Low Complexity PMI Selection for BICM-MIMO Rate Maximization in 5G New Radio Systems

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***Abstract*—This paper presents novel methods for selecting the best precoding matrix index (PMI) from the Type-I codebook adopted in 5G New Radio (5G NR), in terms of the achievable rate for MIMO-BICM systems. To overcome the complexity of dealing with a multi-variable problem with discrete domains, we introduce heuristic algorithms that exploit the Kronecker and DFT structure of the codebook. Our proposed methods utilize a combination of direct estimation and a low-dimensional search to derive the optimal PMI indices, and the singular value (SV) precoder serves as an optimal reference. The approach significantly reduces the number of codebook precoder candidates, resulting in a much lower complexity compared to the exhaustive search methods. Simulation results demonstrate the effectiveness of our proposed algorithms in achieving a performance comparable to the performance obtained by an exhaustive search.**

***Index Terms*—BICM-MIMO, 5G NR, Precoder Matrix Index (PMI), channel state information (CSI), Type-I Codebook.**

I. Introduction

In the closed-loop multiple-input multiple-output (MIMO) technology, the channel state information (CSI) feedback from the user equipment (UE) is crucial to perform precoding according to the downlink CSI and achieve the beamforming gains [1]. Codebook-based precoding is adopted within 5G New Radio (5G NR) systems to reduce the feedback complexity and overhead, where the UE reports the precoder matrix index (PMI) to the gNodeB (gNB). The best PMI is determined by a brute-forth search in the codebook to select the precoder that maximizes the channel capacity or another related metric [2]. However, the codebook size scales with the number of antennas and as the MIMO dimensions increase, the complexity of an exhaustive search for all PMI candidates becomes prohibitive.

Several PMI selection methods have been proposed in the literature that exploit the exhaustive search methods. For instance, in [3], a link adaptation and PMI selection method is presented based on mutual information maximization, while [4] considers joint PMI/RI selection using the MMSE equalizer. A different approach was proposed in [5], where the authors estimated the best PMI directly from the singular vectors of the channel matrix by exploiting the DFT structure, utilizing an iterative linear phase estimation (ILPE) and maximal ratio combining (MRC). However, these methods were designed for LTE and legacy systems, and may not meet the requirements of 5G NR. To address this issue, a recent study [6] proposes a neural network-based method for direct precoder matrix selection from the Type-I codebook in 5G NR, with the goal of maximizing the corresponding channel capacity.

Motivated by this, our objective is to develop a low-complexity algorithm that selects the optimal precoding matrix and its corresponding PMI indices from the Type-I codebook in 5G NR. Specifically, we aim to maximize the bit-interleaved coded modulation (BICM) rate, which is widely used in 5G NR systems for achieving high data transmission. However, the BICM rate cannot be expressed in a closed-form, and the variables are drawn from a set of discrete values, making the problem of BICM rate maximization challenging to solve through conventional precoding design procedures. To address this, we propose novel heuristic techniques that utilize the channel covariance matrix for PMI selection. Our approach involves a two-stage procedure, where some PMI indices are derived directly in the first stage, and the remaining ones are found by evaluating a cost function in the second stage. This reduces the number of candidate codebook precoders that need to be inspected for determining the best PMI indices.

We present two algorithms for PMI selection. The first algorithm is a tensor-based solution that utilizes the Kronecker structure of the codebook precoders. The second algorithm derives the PMI indices by minimizing the chordal distance of the codebook precoder from the optimal singular value (SV)-based precoder as the reference precoder. We compare our algorithms' performance against the exhaustive search over the codebook precoders, maximizing two metrics of the BICM rate and MMSE capacity. Our proposed methods offer a promising solution to the problem of PMI selection for 5G NR systems.

II. System model and problem statement

In a 5G NR system, a gNB with M_T antennas sends N_L layers to a receiver with M_R antennas. The antenna panel has N_1 horizontal and N_2 vertical antennas with cross-polarization, making $M_T = 2N_1N_2$. The payload bit-stream is divided and processed by a BICM scheme with LDPC, a capacity-approaching code. Assuming linear precoding, the frequency-domain received signal can be expressed as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{P}\boldsymbol{s} + \boldsymbol{n} \in \mathbb{C}^{M_R}, \tag{1}$$

where $\boldsymbol{H} \in \mathbb{C}^{M_R \times M_T}$ represents the MIMO channel matrix, $\boldsymbol{P} \in \mathbb{C}^{M_T \times N_L}$ the precoding matrix, and $\boldsymbol{n} \in \mathbb{C}^{M_R}$ the additive noise vector with zero mean and covariance matrix $\sigma^2 \boldsymbol{I}$. The transmitted symbol vector $\boldsymbol{s} \in \mathcal{M}^{N_L}$ is taken from the symbol alphabet $\mathcal{M}$.

With the assumption of $E\left(\boldsymbol{s}\boldsymbol{s}^H\right) = \boldsymbol{I}$, the precoding design problem, which maximizes a given cost function $C(\boldsymbol{P})$ with a power constraint, is defined as

$$\begin{aligned} \max_{\boldsymbol{P} \in \mathcal{V}} \quad & C(\boldsymbol{P}) \\ \text{s.t.} \quad & \operatorname{tr}\left(\boldsymbol{P}^H \boldsymbol{P}\right) = \gamma \end{aligned} \tag{2}$$

where γ is the maximum transmit power and $\mathcal{V}$ is the type-I codebook.

To incorporate the modulation-coding scheme into the precoder optimization process, it is necessary to consider an appropriate optimization metric, such as the BICM mutual information. By using the optimum soft detection at the receiver, the intrinsic log-likelihood ratio (LLR) value for each b_{iq}, which denotes the q^{th} bit in the i^{th} stream, for a given constellation size of $|\mathcal{M}| = 2^Q$, is obtained as

$$L_{iq} = \ln \frac{p\left(b_{iq} = 1 \middle| \boldsymbol{y}, \bar{\boldsymbol{H}}\right)}{p\left(b_{iq} = 0 \middle| \boldsymbol{y}, \bar{\boldsymbol{H}}\right)} = \ln \frac{\sum\limits_{\boldsymbol{s} \in \mathcal{X}_{iq}^{o}} \exp(-\frac{\left\|\boldsymbol{y} - \bar{\boldsymbol{H}}\boldsymbol{s}\right\|^2}{\sigma^2})}{\sum\limits_{\boldsymbol{s} \in \mathcal{X}_{iq}^{1}} \exp(-\frac{\left\|\boldsymbol{y} - \bar{\boldsymbol{H}}\boldsymbol{s}\right\|^2}{\sigma^2})}, \tag{3}$$

where we have defined the equivalent channel $\bar{\boldsymbol{H}} \triangleq \boldsymbol{H}\boldsymbol{P}$, $p\left(b_{iq} \middle| \boldsymbol{y}, \bar{\boldsymbol{H}}\right)$ is the probability mass function of b_{iq} given $\left(\boldsymbol{y}, \bar{\boldsymbol{H}}\right)$ and $\mathcal{X}_{iq}^{b}$ denotes the set of transmit vector given $b_{iq} = b$. Afterwards, the estimated symbols based on LLR values are demodulated, de-interleaved, and decoded, yielding the estimated payload bitstream.

In this BICM-MIMO channel, we neglect the correlation among different binary-input channels and for i.i.d. uniform code bits, the mutual information between bits and the corresponding LLRs is obtained as

$$C_{\text{BICM}}(\boldsymbol{P}) = \sum_{i=1}^{N_L} \sum_{q=1}^{Q} \left(1 - \mathcal{H}\left(b_{iq} \middle| \boldsymbol{y}, \bar{\boldsymbol{H}}\right)\right), \tag{4}$$

where the conditional entropy is given as

$$\mathcal{H}\left(b_{iq} \middle| \boldsymbol{y}, \bar{\boldsymbol{H}}\right) = \mathbb{E}\left(\log_2\left(1 + e^{(1 - 2b_{iq})L_{iq}}\right)\right), \tag{5}$$

in which the expectation is over (b_{iq}, L_{iq}).

To solve the optimization problem (2) with the cost function in (4), one can search through all possible precoder candidates in the codebook. However, evaluating the BICM rate using Monte Carlo simulations or numerical methods can be computationally burdensome. To simplify the process, a less complex baseline metric is needed. One option is to use MIMO demodulators with a linear equalizer followed by per-layer scalar soft demodulators to approximate the log-likelihood criterion for decoding. By using an minimum mean square error (MMSE) equalizer to decouple the layers, the post-MMSE SINR of the l^{th} output layer can be expressed as [7]

$$\gamma_l = \frac{1}{\sigma^2 \left\{\left(\boldsymbol{P}^H \boldsymbol{H}^H \boldsymbol{H} \boldsymbol{P} + \sigma^2 \boldsymbol{I}_{N_L}\right)^{-1}\right\}_{l,l}} - 1, \tag{6}$$

where, $\{.\}_{l,l}$ denotes l^{th} diagonal element of the inverse matrix.Therefore, the total achievable capacity across N_L sub-channels could be utilized as the cost function for exploring the codebook. This is expressed as

$$C_{\text{MMSE}}(\boldsymbol{P}) = \sum_{l=1}^{N_L} \log_2\left(1 + \gamma_l\right). \tag{7}$$

The PMI specified to the precoder that maximizes the post-equalization mutual information will be derived for the CSI feedback.

III. The codebook type-I single-panel structure

The precoder matrix $\boldsymbol{W}$ for type-I single-panel CSI can be expressed as $\boldsymbol{W} = \boldsymbol{W}_1 \boldsymbol{W}_2$, with $\boldsymbol{W}_1 \in \mathbb{C}^{2N_1N_2 \times 2L}$ and $\boldsymbol{W}_2 \in \mathbb{C}^{2L \times \nu}$, where $L = \lceil \frac{\nu}{2} \rceil$. $\boldsymbol{W}_1$ targets wideband and long-term channel properties, while $\boldsymbol{W}_2$ represents the subband and frequency dependent part of the channel. Type-I codebooks support precoding matrices up to rank 8, reported as the rank indicator $\nu \in \{1, 2, ..., 8\}$ to the gNB [8].

The matrix $\boldsymbol{W}_1$ defines a beam or group of beams pointing in various directions and can be expressed as:

$$\boldsymbol{W}_1 = \boldsymbol{I}_2 \otimes \boldsymbol{B}, \tag{8}$$

where, the first column of $\boldsymbol{B} \in \mathbb{C}^{N_1N_2 \times L}$, denoted by $\boldsymbol{v}_{l,m}$ (or $\tilde{\boldsymbol{v}}_{l,m}$ for $\nu \in \{3, 4\}$ and $M_T \geq 16$), can be expressed as the Kronecker product of two column vectors, given as

$$\begin{cases} \tilde{\boldsymbol{v}}_{l,m} = \tilde{\boldsymbol{v}}_l \otimes \boldsymbol{u}_m, & \text{if } \nu = 3 \text{ or } 4 \text{ and } M_T \geq 16 \\ \boldsymbol{v}_{l,m} = \boldsymbol{v}_l \otimes \boldsymbol{u}_m, & \text{otherwise,} \end{cases} \tag{9}$$

where

$$\begin{aligned} \boldsymbol{v}_l &= \begin{bmatrix} 1 & e^{j\frac{2\pi l}{O_1 N_1}} & \dots & e^{j\frac{2\pi l(N_1 - 1)}{O_1 N_1}} \end{bmatrix}^T, \\ \boldsymbol{u}_m &= \begin{bmatrix} 1 & e^{j\frac{2\pi m}{O_2 N_2}} & \dots & e^{j\frac{2\pi m(N_2 - 1)}{O_2 N_2}} \end{bmatrix}^T, \\ \tilde{\boldsymbol{v}}_l &= \begin{bmatrix} 1 & e^{j\frac{4\pi l}{O_1 N_1}} & \dots & e^{j\frac{4\pi l(N_1/2 - 1)}{O_1 N_1}} \end{bmatrix}^T, \end{aligned} \tag{10}$$

and the values of l and m have been determined in [9]. The parameters O_1 and O_2 are the over-sampling factors for the horizontal and the vertical directions, respectively. For higher ranks, $\nu > 1$, the rest of the columns could be defined similarly as $\boldsymbol{v}_{l',m'} = \boldsymbol{v}_{l'} \otimes \boldsymbol{u}_{m'}$, $\boldsymbol{v}_{l'',m''} = \boldsymbol{v}_{l''} \otimes \boldsymbol{u}_{m''}$ and $\boldsymbol{v}_{l''',m'''} = \boldsymbol{v}_{l'''} \otimes \boldsymbol{u}_{m'''}$, using coefficients l', l'', l''' and m', m'', m''', respectively.

The codebook precoder is generated using predefined indices known as PMI, which consist of either three or four indices depending on the supported rank. The precoder construction involves using the indices $i_{1,1}$ and $i_{1,2}$ to determine l and m, respectively. Moreover, the index $i_{1,3}$ is mapped to k_1 and k_2 to derive additional parameters l' and m' for ranks 2, 3, and 4. Similarly, for $\nu > 4$, discrete Fourier transform coefficients are derived by adding constant values to l and m. Assuming constants α and β are added to l and m, respectively, to obtain the DFT coefficients, we can define $\boldsymbol{v}_\alpha$ and $\boldsymbol{u}_\beta$ in a similar manner as shown in (10). Then, the second and subsequent columns of $\boldsymbol{B}$ can be derived from the first column by calculating element-wise Hadamard product between $\boldsymbol{v}_l$ (or $\boldsymbol{u}_m$) and $\boldsymbol{v}_\alpha$ (or $\boldsymbol{u}_\beta$). Thus, we have

$$\begin{aligned}\boldsymbol{v}_x &= \boldsymbol{v}_l \odot \boldsymbol{v}_\alpha = \boldsymbol{D}_\alpha \boldsymbol{v}_l,\\ \boldsymbol{u}_y &= \boldsymbol{u}_m \odot \boldsymbol{u}_\beta = \boldsymbol{D}_\beta \boldsymbol{u}_m.\end{aligned} \tag{11}$$

Here, $y \in \{m', m'', m'''\}$, $x \in \{l', l'', l'''\}$, $\boldsymbol{D}_\beta = \text{Diag}(\boldsymbol{u}_\beta)$ and $\boldsymbol{D}_\alpha = \text{Diag}(\boldsymbol{v}_\alpha)$, where $\text{Diag}(.)$ creates a diagonal matrix by placing the vector argument along its main diagonal. Finally, the corresponding column can be obtained by taking the Kronecker product of these two vectors as

$$\boldsymbol{v}_x \otimes \boldsymbol{u}_y = (\boldsymbol{D}_\alpha \boldsymbol{v}_l) \otimes (\boldsymbol{D}_\beta \boldsymbol{u}_m) = \boldsymbol{D}_{\alpha\beta} \boldsymbol{v}_{l,m} \tag{12}$$

where, $\boldsymbol{D}_{\alpha\beta} = \boldsymbol{D}_\alpha \otimes \boldsymbol{D}_\beta$. This shows the dependencies between the first column and the remaining columns of $\boldsymbol{B}$.

The $\boldsymbol{W}_2$ matrix chooses a group of DFT vectors from $\boldsymbol{W}_1$ and applies phase shifts over panels and/or polarizations. It can be expressed as the following Khatri-Rao product

$$\boldsymbol{W}_2 = \boldsymbol{\Psi} \diamond \boldsymbol{E}, \tag{13}$$

where the permutation matrix $\boldsymbol{E} \in \{0,1\}^{L\times\nu}$ selects the intended beam from the DFT column vectors of $\boldsymbol{B}$. The phase shifting between two polarizations is performed by $\boldsymbol{\Psi} = \begin{bmatrix}\boldsymbol{1}_\nu^T \\ \tilde{\boldsymbol{\varphi}}_\nu^T\end{bmatrix} \in \mathbb{C}^{2\times\nu}$, where $\boldsymbol{1}_\nu$ is a column vector of length ν with all elements equal to one, and another length-ν vector $\tilde{\boldsymbol{\varphi}}_\nu$ containing elements from the set $\{\pm 1, \pm\varphi_n\}$, with $\varphi_n = e^{\frac{j\pi n}{2}}$.

IV. Proposed algorithms for Precoder Design

To deal with this non-convex combinatorial optimization problem in (2), we have to devise a heuristic solution to reduce the computational complexity of the search and find optimal or near optimal solutions.

Let us have N observations of a complex-valued sinusoidal signal buried in noise

$$x(k) = a e^{j\phi k} + \epsilon(k), \qquad k = 0, 1, ..., N-1 \tag{14}$$

where a is a complex-valued amplitude, and $\phi \in (-\pi, \pi)$ is the normalized phase and ϵ is the noise term. Using N samples of the noisy signal $x(k)$, the covariance method of linear prediction is employed for the phase estimation and the first order estimator is

$$\hat{\phi} = \angle\left(\sum_{k=0}^{N-2} x^*(k)x(k+1)\right) \mod (2\pi), \tag{15}$$

which is called linear frequency estimation in [10]- [11] and linear phase estimation (LPE) in [5].

By defining $\theta_l \triangleq \frac{2\pi l}{N_1 O_1}$ and $\theta_m \triangleq \frac{2\pi m}{N_2 O_2}$, two consistent DFT vectors in $\boldsymbol{v}_{l,m}$ can be represented as $\boldsymbol{v}_l = \begin{bmatrix}1 & e^{j\theta_l} & \cdots & e^{j(N_1-1)\theta_l}\end{bmatrix}^T$ and $\boldsymbol{v}_{l,m}$ can be represented as $\boldsymbol{u}_m = \begin{bmatrix}1 & e^{j\theta_m} & \cdots & e^{j(N_2-1)\theta_m}\end{bmatrix}^T$. Therefore, given a vector of noisy observations, our problem is to estimate the unknown phases θ_l, θ_m and φ_n, for which we propose to apply LPE technique.

A. Tensor Decomposition based PMI Selection

Our first proposed approach starts by noticing that the codebook precoder has a Kronecker structure which can be written from (8) and (13) as

$$\boldsymbol{W} = (\boldsymbol{I}_2 \otimes \boldsymbol{B})(\boldsymbol{\Psi} \diamond \boldsymbol{E}) = \boldsymbol{\Psi} \diamond \tilde{\boldsymbol{B}}, \tag{16}$$

where $\tilde{\boldsymbol{B}} = \boldsymbol{B}\boldsymbol{E}$. We observe that the precoder can be considered as the transposed 1-mode unfolding of a 3-way tensor of $\boldsymbol{\mathcal{W}} \in \mathbb{C}^{N_L \times N_1 N_2 \times 2}$ i.e., $\boldsymbol{W} = [\boldsymbol{\mathcal{W}}]_{(1)}^T$. It admits a canonical Polyadic (CP) decomposition model as

$$\boldsymbol{\mathcal{W}} = \boldsymbol{\mathcal{I}}_{3,N_L} \times_1 \boldsymbol{I}_{N_L} \times_2 \tilde{\boldsymbol{B}} \times_3 \boldsymbol{\Psi}, \tag{17}$$

where $\boldsymbol{\mathcal{I}}_{3,N_L} \in \mathbb{C}^{N_L \times N_L \times N_L}$ is a super-diagonal tensor with ones on the super diagonal. Therefore, the precoding matrix could be reformulated as

$$\boldsymbol{W} = \left(\boldsymbol{\Psi} \otimes \tilde{\boldsymbol{B}}\right)[\boldsymbol{\mathcal{I}}_{3,N_L}]_{(1)}^{\mathrm{T}}. \tag{18}$$

Since the SV-based precoding technique achieves capacity-approaching performance, the eigen vectors can serve as a basis to map to the codebook structure. We reshape the channel matrix into a three-way tensor based on the codebook precoder structure and utilize factor matrices to design sub-matrices $\boldsymbol{\Psi}$ and $\tilde{\boldsymbol{B}}$. The channel matrix's tensor representation, based on the higher-order SV decomposition (HOSVD) [12], can be expressed as

$$\boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{S}} \times_1 \boldsymbol{U}_1 \times_2 \boldsymbol{U}_2 \times_3 \boldsymbol{U}_3, \tag{19}$$

where $\boldsymbol{\mathcal{S}} \in \mathbb{C}^{M_R \times N_1 N_2 \times 2}$ is the core tensor and $\boldsymbol{U}_1 \in \mathbb{C}^{M_R \times M_R}$, $\boldsymbol{U}_2 \in \mathbb{C}^{N_1 N_2 \times N_1 N_2}$, and $\boldsymbol{U}_3 \in \mathbb{C}^{2\times 2}$ are the factor matrices. For $\nu \in \{3, 4\}$ with $M_T \geq 16$, we have $\boldsymbol{\mathcal{S}} \in \mathbb{C}^{M_R \times N_1 N_2/2 \times 4}$, $\boldsymbol{U}_2 \in \mathbb{C}^{N_1 N_2/2 \times N_1 N_2/2}$, and $\boldsymbol{U}_3 \in \mathbb{C}^{4\times 4}$. The channel matrix $\boldsymbol{H}$ is equivalent to 1-mode unfolding of $\boldsymbol{\mathcal{H}}$ that can be obtained as

$$\boldsymbol{H} = [\boldsymbol{\mathcal{H}}]_{(1)} = \boldsymbol{U}_1 [\boldsymbol{\mathcal{S}}]_{(1)} (\boldsymbol{U}_3 \otimes \boldsymbol{U}_2)^T. \tag{20}$$

By using the mixed-product property of the Kronecker product on (19) and considering a precoder matrix $\boldsymbol{P}$ from the codebook with the two-stage form of (18), we get the equivalent channel matrix as

$$\begin{aligned}\boldsymbol{H}\boldsymbol{P} &= \boldsymbol{U}_1 [\boldsymbol{\mathcal{S}}]_{(1)} (\boldsymbol{U}_3 \otimes \boldsymbol{U}_2)^T \left(\boldsymbol{\Psi} \otimes \tilde{\boldsymbol{B}}\right)[\boldsymbol{\mathcal{I}}_{3,N_L}]_{(1)}^{\mathrm{T}} \\ &= \boldsymbol{U}_1 [\boldsymbol{\mathcal{S}}]_{(1)} \left(\left(\boldsymbol{U}_3^T \boldsymbol{\Psi}\right) \otimes \left(\boldsymbol{U}_2^T \tilde{\boldsymbol{B}}\right)\right)[\boldsymbol{\mathcal{I}}_{3,N_L}]_{(1)}^{\mathrm{T}}.\end{aligned} \tag{21}$$

In order to send parallel sub streams, it is desired to have $\boldsymbol{U}_3^T \boldsymbol{\Psi} = \boldsymbol{I}_2$ and $\boldsymbol{U}_2^T \tilde{\boldsymbol{B}} = \boldsymbol{I}_{N_1 N_2}$. To meet this goal, the first column of $\boldsymbol{U}_2$ is considered to estimate the first column of $\tilde{\boldsymbol{B}}$, i.e., $\boldsymbol{v}_{l,m}$. Denoting the conjugate of the first column of $\boldsymbol{U}_2$ as $\dot{\boldsymbol{u}}_1$, we apply the LPE technique (15) for the relevant elements of each unknown phase θ_l and θ_m and obtain

$$\begin{aligned}\hat{\theta}_m &= \angle\left(\sum_{k=1}^{N_2-1} \dot{\boldsymbol{u}}_1^*(k)\, \dot{\boldsymbol{u}}_1(k+1)\right) \mod (2\pi), \\ \hat{\theta}_l &= \angle\left(\sum_{k=1}^{N_2} \dot{\boldsymbol{u}}_1^*(k)\, \dot{\boldsymbol{u}}_1(N_2+k)\right) \mod (2\pi).\end{aligned} \tag{22}$$

With defining the conjugate of first column of $\boldsymbol{U}_3$ as $\ddot{\boldsymbol{u}}_1$, we can estimate φ_n by inserting $\ddot{\boldsymbol{u}}_1$ into (15) as

$$\hat{\varphi}_n = \angle\left(\sum_{k=1}^{N_1} \ddot{\boldsymbol{u}}_1^*(k)\,\ddot{\boldsymbol{u}}_1(N_1N_2+k)\right) \mod (2\pi). \quad (23)$$

At this point, the DFT coefficients can be derived by a hard decision on the estimated phases as

$$\hat{m} = \left\lfloor \frac{O_2N_2}{2\pi}\hat{\theta}_m \right\rceil, \hat{l} = \left\lfloor \frac{O_1N_1}{2\pi}\hat{\theta}_l \right\rceil, \hat{n} = \left\lfloor \frac{2}{\pi}\hat{\varphi}_n \right\rceil. \quad (24)$$

It is required to map the estimated DFT coefficients $\hat{l}$, $\hat{m}$ and $\hat{n}$ to the corresponding PMI indices $i_{1,1}$, $i_{1,2}$ and i_2. The mapping rule depends on the number of transmit layers and the codebook mode and antenna configurations (tables 5.2.2.2.1-5 to 5.2.2.2.1-12 in [13]).

To find the best value of $i_{1,3}$ for achieving the best BICM rate, we evaluate codebook precoders $\hat{\boldsymbol{P}}_{i_{1,3}}$ for each permitted value of $i_{1,3}$, using the estimated parameters $\hat{i}_{1,1}$, $\hat{i}_{1,2}$ and $\hat{i}_2$. The optimal value of $i_{1,3}$ is selected as the PMI index corresponding to the precoder that maximizes the BICM rate

$$\hat{i}_{1,3} = \arg \max_{0 \le i_{1,3} \le I_{1,3}} C_{\text{BICM}}(\hat{\boldsymbol{P}}_{i_{1,3}}). \quad (25)$$

However, since the BICM rate cannot be directly evaluated through a closed form and the evaluation requires high complexity, alternative methods are necessary for selecting the optimal parameter $i_{1,3}$. One approach is to use mutual information (or capacity) as a selection criterion, which can be calculated using the equation $I(\boldsymbol{P}) = \log_2 \det\left(\boldsymbol{I}_{N_L} + \frac{1}{\sigma^2}\boldsymbol{P}^H\boldsymbol{H}^H\boldsymbol{H}\boldsymbol{P}\right)$ for a given $\boldsymbol{H}$ and fixed $\boldsymbol{P}$. Therefore, the selection criterion for the best $i_{1,3}$ is to maximize $\det\left(\hat{\boldsymbol{P}}_{i_{1,3}}^H\boldsymbol{H}^H\boldsymbol{H}\hat{\boldsymbol{P}}_{i_{1,3}}\right)$, which maximizes the mutual information.

Another appropriate selection metric for some system configurations could be maximization of the transmit power and the index $i_{1,3}$ which maximizes the transmit power, given by $\|\hat{\boldsymbol{P}}_{i_{1,3}}^H\boldsymbol{H}^H\boldsymbol{H}\hat{\boldsymbol{P}}_{i_{1,3}}\|_F$.

Based on the above steps we acquire the best PMI indices without requiring to search over all candidate precoders in the codebook and calculate the metric for each one. Our proposed method eliminates the search phase completely by directly deriving the PMI indices or reduce the number of search parameters to one index.

B. Chordal Distance Minimization PMI Selection

Let us define the singular value decomposition of the channel matrix as $\boldsymbol{H} = \boldsymbol{K}\boldsymbol{\Sigma}\boldsymbol{L}^H$, where $\boldsymbol{K} \in \mathbb{C}^{M_R\times M_R}$ and $\boldsymbol{L} \in \mathbb{C}^{M_T\times M_T}$ are unitary matrices and $\boldsymbol{\Sigma} \in \mathbb{C}^{M_R\times M_T}$ is the diagonal matrix containing the singular values of $\boldsymbol{H}$. We introduce the chordal distance (CD) as a metric to obtain the optimal rank-N_L SV-based precoder $\tilde{\boldsymbol{L}}$, represented by the first N_L columns of $\boldsymbol{L}$. For non-coherent MIMO systems, the chordal distance measures the distance between two codewords of an orthogonal code [14], and in this section represents the distance we aim to minimize between the optimal precoder and the codebook-based precoder. It is defined as

$$\begin{aligned}\|\boldsymbol{P}\boldsymbol{P}^H - \tilde{\boldsymbol{L}}\tilde{\boldsymbol{L}}^H\|_F^2 &= \|\boldsymbol{P}\boldsymbol{P}^H\|_F^2 + \|\tilde{\boldsymbol{L}}\tilde{\boldsymbol{L}}^H\|_F^2 - 2tr\left(\boldsymbol{P}\boldsymbol{P}^H\tilde{\boldsymbol{L}}\tilde{\boldsymbol{L}}^H\right)\\ &= \frac{1}{N_L} + N_L - 2tr\left(\boldsymbol{P}^H\tilde{\boldsymbol{L}}\tilde{\boldsymbol{L}}^H\boldsymbol{P}\right), \quad (26)\end{aligned}$$

where we have used this property that $\tilde{\boldsymbol{L}}$ and the codebook precoder $\boldsymbol{P}$ are unitary matrices. Hence, the optimization problem of the chordal distance minimization will be equivalent to maximization of the third term on the right-hand side of (26), formulated as

$$\max_{\boldsymbol{P}\in\mathcal{V}} \|\tilde{\boldsymbol{L}}^H\boldsymbol{P}\|_{\text{F}}^2. \quad (27)$$

Given the coupled columns of the codebook precoders, jointly optimizing the PMI indices is a challenging task. Thus, we solve the optimization problem to find the optimal $\boldsymbol{v}_{l,m}$ for a given i_2 and $i_{1,3}$ (when $N_L \in 2,3,4$), and then estimate $i_{1,1}$ and $i_{1,2}$. Subsequently, we evaluate the corresponding codebook precoders for different values of i_2 and $i_{1,3}$ to find the best (maximum) value of the considered cost functions using the procedure explained in the previous section. From (16), we can rewrite the objective function as

$$\tilde{\boldsymbol{L}}^H\boldsymbol{P} = \tilde{\boldsymbol{L}}^H\left(\boldsymbol{\Psi}\diamond\tilde{\boldsymbol{B}}\right) = \tilde{\boldsymbol{L}}^H\left(\boldsymbol{\Psi}\otimes\boldsymbol{I}_{N_1N_2}\right)\left(\boldsymbol{I}_{N_L}\diamond\tilde{\boldsymbol{B}}\right). \quad (28)$$

Using (12), the ith column of the codebook precoder can be expressed as $\left[\boldsymbol{D}_i\boldsymbol{v}_{l,m}^T \quad \pm\varphi_n\boldsymbol{D}_i\boldsymbol{v}_{l,m}^T\right]^T$. Then, if $\boldsymbol{\lambda}_{ij} \in \mathbb{C}^{N_1N_2\times 1}$ represents the i'th sub-vector of length N_1N_2 from j'th column of $\tilde{\boldsymbol{L}}^H(\boldsymbol{\Psi}\otimes\boldsymbol{I}_{N_1N_2})$, we can reformulate the objective function as

$$\|\tilde{\boldsymbol{L}}^H\boldsymbol{P}\|_F^2 = \sum_{i=1}^{N_L}\sum_{k=1}^{N_L}|\boldsymbol{\lambda}_{ik}^T\boldsymbol{D}_k\boldsymbol{v}_{l,m}|^2 = \boldsymbol{v}_{l,m}^H\boldsymbol{X}\boldsymbol{v}_{l,m}, \quad (29)$$

where we have defined $\boldsymbol{X} = \sum_{i=1}^{N_L}\sum_{k=1}^{N_L}\boldsymbol{D}_k^H\boldsymbol{\lambda}_{ik}^*\boldsymbol{\lambda}_{ik}^T\boldsymbol{D}_k$. This results in the simplified optimization problem as shown below

$$\arg\max_{\boldsymbol{v}_{l,m}} \quad \boldsymbol{v}_{l,m}^H\boldsymbol{X}\boldsymbol{v}_{l,m}, \quad (30)$$

where $\boldsymbol{X} \in \mathbb{C}^{N_1N_2\times N_1N_2}$ is a positive semi-definite matrix. By introducing the eigenvalue decomposition of $\boldsymbol{X}$ as $\boldsymbol{X} = \boldsymbol{U}_x\boldsymbol{\Sigma}_x\boldsymbol{U}_x$, the optimal continuous solution of this problem is straightforward and the maximizing $\boldsymbol{v}_{l,m}$ is given by the dominant (first) eigenvector of $\boldsymbol{X}$ i.e., $\hat{\boldsymbol{v}}_{l,m} = \boldsymbol{U}_{x(:,1)}$. Now, to derive the PMI indices $i_{1,1}$ and $i_{1,2}$ from the estimated $\hat{\boldsymbol{v}}_{l,m}$, we use the LPE technique in (15) to obtain the DFT coefficients $\hat{l}$ and $\hat{m}$ of the underlying $\boldsymbol{v}_l$ and $\boldsymbol{u}_m$ vectors, respectively. The process is repeated for various values of i_2 and $i_{1,3}$. The optimal combination of these values is determined by evaluating a cost function similar to (25). This can be achieved with lower complexity by maximizing the capacity or power, or by minimizing the chordal distance.

V. Simulation Results

We present simulation results in this section to evaluate the performance of our proposed PMI selection algorithms for the type-I single-panel codebook in BICM-MIMO systems. We compare the results of the proposed PMI selection schemes, employing different cost functions for determining the i_2 and $i_{1,3}$ indices, with two exhaustive search methods for selecting the optimal PMI that maximizes the achievable BICM rate and the MMSE capacity. We assume an LDPC coding gain of 0.65 with a codeword length of 4800 bits, where the input signal vector elements are drawn from an M-QAM constellation.

We consider a highly correlated Kronecker Rayleigh fading channel model with two-dimensional cross-polarized antennas at the gNB and the UE, as specified in [13]. The SNR is defined as SNR = $\frac{\gamma}{\sigma^2}$, where $\gamma = 1$. In a 24×4 BICM-MIMO system with $(N_1, N_2) = (4, 3)$, for each given SNR value, we conduct a comprehensive parameter tuning analysis to determine the optimal values for three key parameters: RI (which determines the number of layers, selected from $N_L \in \{1, 2, 3, 4\}$), modulation order (chosen from QPSK, 16-QAM, and 64-QAM schemes), and the PMI index (in type-I single panel codebook).

The simulation results demonstrate that the proposed PMI selection algorithms can achieve similar performance to the exhaustive search method based on BICM rate. Furthermore, in the high-SNR range, the proposed algorithms outperform the MMSE capacity based exhaustive search method. One key advantage of our proposed algorithms is the significant reduction in the number of search parameters, which can lead to much lower complexity. For instance, in a 12×4 MIMO system, the complexity is reduced by up to 100 times compared to the baseline exhaustive search solution. Another advantage is that the proposed algorithms are applicable for each given RI and can be utilized for RI selection and link adaption purposes. Additionally, by employing cost functions such as capacity, power, or chordal distance instead of the BICM rate, the proposed algorithms do not depend on the SNR value, which further reduces the computation complexity.

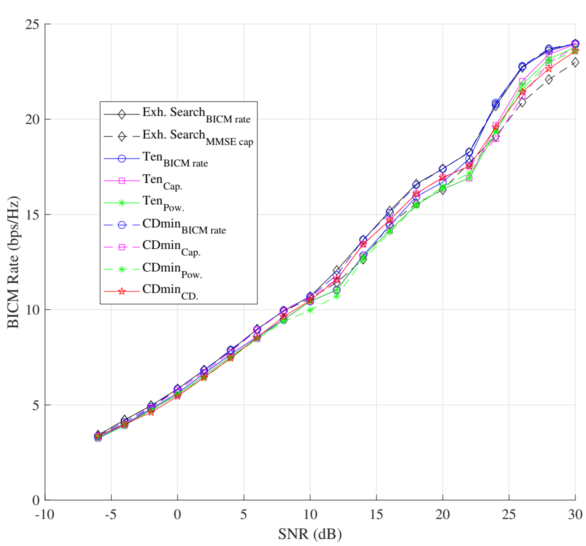

Fig. 1. BICM rate comparison of different algorithms for a 24×4 highly correlated MIMO-BICM system, with varying modulation orders and number of layers.

VI. Conclusion

For the MIMO-BICM system model based on 5G NR networks, in order to find the optimal PMI in terms of the BICM rate from the Type-I codebook, we proposed two low complexity precoding matrix selection techniques. The first solution finds the best PMI is the tensor-based algorithm exploiting the Kronecker structure of the codebook precoders and the second algorithm solves a chordal minimization between the codebook precoder and the optimal SVD based precoder.

The simulation results indicate that the proposed solutions can outperform the exhaustive search method based on the MMSE capacity and can reach the exhaustive search based on the BICM rate in Rayleigh correlated channels.

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References

[1] D. J. Love and R. W. Heath, "Limited feedback unitary precoding for spatial multiplexing systems," *IEEE Transactions on Information Theory*, vol. 51, no. 8, pp. 2967–2976, Aug. 2005.

[2] T. Tang, R. Doostnejad, and T. J. Lim, "Mean mutual information per coded bit based precoding in MIMO-OFDM systems," in *Proc. IEEE 72nd Vehicular Technology Conference-Fall*, 2010, pp. 1–5.

[3] S. Schwarz, M. Wrulich, and M. Rupp, "Mutual information based calculation of the precoding matrix indicator for 3GPP UMTS/LTE," in *Proc. International ITG Workshop on Smart Antennas*, 2010, pp. 52–58.

[4] W. Chen and S. Jin, "Performance evaluation of closed loop transmission for LTE-A uplink MIMO," in *Proc. 7th International Conference on Wireless Communications, Networking and Mobile Computing*, 2011, pp. 1–4.

[5] F. Penna, H. Cheng, and J. Lee, "A search-free algorithm for precoder selection in FD-MIMO systems with DFT-based codebooks," in *Proc. IEEE 86th Vehicular Technology Conference*. IEEE, 2017, pp. 1–6.

[6] T. Akyıldız and T. M. Duman, "Search-free precoder selection for 5G new radio using neural networks," in *Proc. IEEE International Black Sea Conference on Communications and Networking (BlackSeaCom)*, 2020, pp. 1–6.

[7] I. B. Collings, M. R. G. Butler, and M. McKay, "Low complexity receiver design for MIMO bit-interleaved coded modulation," in *Proc. 8th IEEE International Symposium on Spread Spectrum Techniques and Applications (ISSSTA)*, 2004, pp. 12–16.

[8] S. Ahmadi, *5G NR: Architecture, technology, implementation, and operation of 3GPP new radio standards*. Academic Press, 2019.

[9] 3rd Generation Partnership Project, "3GPP TS 38.214 V16.7.0 (2021-09): Technical Specification," 3rd Generation Partnership Project; Technical Specification Group Radio Access Network, Technical Specification TS 38.214, September 2021, release 16.

[10] L. Jackson, D. Tufts, F. Soong, and R. Rao, "Frequency estimation by linear prediction," in *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 3. IEEE, 1978, pp. 352–356.

[11] P. Händel and I. Kiss, "On correlation based single tone frequency estimation," in *Proc. Finnish Signal Processing Symposium*, 1997, pp. 32–36.

[12] M. Haardt, F. Roemer, and G. Del Galdo, "Higher-order svd-based subspace estimation to improve the parameter estimation accuracy in multidimensional harmonic retrieval problems," *IEEE Transactions on Signal Processing*, vol. 56, no. 7, pp. 3198–3213, Jul. 2008.

[13] ETSI, "LTE; evolved universal terrestrial radio access (E-UTRA); user equipment (UE) radio transmission and reception; part 101: User equipment (UE) radio transmission and reception (release 16)," ETSI, Tech. Rep. 136 101 v16.7.0, 2020.

[14] A. Barg and D. Y. Nogin, "Bounds on packings of spheres in the grassmann manifold," *IEEE Transactions on Information Theory*, vol. 48, no. 9, pp. 2450–2454, Sep. 2002.