Fast Subspace-based Semi-Blind Channel Estimation for MIMO-OFDM Communications

Kabiru N. Aliyu*, Abdulmajid Lawal*, Karim Abed-Meraim†, Azzedine Zerguine*
*EE Dept. and the Center for Communication Systems and Sensing, KFUPM, Dhahran, 31261, Saudi Arabia,
emails: {g201705110, abdulmajid.lawal, azzedine}@kfupm.edu.sa
†PRISME lab., Univ. Orléans, France, email: karim.abed-meraim@univ-orleans.fr

***Abstract*—This work addresses the issue of semi-blind (SB) subspace-based channel estimation when MIMO-OFDM communications systems are considered. The suggested solution primarily decreases the computational cost while ensuring reliable channel estimates. Covariance matrices and noise subspace are calculated for each subcarrier in a parallel manner by utilizing the OFDM modulation's orthogonality property. Pilots and data are then jointly used in a SB scheme, via a global (hybrid) cost function, to improve further the channel estimation accuracy as well as the information data throughput. To support our study, several numerical simulations are performed to investigate the requirements for channel identifiability and assess the performance and limitations of our SB method.**

***Index Terms*—MIMO, OFDM, subspace method, semi-blind channel estimation.**

I. Introduction

In recent years, much research has been done on wide-band wireless communications systems in response to applications that demand high data rates and high-quality services. To increase the capacity, data rate and quality of services, multiple inputs multiple outputs (MIMO) systems have been studied extensively and considered as promising technology for achieving spectrally efficient communications due to the spatial diversity and multiplexing gain [1], [2]. Likewise, using orthogonal frequency division multiplexing (OFDM), the entire channel can be transformed into a series of narrow-band, low-rate, parallel flat fading channels that allow simultaneous transmission of many data symbols immune to multipath while preserving a high data rate. Moreover, introducing cyclic prefix (CP) and Fourier transform removes the inter-carrier interference (ICI), inter-symbol interference (ISI) and reduces the number of required modulators and demodulators in the system [3]. MIMO and OFDM technology are combined to form a MIMO-OFDM system that became a leading technology in 4G and 5G wireless communications systems [4], [5].

The overall performance of MIMO-OFDM wireless technology depends heavily on the channel state information (CIS) estimation quality, which is difficult to obtain when large MIMO systems are considered [6]. The literature presented many channel estimation techniques for the MIMO-OFDM system. They can generally be divided into pilot-based, blind, and semi-blind (SB) approaches. The pilot-based estimation technique is the most widely used in wireless communications due to its simplicity, stability, and high accuracy. Thus, it uses extra resources to transmit known periodic pilot symbols that decrease the system's spectral efficiency. Blind channel estimation preserves spectral efficiency by using fully statistical properties of the transmitted data in which no pilot symbols are required. However, due to eigenvalue-eigenvector decomposition (EVD), the blind approach experiences an ambiguity problem [7]. Furthermore, semi-blind estimation combines the pilot-based and blind methods to address the ambiguity problem in the blind approach and gets the desired channel response [8].

Two main types of semi-blind approaches can be identified in the literature. The first type includes studies combining pilots and data symbols to enhance channel estimation performance. For instance, authors in [9] used subspace-based properties to estimate the channel. Meanwhile, authors in [10] suggest the decomposition of the channel matrix into a whitening and unitary rotational matrix. The second type concentrates on minimizing the number of pilot symbols to enhance spectral efficiency and decrease the transmission power using zero-samples to replace pilot symbols that can be replaced without deteriorating the estimation performance (green communications), see [6], [11], [12].

This work proposes a semi-blind channel estimation approach in the context of the MIMO-OFDM communications systems. The proposed algorithm aims to improve the convergence rate by decreasing computation complexity and providing reliable channel estimates using a small sample size. The proposed algorithm uses the orthogonal properties and covariance matrix structure of the received OFDM symbols for each subcarrier to estimate certain noise subspace vectors, then employs a global cost function to find out the channel coefficient. In addition, the performance evaluation of the proposed algorithm is provided and analysed in terms of number of considered subcarriers and number of pilots.

II. Problem Formulation

A. MIMO-OFDM system model

Figure 1 presented a MIMO-OFDM system composed of N_t, N_r transmit and receive antennas employing K subcarriers, whereby $N_r > N_t$. The data symbol transmitted comprises of K samples after carrying out K-point inverse

FFT, and it is extended by adding the last L samples as CP in its front. The maximum multipath channel delays N are assumed to be smaller than the CP size, i.e., $(N \leq L)$. At the receiving end, the r-th antenna receives OFDM symbols, after taking out the CP and performing the K-point FFT, the received symbol at the k-th subcarrier can be written as:

$$\mathbf{y}_k(t) = \mathcal{H}_k \mathbf{s}_k(t) + \boldsymbol{\omega}_k(t), \quad t = 1, \cdots, N_s \tag{1}$$

where $\mathcal{H}_k \in \mathbb{C}^{N_r \times N_t}$ is the channel matrix written as:

$$\mathcal{H}_k = \sum_{l=0}^{N-1} \mathbf{H}(l) e^{-j2\pi \frac{k}{K} l} = \mathbf{H}(\mathbf{e}_k^T \otimes \mathbf{I}_{N_t}), \tag{2}$$

where $\mathbf{e}_k = \left[1, e^{-j2\pi\frac{k}{K}} \ldots e^{-j2\pi\frac{k(N-1)}{K}}\right]$, $\mathbf{H} = \left[\mathbf{H}(0), \mathbf{H}(1) \ldots, \mathbf{H}(N-1)\right]$ and $\mathbf{H}(l) \in \mathbb{C}^{N_r \times N_t}$ is the l-th channel's tap matrix of the considered MIMO system.

The $N_t \times 1$ OFDM transmitted data $\mathbf{s}_k$ of the k-th subcarrier is considered to be i.i.d and has a covariance $\mathbf{R}_s = E[\mathbf{s}_k(t)\mathbf{s}_l^H(t)] = \delta_{kl} diag(\sigma_{s,1}^2, \ldots, \sigma_{s,N_t}^2)$ where δ_{kl} is the Kronecker delta operator and the i.i.d. $\boldsymbol{\omega}_k(t)$ is additive white Circular Gaussian (CG) noise having covariance $\mathbf{R}_\omega = E[\boldsymbol{\omega}_k(t)\boldsymbol{\omega}_l^H(t)] = \sigma_{\omega,k}^2 \delta_{kl} \mathbf{I}_{N_r}$. We assume that for all z, $\mathcal{H}(z) = \sum_{l=0}^{N-1} \mathbf{H}(l) z^{-l}$ is a full rank matrix.

B. Pilot-based channel estimation

The pilot-based technique depends on the training sequence and can be derived from the vectorized form of equation (1) when considering all sub-carriers as:

$$\mathbf{y} = \tilde{\mathbf{S}}\mathbf{h} + \boldsymbol{\omega}, \tag{3}$$

where $\mathbf{h} = \left[\mathbf{h}_1^T \cdots \mathbf{h}_{N_r}^T\right]^T$ with $\mathbf{h}_i = [\mathbf{H}_{:,i}^T(0), \cdots, \mathbf{H}_{:,i}^T(N)]^T$, is a MIMO channel taps with a size $(N_r N_t N \times 1)$. $\mathbf{S}_i = diag\{s_i\}$ is a $(K \times K)$ diagonal matrix; $\mathbf{S} = \left[\mathbf{S}_1\mathbf{F} \cdots \mathbf{S}_{N_t}\mathbf{F}\right]$ having $K \times NN_t$ size; and $\tilde{\mathbf{S}} = \mathbf{I}_{N_r} \otimes \mathbf{S}$ with $N_r K \times NN_t N_r$ size, and $\mathbf{F}$ is a submatrix of the Fourier matrix formed by its N first columns. The pilot-based estimator $\hat{\mathbf{h}}_{PB}$ can be found,

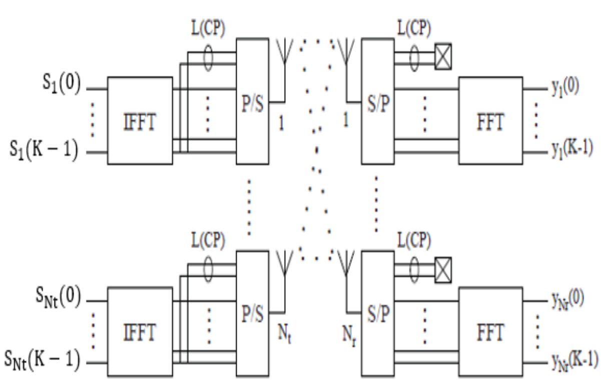

Fig. 1. MIMO-OFDM communications system

in our context, by minimizing the least squares cost function:

$$C(\mathbf{h}) = \|\tilde{\mathbf{y}}_p - \tilde{\mathbf{S}}_p \mathbf{h}\|^2, \tag{4}$$

where $\tilde{\mathbf{y}}_p = \left[\mathbf{y}(1)^T \cdots \mathbf{y}(N_p)^T\right]^T$ (index p stands for 'pilot' and N_p is the number of used pilots), and $\tilde{\mathbf{S}}_p = \left[\tilde{\mathbf{S}}(1)^T \cdots \tilde{\mathbf{S}}(N_p)^T\right]^T$. The $\hat{\mathbf{h}}_{PB}$ estimator can be expressed as:

$$\hat{\mathbf{h}}_{PB} = \left(\tilde{\mathbf{S}}_p^H \tilde{\mathbf{S}}_p\right)^{-1} \tilde{\mathbf{S}}_p^H \tilde{\mathbf{y}}_p. \tag{5}$$

The latter, will be used in the sequel for comparison as well as for the proposed semi-blind approach.

C. Subspace-based semi-blind method

Let's start by introducing the blind subspace channel identification method, recently provided in [13]. Indeed, numerous subspace methods have been introduced in the literature for channel equalization in MIMO-OFDM, but the one by Ouahbi *et al.* has the advantage of low-cost, parallelizable architecture, and efficiency for small or moderate sample sizes. The subspace methods are based on exploiting the orthogonality of the signal and noise subspace using appropriate covariance matrices of the received signal [14]–[16]. In [13], the noise subspaces are derived from the received signal covariance matrix $\mathbf{C}_k$ at the k-th subcarrier, given by:

$$\mathbf{C}_k = E(\mathbf{y}_k \mathbf{y}_k^H) = \mathcal{H}_k \mathbf{R}_s \mathcal{H}_k^H + \sigma_{\omega,k}^2 \mathbf{I}_{N_r}, \tag{6}$$

which is estimated from N_s received data by sample averaging as $\hat{\mathbf{C}}_k = \frac{1}{N_s} \sum_{t=0}^{N_s-1} \mathbf{y}_k(t)\mathbf{y}_k(t)^H$.

The noise subspace matrix at the k-th subcarrier, denoted $\mathbf{U}_k \in \mathbb{C}^{N_r - N_t}$, is obtained from the least $(N_r - N_t)$ eigenvectors of $\mathbf{C}_k$. The channel coefficients can be estimated blindly using the orthogonal relationship of signal and noise subspace according to:

$$\mathbf{U}_k^H \mathcal{H}_k = 0, \quad \forall k \tag{7}$$

However, using the following properties:

$$\text{vec}(\mathbf{U}_k^H \mathcal{H}_k) = (\mathbf{I}_{N_t} \otimes \mathbf{U}_k^H)\text{vec}(\mathcal{H}_k), \tag{8}$$

where vec(.) denotes the vectorization operator, $\otimes$ is the Kronecker product, and

$$\text{vec}(\mathcal{H}_k) = (\mathbf{e}_k \otimes \mathbf{I}_{N_t} \otimes \mathbf{I}_{N_r})\mathbf{h}, \tag{9}$$

we can rewrite equation (7) as:

$$\text{vec}(\mathbf{U}_k^H \mathcal{H}_k) = (\mathbf{e}_k \otimes \mathbf{I}_{N_t} \otimes \mathbf{U}_k^H)\mathbf{h} = \mathcal{A}_k \mathbf{h} = 0. \tag{10}$$

Therefore, by considering the K subcarriers and solving for (7) in the least squares sense, the following cost function is minimized for blind channel estimation:

$$C(\mathbf{h}) = \sum_{k=0}^{K-1} \|(\mathbf{e}_k \otimes \mathbf{I}_{N_t} \otimes \mathbf{U}_k^H)\mathbf{h}\|_F^2 = \mathbf{h}^H \mathbf{Q} \mathbf{h}, \tag{11}$$

where

$$\mathbf{Q} = \sum_{k=0}^{K-1} \mathbf{E}_k \otimes \mathbf{I}_{N_t} \otimes \boldsymbol{\Lambda}_k, \tag{12}$$

where $\mathbf{E}_k = \mathbf{e}_k^H \mathbf{e}_k$, and $\boldsymbol{\Lambda}_k = \mathbf{U}_k \mathbf{U}_k^H$ is the projection matrix onto the noise subspace of the k-th subcarrier.

In the semi-blind technique, the cost function comprises both blind and pilot based terms, according to:

$$C(\underline{\mathrm{h}}) = \|\tilde{\mathbf{y}}_p - \tilde{\mathbf{S}}_p\mathbf{h}\|^2 + \alpha \sum_{k=0}^{K-1} \|(\mathbf{e}_k \otimes \mathbf{I}_{N_t} \otimes \mathbf{U}_k^H)\mathbf{h}\|_F^2 \quad (13)$$

where $\alpha > 0$ is a weighting factor. The semi-blind channel estimation is obtained by minimization of (13) leading to:

$$\hat{\mathbf{h}} = \left(\tilde{\mathbf{S}}_p^H \tilde{\mathbf{S}}_p + \alpha\mathbf{Q}\right)^{-1} \tilde{\mathbf{S}}_p^H \tilde{\mathbf{y}}_p. \quad (14)$$

III. Discussion

We provide here some comments to get more insights on the proposed SB method:

- The blind approach provides an estimate of the channel transfer function up to an unknown constant matrix [14], i.e., $\mathcal{H}_B(z) = \mathcal{H}(z)\mathbf{A}$ where $\mathcal{H}_B(z)$ denotes the blindly estimated transfer function and $\mathbf{A}$ is an unknown $(N_r - N_t) \times (N_r - N_t)$ constant matrix. Hence, the SB solution allows us to get rid of this indeterminacy with only $(N_r - N_t)^2$ pilot samples[1].
- Another advantage of the SB approach resides in its computational cost gain. Indeed, the blind method needs to minimize cost function (11) under appropriate constraints to avoid spurious undesired solutions (see [13], [14] for details) while the SB one relies simply on the LS solution in (14).
- In addition, the SB method allows us to reduce the number of subcarriers considered in the cost function (13) without compromising the solution uniqueness, which significantly reduces the cost.
- Contrary to other existing subspace methods for MIMO-OFDM, the considered one relies on small size covariance matrices (of size $N_r \times N_r$) which helps not only reducing the cost but also improving the estimation accuracy when the number of OFDM symbols is relatively small[2].
- Finally, the SB methods are robust to channel order over-estimation errors [17] and help to reduce the pilot size (hence increasing the information throughput) without affecting the estimation quality [18].

IV. Simulation Results

In this section, the performance of the proposed semi-blind method is analyzed and presented. The proposed algorithm is evaluated using Normalized Mean-Square Error (NMSE) expressed as:

$$\mathrm{NMSE} = \frac{1}{N_{mc}} \sum_{mc=1}^{N_{mc}} \frac{\|\hat{\mathbf{h}}_{mc} - \mathbf{h}_{exact}\|^2}{\|\mathbf{h}_{exact}\|^2}, \quad (15)$$

where $N_{mc} = 200$ Monte-Carlo simulation is considered, $\hat{\mathbf{h}}_{mc}$, and $\mathbf{h}_{exact}$ is the estimated and the actual channel coefficients, in which the actual channels are randomly generated and considered to be i.i.d, unit variance, zero-mean Gaussian random vectors. The transmitted symbols are modulated using the 4-QAM modulation technique in which $N_t = 2$ transmitting antennas and $N_r = 4$ receiving antennas are used. The number of channel taps used is $N = 3$, the size of the pilot sequence is $N_p = 3$, and the total OFDM subcarriers are $K = 64$. The average Signal-to-Noise Ratio (SNR) is defined as:

$$\mathrm{SNR} = \frac{1}{K} \sum_{k=1}^{K-1} \|\mathcal{H}_k \mathbf{S}_k\|_F^2 / \|\mathbf{N}_k\|_F^2. \quad (16)$$

the weighing factor α is defined based on [19].

Figure 2 presents the NMSE vs number of OFDM symbols for different SNR values. It's clearly shown that as the SNR increases, the NMSE significantly improves. However, the increase in OFDM symbols doesn't significantly affect the algorithm's performance for anything above 50 OFDM symbols. On the other hand, Fig. 3 depicts that the increase in the number of used subcarriers affects the algorithm's performance when the SNR is above 20 dB. However, one can see that for low or moderate SNRs, one can decrease the number of used subcarriers without much affecting the estimation accuracy.

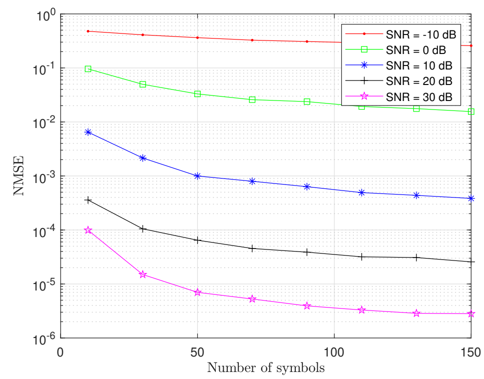

Fig. 2. NMSE vs. number of OFDM symbols N_s for different SNR with $K = 64$.

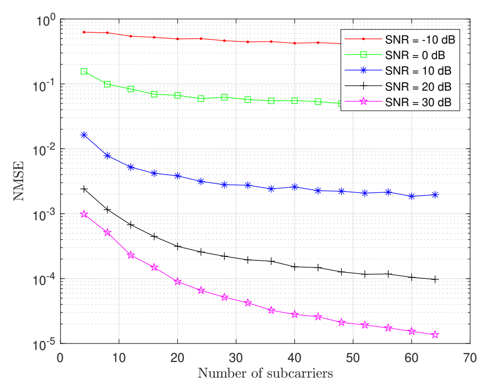

Fig. 3. NMSE vs. number of used subcarriers K for different SNR with $N_s = 30$.

This is further illustrated by Fig. 4 which shows how the proposed algorithm performs, in terms of NMSE vs SNR,

[1]Note that one OFDM symbol corresponds to K samples.

[2]Usually, the 'accurate' estimation of a $N \times N$ matrix by sample averaging requires at least few tens of N i of samples.

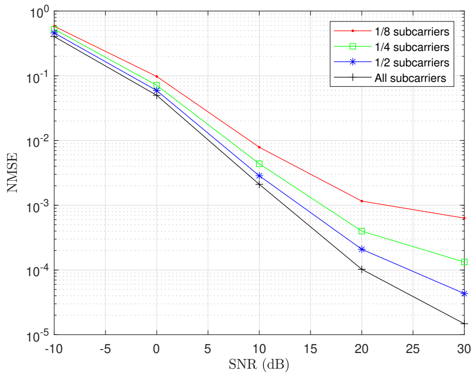

Fig. 4. NMSE vs. SNR when using some part of available subcarriers with $N_s = 30$.

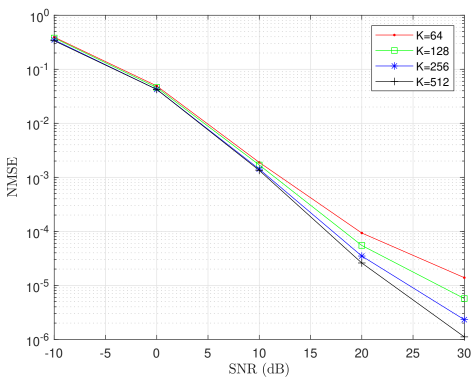

Fig. 5. NMSE vs. SNR with different K values and $N_s = 30$.

when some subcarriers are considered to evaluate matrix $\mathbf{Q}$ in equation (12). It can be noticed that one can rely on selecting some subcarriers for estimating channel coefficients based on sufficient (targeted) accuracy, in contrast to the scenario of using all subcarriers.

Finally, Fig. 5 shows the NMSE vs SNR for different sizes of subcarriers while maintaining $N_s = 30$. As the size of the subcarriers increases, the proposed approach has a similar performance for anything less than 20 dB. Meanwhile, the performance improves significantly when the SNR exceeds 20 dB.

V. Conclusion

This work considers MIMO-OFDM communications systems and proposes a new semi-blind subspace-based approach. The proposed solution intends to increase the convergence rate by reducing the computational complexity while preserving accurate channel estimates with fewer received symbols. The suggested technique uses the orthogonality characteristic of OFDM systems to estimate each subcarrier's covariance matrix and noise subspace separately. The efficiency of the suggested method was illustrated by the numerical simulations presented.

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