In real-time scenarios, achieving high throughput in the communication channel is critical. To this end, researchers have been focusing on wideband scenarios for underwater acoustic communications. Several methods have been proposed, using preambles, postambles, or a combination of both [1]. Using a preamble only is to be preferred as it allows the receiver to demodulate the data without having to buffer the entire data package. Moreover, a preamble is also used for signal detection and synchronisation.

One of the simplest estimation methods is based on computing the cross-correlation between the received preamble and a delayed and Doppler-scaled version of the known preamble [1], [2]. The time delay and Doppler scale are estimated by finding the maximum of the wideband cross-ambiguity function. Despite its simplicity, the method is computationally intensive. Moreover, if the channel consists of more than one path, cross-correlating only once is not optimal anymore, as the different paths will influence each other's peak location.

In order to improve the estimation result, a matching pursuit decomposition algorithm was proposed [3]. Here, the channel parameters are found in a greedy way. First the received signal is correlated with a dictionary of delayed and scaled signals. After identifying the signal having maximum correlation (corresponding to the strongest path), this signal is subtracted from the received signal, thereby forming a residual signal. The correlation process is then applied repeatedly to the residual signal, until a predefined number of paths are identified. Although this method leads to improved estimation results, it remains computationally intensive (or even more so). Berger *et al.* [4] proposed a similar approach based on compressed sensing.

Mason *et al.* [5] proposed the use of a preamble consisting of a cyclic prefix and two identical OFDM symbol frames. Since the Doppler scale of the channel causes dilation/compression of the preamble, the repetition period of the two OFDM frames in the preamble changes. The received preamble is correlated with preambles having different repetition periods, from which the (time delay and) Doppler scale is estimated. However, this method only works under the assumption of a single Doppler scale for all paths (sometimes called the shallow water assumption).

Zhao *et al.* [7] introduced a method based on the fractional Fourier transform (FrFT). By using a linear frequency modulated (LFM) preamble, the channel parameters are estimated using a two-step approach. This method shows a significant improvement in estimation accuracy compared to the matching pursuit method of [3], while keeping the computational complexity acceptable. Additionally, the method can be used to estimate multiple paths with different Doppler scales.

In this paper, we propose an improvement of the above mentioned FrFT method in terms of computational complexity. In addition, we show that the FrFT method is similar to matched filtering; both methods can be viewed as integral transforms having the same kernel. Our findings indicate that, contrary to the prevailing belief in literature [7], [8], matched

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[1]Note that the Doppler scale is the factor by which the frequencies in the signal are multiplied. One could thus interpret the Doppler scale as a frequency-dependent Doppler shift.

filtering outperforms the FrFT method for channel parameter estimation.

II. Linear frequency modulated preamble

A linear frequency modulated (LFM) preamble is a signal of the form

$$s(t) = w(t)e^{j\varphi_s(t)},$$

where $w(t)$ is a rectangular window having support $(0, T)$ and $\varphi_s(t)$ is the phase given by

$$\varphi_s(t) = 2\pi f_0 t + \pi k t^2 + \varphi_0.$$

Here f_0, k and φ_0 are the starting frequency, the modulation slope and the initial phase, respectively. With this, the (time-varying) instantaneous (ordinary) frequency of $s(t)$ is given by

$$f_s(t) = \frac{1}{2\pi}\frac{\mathrm{d}\varphi_s(t)}{\mathrm{d}t} = f_0 + kt, \quad t \in (0, T).$$

Within the time interval $(0, T)$, the frequency will linearly increase from the starting frequency f_0 to the stopping frequency $f_1 = f_0 + kT$.

During propagation, the preamble is affected by the channel. Each path will undergo a certain attenuation, delay and Doppler scale. Since we are primarily interested in estimating the delay and Doppler scale, we will ignore the attenuation. Typically, the parameters of the different paths are found one by one (matching pursuit based), so that we will focus in this paper on the estimation of the channel parameters of one single path. That is, the received signal $r(t)$ is of the form

$$r(t) = s(\alpha_0(t - \tau_0)) + n(t),$$

where τ_0 and α_0 denote the delay and Doppler scale of the channel, respectively, and $n(t)$ denotes an additive noise term.

Ignoring the additive noise component, the received signal is still an LFM signal, as

$$\begin{aligned} s(\alpha_0(t-\tau_0)) &= \\ &= w(\alpha_0(t-\tau_0))e^{j(2\pi f_0\alpha_0(t-\tau_0)+\pi k\alpha_0^2(t-\tau_0)^2+\varphi_0)} \\ &= w(\alpha_0(t-\tau_0))e^{j(2\pi f_0' t+\pi k' t^2+\varphi_0')}, \end{aligned}$$

where

$$\begin{aligned} f_0' &= f_0\alpha_0 - k\alpha_0^2\tau_0 \\ k' &= k\alpha_0^2 \\ \varphi_0' &= -2\pi f_0\alpha_0\tau_0 + \pi k\alpha_0^2\tau_0^2 + \varphi_0. \end{aligned}$$

Note that the support of the received signal is $(\tau_0, \tau_0 + T/\alpha_0)$ and that the instantaneous frequency is given by

$$f_r(t) = \frac{1}{2\pi}\frac{\mathrm{d}\varphi_r(t)}{\mathrm{d}t} = f_0' + k't = f_0\alpha_0 - k\alpha_0^2\tau_0 + k\alpha_0^2 t.$$

III. Channel Parameter Estimation

In this section we discuss two methods to estimate the channel parameters, namely a two-dimensional (delay and Doppler scale) matched filter and the FrFT.

A. Continuous Matched Filter

A two-dimensional matched filter correlates the received signal $r(t)$ with a predefined delayed and Doppler scaled version of the LFM signal $s(t)$. That is, the output of the matched filter is defined as

$$\begin{aligned} y_{\mathrm{MF}}(\tau,\alpha) &= \int_{-\infty}^{+\infty} s^*(\alpha(t-\tau))r(t)\mathrm{d}t \\ &= \int_{\tau}^{\tau+\frac{1}{\alpha}T} e^{-j(2\pi f_0\alpha(t-\tau)+\pi k(\alpha(t-\tau))^2+\varphi_0)} r(t)\mathrm{d}t. \end{aligned} \quad (1)$$

Note that when the noise in the channel is ignored, (1) is equivalent to the well known wideband ambiguity function [9] up to a multiplicative factor $\sqrt{\alpha}$. We can define an estimator for the channel parameters based on the two-dimensional matched filter as

$$(\hat{\alpha}_0, \hat{\tau}_0) = \arg\max_{\alpha,\tau} |y_{\mathrm{MF}}(\tau,\alpha)|.$$

B. Existing method using the FrFT

Zhao *et al.* [7] proposed a channel estimation method using a two-step approach; the FrFT in conjunction with a one-dimensional (delay) matched filter. We will see that the FrFT is an optimization done over two variables, of which in the end only one is used. For that reason a (redundant) second step is needed. More details about this redundancy will be described in Section III-C. For the sake of completeness, we present a summary of the procedure of Zhao *et al.* [7] here.

1) FrFT step: The FrFT is defined as [10]

$$\begin{aligned} &\mathcal{F}_\phi[r](u) = \\ &\sqrt{1 - j\cot(\phi)} \int_{-\infty}^{\infty} e^{j\pi\left((t^2+u^2)\cot(\phi) - 2ut\csc(\phi)\right)} r(t)\mathrm{d}t. \end{aligned} \quad (2)$$

Fig. 1 shows an example of the FrFT for three LFM signals in the time-frequency domain that have been disturbed by different channel parameters. The FrFT can be understood as projecting the signal onto a new axis that forms a (so-called fractional) angle ϕ with respect to the time axis. In the first step, for each (disturbed) LFM signal, the "optimal fractional angle" ϕ is estimated by maximizing its projection on the fractional Fourier axis. Although an iterative approach for finding ϕ was proposed in [7], the goal is to find

$$(\hat{\phi}_0, \hat{u}_0) = \arg\max_{\phi,u} |\mathcal{F}_\phi[r](u)|. \quad (3)$$

As visualized in Fig. 2, see the upper (blue) triangle, the optimal fractional angle is directly related to the Doppler scale by[2]

$$-\cot(\phi_0) = k' = \alpha_0^2 k. \quad (4)$$

Hence, the relation between the estimated parameters $\hat{\phi}_0$ and $\hat{\alpha}_0$ is given by

$$\hat{\alpha}_0 = \sqrt{-\frac{\cot(\hat{\phi}_0)}{k}}. \quad (5)$$

[2]Note that ϕ is defined positive counter clockwise, hence the minus sign in (4).

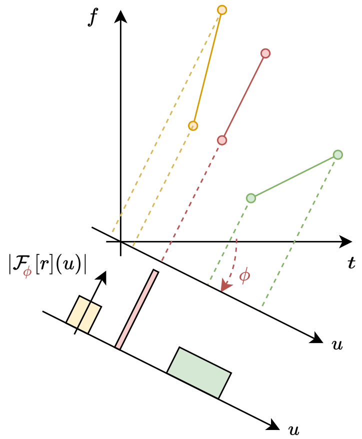

Fig. 1: Fractional Fourier Transform of a LFM signal.

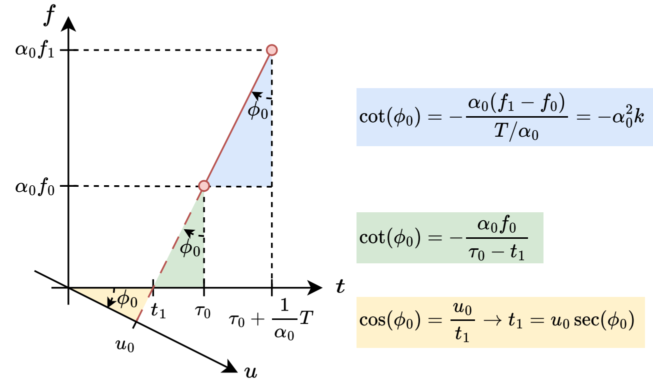

Fig. 2: Parameter relations based on the received LFM signal, with parameters $\tau = \tau_0$, $\alpha = \alpha_0$, $\phi = \phi_0$ and $u = u_0$.

2) Matched filter step: In [7], given the estimated Doppler scale $\hat{\alpha}_0$, the delay was proposed to be estimated using a one-dimensional matched filter as

$$\hat{\tau}_0 = \arg\max_{\tau} |y_{\text{MF}}(\tau, \hat{\alpha}_0)|.$$

C. Proposed Improvement

We can eliminate the second (matched filter) step since the information about the delay τ_0 is already available in the location u_0 of the peak of the FrFT spectrum. By eliminating this second step, the computational costs of the estimation procedure can be reduced. Fig. 2, in particular the middle (green) and lower (yellow) triangles, shows the geometrical relationship between u_0 and τ_0. We have

$$-\cot(\phi_0) = \frac{\alpha_0 f_0}{\tau_0 - t_1} = \frac{\alpha_0 f_0}{\tau_0 - u_0 \sec(\phi_0)}. \quad (6)$$

Note that Huang *et al.* [8] also mentioned the relationship between u_0 and τ_0 and that they defined an expression akin to (III-C) (cf. [8], eq. (16)). However, (6) is simpler and does not involve sampling parameters.

Summarizing, after solving (3), we obtain $\hat{\alpha}_0$ from (5), and $\hat{\tau}_0$ using (6) as

$$\hat{\tau}_0 = \hat{u}_0 \sec(\hat{\phi}_0) - \hat{\alpha}_0 f_0 \tan(\hat{\phi}_0).$$

IV. Relation Between MF and FrFT

In this section we will show that, in contrast to what is reported in [7] and [8], the matched filter and the FrFT approach are highly related and will result in similar results. Additionally, we will explain that the reported improvements (in [7] and [8]) of the FrFT method over matched filtering are only due to a difference in gridding and the use of iterative approaches, and is not an inherent property of the FrFT.

A. Derivation of the relation

By inspection of (1), we conclude that

$$\begin{aligned} |y_{\text{MF}}(\tau, \alpha)| &= \left| \int_{\tau}^{\tau + \frac{1}{\alpha}T} e^{-j(2\pi f_0 \alpha(t-\tau) + \pi k(\alpha(t-\tau))^2 + \varphi_0)} r(t) \mathrm{d}t \right| \\ &= \left| \int_{\tau}^{\tau + \frac{1}{\alpha}T} e^{-j\pi(2(\alpha f_0 - k\alpha^2\tau)t + k\alpha^2 t^2)} r(t) \mathrm{d}t \right| \\ &= \left| \int_{\tau}^{\tau + \frac{1}{\alpha}T} K_{\text{MF}}(t, \tau, \alpha) r(t) \mathrm{d}t \right|, \end{aligned}$$

where

$$K_{\text{MF}}(t, \tau, \alpha) = e^{-j\pi(2(\alpha f_0 - k\alpha^2\tau)t + k\alpha^2 t^2)},$$

is the kernel function of the integral transform

$$(T_{\text{MF}} r)(\tau, \alpha) = \int_{\tau}^{\tau + \frac{1}{\alpha}T} r(t) K_{\text{MF}}(t, \tau, \alpha) \mathrm{d}t.$$

Similarly, by inspection of (2), we have

$$\begin{aligned} &\left|\mathcal{F}_{\phi}[r](u)\right| = \\ &= \left| \sqrt{1 - j\cot(\phi)} \int_{-\infty}^{\infty} e^{j\pi\left((t^2+u^2)\cot(\phi) - 2ut\csc(\phi)\right)} r(t) \mathrm{d}t \right| \\ &= \left| \sqrt{1 - j\cot(\phi)} \right| \left| \int_{-\infty}^{\infty} e^{-j\pi(2ut\csc(\phi) - \cot(\phi)t^2} r(t) \mathrm{d}t \right| \\ &= \left| \sqrt{1 - j\cot(\phi)} \right| \left| \int_{-\infty}^{\infty} K_{\text{FT}}(t, \phi, u) r(t) \mathrm{d}t \right|, \end{aligned}$$

where

$$K_{\text{FT}}(t, \phi, u) = e^{-j\pi(2ut\csc(\phi) - \cot(\phi)t^2)},$$

is the kernel function of the integral transform

$$(T_{\text{FT}} r)(\phi, u) = \int_{-\infty}^{\infty} r(t) K_{\text{FT}}(t, \phi, u) \mathrm{d}t.$$

It turns out that the two kernel functions are identical, that is $K_{\text{MF}}(t, \alpha, \tau) = K_{\text{FT}}(t, \phi, u)$.

To show that these two kernel functions are identical, we observe from Fig. 2, by combining the two expressions for $\cot(\phi)$, that

$$\alpha f_0 - k\alpha^2\tau = -k\alpha^2 t_1.$$

Moreover, since $t_1 = u\sec(\phi)$, we obtain that

$$\alpha f_0 - k\alpha^2\tau = -k\alpha^2 u\sec(\phi) = u\csc(\phi),$$

where the last equality follows from (4). Hence, we have

$$\begin{aligned} K_{\text{MF}}(t,\tau,\alpha) &= e^{-j\pi\left(2(\alpha f_0 - k\alpha^2\tau)t + k\alpha^2 t^2\right)} \\ &= e^{-j\pi(2ut\csc(\phi) - \cot(\phi)t^2)} \\ &= K_{\text{FT}}(t,\phi,u), \end{aligned} \quad (7)$$

as required.

B. Interpretation of the relation

Since both kernel functions are identical, computing the FrFT can be viewed as matched filtering the received signal with an LFM signal. Except for the scaling by $|\sqrt{1 - j\cot(\phi)}|$, which is known, the two methods only differ from each other in the integration interval. That is, the matched filter method correlates the received signal with a *windowed* LMF signal (having support $(\tau, \tau + T/\tau)$), whereas the FrFT method correlates the received signal with an LFM signal having a support equal to the interval over which the correlation is computed, typically the symbol length plus the size of a guard interval. As a consequence, the FrFT method also correlates the received signal outside the region of interest, which contains noise and possible prefix information. This information will, in general, negatively influence the estimation performance, as confirmed by the simulation results presented in the next section.

V. Numerical Simulation

In order to numerically validate the above findings, we performed computer simulations. To ensure a fair comparison between the two methods, we used the proposed FrFT-based estimation scheme as described in Section III-C and evaluated both methods on a (two-dimensional) grid where the grid points were related through (4) and (6).

The simulations were done using an LFM signal and channel (single-path model) with parameters as the ones used in [7], which are summarized in Table I. These parameters are representative for an underwater acoustic communication scenario. For the guard interval we used zeros. The guard interval is longer than the maximum delay in the channel such that there is no interference from the other communication frames.

TABLE I
Parameter settings for the computer simulations.

True LFM signal		Grids		Channel	
f_0	5 [kHz]	M	21	α	$\mathcal{U}(1, 1.02)$
f_1	15 [kHz]	L	4000	τ	$\mathcal{U}(0, 0.25T)$ [s]
T	50 [ms]	$\Delta_{\hat{\alpha}}$	0.001	n(t)	$\mathcal{CN}(0, \sigma_n^2)$
T_s	$1.25 \cdot 10^{-2}$ [ms]	$\Delta_{\hat{\tau}}$	T_s		

We performed 100 Monte Carlo runs. The resulting normalized mean squared-errors of the Doppler scale $\mathbb{E}\|\alpha_0 - \hat{\alpha}\|^2$ and time delay $\mathbb{E}\|\tau_0 - \hat{\tau}\|^2$ are shown in Fig. 3a and b, respectively, where $\mathbb{E}(\cdot)$ denotes statistical expectation. From the results we can draw the following conclusions.

- Overall, the matched filter shows slightly better results. Previous papers, including [7] and [8], did not use equal grids for both methods or used iterative methods for only one of them, leading to an incorrect conclusion that the FrFT approach outperforms the matched filtering method.
- For high SNR values, the two methods give identical results, confirming the relationship shown in Section IV.
- The estimation errors reach a plateau at high SNR values, which is due to a finite sampling of the parameter space. Refining the sampling grid will lower both estimation errors.

The computational complexity of both methods is comparable since both have efficient implementations; the fast Fourier transform (FFT) for matched filtering and the fast fractional Fourier transform for the FrFT [11]. As a consequence, both methods have a computational complexity of $\mathcal{O}(L\log L)$, where L is the FFT or FrFT size.

VI. Conclusion

In this paper, we proposed an improvement on the existing method from [7], which is based on the FrFT, to estimate the channel parameters of an underwater acoustic channel in a wideband scenario. Additionally, we compared the channel parameter estimation performance of the FrFT and matched filtering using an LFM preamble. We showed that the two methods are highly related in the sense that, as viewed as integral transforms, both have identical kernels and only differ from each other in the integration interval, which is sub-optimal in the FrFT case. Computer simulations supported the conclusion that matched filtering is superior to the FrFT approach. This is in contrast to previously reported results in literature, which were obtained using unequal sampling grids and iterative methods.

VII. Acknowledgement

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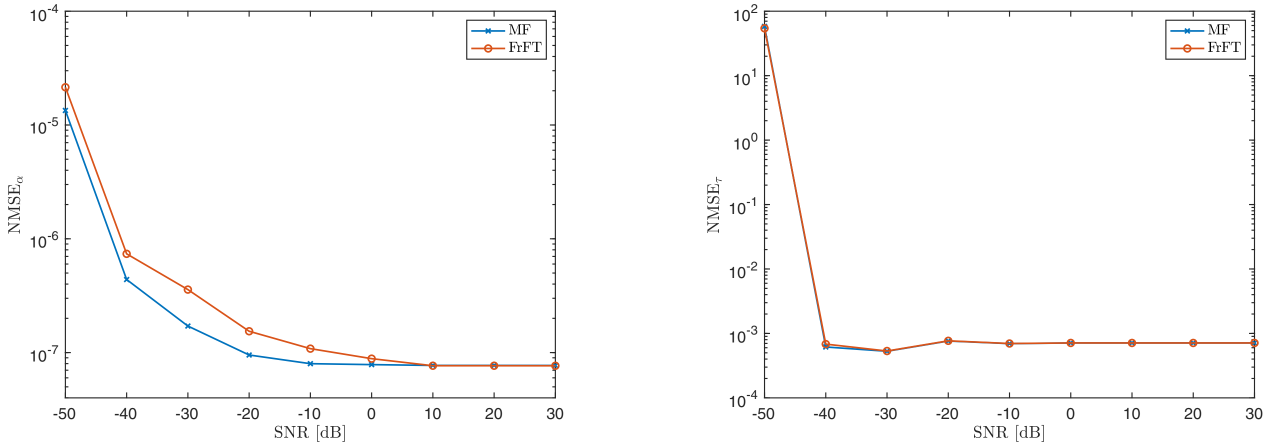

(a) Doppler scale

(b) Time delay

Fig. 3: Channel parameter estimation performance of the FrFT and matched filter.

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