# Multipair DF Relaying with Network-Assisted Full-Duplex Cell-Free Massive MIMO 

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#### Abstract

We consider a multipair decode-and-forward network-assisted full-duplex (NAFD) cell-free massive multipleinput multiple-output relaying system, where access points (APs) with a downlink (DL) mode serve destination nodes and those with an uplink (UL) mode serve source nodes, at the same time. Aiming at maximizing the sum of the spectral efficiency (SE) of all the transmission pairs, we formulate a mixed-integer nonconvex optimization problem to jointly design the AP mode assignment, power control, and large-scale fading decoding coefficients. This problem is subject to minimum per-pair SE requirements, per-AP power control, and per-source-node power constraints. By employing the successive convex approximation technique, we propose an algorithm to obtain a stationary solution to the formulated problem. Numerical results show that the NAFD approach can increase $90 \%$-likely per-pair SE of the considered system by up to $63 \%$ compared with those of the traditional half-duplex and heuristic baseline schemes, respectively.


## I. Introduction

Cell-free massive multiple-input multiple-output (CFmMIMO) is considered as a promising solution for beyond 5G wireless systems due to its potential to provide uniformly good service to all users [1]. In a cell-free massive MIMO system, a large number of access points (APs) simultaneously serve a large number of users in the same frequency band. A CFmMIMO system inherits the macro-diversity gain from the distributed systems, and the favorable propagation and channel hardening from colocated massive MIMO systems. Therefore, CFmMIMO can offer a very high spectral efficiency (SE) and connectivity with simple signal processing and resource allocation schemes [2].

Massive MIMO can also be an emerging solution for multipair relaying systems [3], [4]. In these systems, the direct links of multiple pairs of source and destination nodes can be weak because of large path loss and heavy shadowing. Therefore, these pairs communicate simultaneously with the help of relays equipped with massive antenna arrays. In the literature, the transmissions between the source and destination nodes can be performed using either one-way or two-way transmission schemes, either amplify-and-forward (AF) or decode-andforward (DF) protocols, and either half-duplex (HD) or fullduplex (FD) relays. While multipair massive MIMO relaying systems are well studied (see [3], [5] and references therein), we
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Fig. 1. A multipair relaying system with NAFD CFmMIMO.
only know of one paper on multipair relaying with CFmMIMO [6]. In [6], a two-way DF relaying protocol is considered which can obtain full gain only when the users know the instantaneous channels for self-interference cancellation.

In this work, we consider a multipair one-way DF networkassisted full-duplex (NAFD) CFmMIMO relaying system. The idea of NAFD CFmMIMO was studied in [7]-[9] for traditional communication scenarios. In this system, the standard HD APs are used to virtually realize in-band FD transmissions without the cost of eliminating self-interference (SI) in the hardware. Specifically, the APs that are assigned an uplink (UL) mode receive the signals from the source nodes and operate simultaneously with the APs that are assigned a downlink (DL) mode and transmit signals to the destination nodes. To the best of our knowledge, there are no studies of NAFD CFmMIMO systems for multipair DF relaying in the existing literature.

The contributions of this paper are summarized as follows. We formulate the problem of optimizing the transmission mode assignments (i.e., UL or DL) of HD APs, power control, and large-scale fading decoding (LSFD) weights for maximizing the achievable sum SE in a multipair NAFD cell-free massive MIMO relaying system. The problem is subject to per-AP, per-source-node transmit power constraints, and per-pair SE requirements. Importantly, our formulated problem only needs to be solved when the large-scale fading changes. We then propose an algorithm to solve the challenging formulated mixed-integer non-convex problem using successive convex approximation techniques. Numerical results confirm that our proposed NAFD approach significantly improves the SE of the system compared to the traditional HD and heuristic approaches.

## II. System Model and Achievable Rate

We consider a multipair DF relaying system where $K$ communication pairs $\left(\mathrm{S}_{k}, \mathrm{D}_{k}\right), \forall k \in \mathcal{K} \triangleq\{1, \ldots, K\}$, are served in the same time-frequency resource with the help of $M$ access points (APs) (acting as the relay nodes) under time-division duplexing operation. The direct links between $\mathrm{S}_{k}$ and $\mathrm{D}_{k}$ are ignored due to large path loss and/or heavy shadowing, as in [3]. Each coherence block includes two phases: UL training for channel estimation, and payload data transmission. Each AP is connected to the central processing unit (CPU) via a high-capacity backhaul link.

All source nodes $S_{k}$ and destination nodes $D_{k}$ are equipped with a single antenna. Each AP is equipped with $N$ transmit antennas for a downlink (DL) mode and $N$ receive antennas for an uplink (UL) mode, with a different radio-frequency chain for each mode. Each AP is assigned to operate either in DL or UL mode to achieve the highest sum SE of transmission pairs, as discussed in Section III. Importantly, the selection of AP modes is performed on the large-scale fading time scale which changes slowly with time. The mode assignment variables of AP $m, \forall m \in \mathcal{M} \triangleq\{1, \ldots, M\}$, are defined as

$$
\begin{align*}
& a_{m} \triangleq\left\{\begin{array}{ll}
1, & \text { if AP } m \text { operates in the DL mode } \\
0, & \text { otherwise }
\end{array}, \forall m,\right.  \tag{1}\\
& b_{m} \triangleq\left\{\begin{array}{ll}
1, & \text { if AP } m \text { operates in the UL mode } \\
0, & \text { otherwise }
\end{array}, \forall m .\right. \tag{2}
\end{align*}
$$

Since AP $m$ only operates in either the DL or UL mode, we have

$$
\begin{equation*}
a_{m}+b_{m}=1, \forall m \tag{3}
\end{equation*}
$$

1) Uplink Training for Channel Estimation: Denote by $\mathrm{g}_{\mathrm{SR}, m \ell} \in \mathbb{C}^{N \times 1}$ and $\mathrm{g}_{\mathrm{RD}, m k} \in \mathbb{C}^{N \times 1}$ the channel vectors from $\mathrm{S}_{\ell}$ and $\mathrm{D}_{k}$ to AP $m$, respectively. These channels are modeled as $\mathbf{g}_{\mathrm{SR}, m \ell} \sim \mathcal{C N}\left(\beta_{\mathrm{SR}, m \ell}, \mathbf{I}\right)$ and $\mathbf{g}_{\mathrm{RD}, m k} \sim \mathcal{C N}\left(\beta_{\mathrm{RD}, m k}, \mathbf{I}\right)$, where $\beta_{\mathrm{SR}, m \ell}$ and $\beta_{\mathrm{RD}, m k}$ represent large-scale fading. In each coherence block, of length $\tau_{c}$, all $\mathrm{S}_{\ell}$ and $\mathrm{D}_{k}$ send their pairwise orthogonal pilot sequences of length $\tau_{p}$ to all the APs; this requires $\tau_{p} \geq 2 K$. At AP $m, \mathbf{g}_{\mathrm{SR}, m \ell}$ and $\mathbf{g}_{\mathrm{RD}, m k}$ are estimated by using the received pilot signals and minimum mean-square error (MMSE) estimation. By following [2], the MMSE estimates of $\mathbf{g}_{\mathrm{SR}, m \ell}$ and $\mathbf{g}_{\mathrm{RD}, m k}$ are $\hat{\mathbf{g}}_{\mathrm{SR}, m \ell}$ and $\hat{\mathbf{g}}_{\mathrm{RD}, m k}$, respectively, where $\hat{\mathrm{g}}_{\mathrm{SR}, m \ell} \sim \mathcal{C N}\left(\mathbf{0}, \sigma_{\mathrm{SR}, m \ell}^{2} \mathbf{I}\right), \hat{\mathrm{g}}_{\mathrm{RD}, m k} \sim \mathcal{C N}\left(\mathbf{0}, \sigma_{\mathrm{RD}, m k}^{2} \mathbf{I}\right)$, with $\sigma_{\mathrm{SR}, m \ell}^{2} \triangleq \frac{\tau_{p} \rho_{p} \beta_{\mathrm{SR}, m \ell}^{2}}{\tau_{p} \rho_{p} \beta_{\mathrm{SR}, m \ell}+1}, \sigma_{\mathrm{RD}, m k}^{2} \triangleq \frac{\tau_{p} \rho_{p} \beta_{\mathrm{RD}, m k}^{2}}{\tau_{p} \rho_{p} \beta_{\mathrm{RD}}, m k+1}$, and $\rho_{p}$ is the normalized transmit power of each pilot symbol. Note that the AP mode selection does not affect the channel estimation.
2) Pairing Payload Data Transmission: At time instant $i$, all $K$ source nodes $\mathrm{S}_{k}, \forall k \in \mathcal{K}$, transmit their signals $\sqrt{\rho_{u} \zeta_{k}} s_{k}[i]$ to the UL APs, while each DL AP $m$ transmits $\mathbf{x}_{m}[i] \in \mathbb{C}^{N \times 1}$ to all the destination nodes $\mathrm{D}_{k}, \forall k \in \mathcal{K}$. Here, $s_{k}[i]$, where $\mathbb{E}\left\{\left|s_{k}[i]\right|^{2}\right\}=1$, is the data symbol, $\rho_{u}$ is the maximum normalized transmit power of $\mathrm{S}_{k}$, and $\zeta_{k}$ is an uplink power control coefficient satisfying

$$
\begin{equation*}
0 \leq \zeta_{k} \leq 1, \forall k \tag{4}
\end{equation*}
$$

The transmit signal $\mathbf{x}_{m}[i]$ at AP $m$ is the precoded version of the signals detected from $K$ source nodes. For ease of distributed implementation, each AP is assumed to use its local channel estimates to perform maximum-ratio (MR) processing
(i.e., conjugate beamforming) to precode the signals [2]. Let $s_{k}[i-d]$ be symbol detected in the UL transmission associated with source $\mathrm{S}_{k}$ after a processing delay $d$. Then, we have

$$
\begin{equation*}
\mathbf{x}_{m}[i]=\sqrt{\rho_{d}} \sum_{k \in \mathcal{K}} \vartheta_{m k} \hat{\mathbf{g}}_{\mathrm{RD}, m k}^{*} s_{k}[i-d] \tag{5}
\end{equation*}
$$

where $\rho_{d}$ is the maximum normalized transmit power at each AP and $\vartheta_{m k}, \forall m, k$, are downlink power control coefficients. We have

$$
\begin{equation*}
\left(\vartheta_{m k}=0, \forall k, \text { if } a_{m}=0\right), \quad \forall m \tag{6}
\end{equation*}
$$

to guarantee that if AP $m$ does not operate in the DL mode, all transmit powers $\rho_{d} \eta_{m k}, \forall k$, are zero, and $\mathbf{x}_{m}[i]=\mathbf{0}$. Each AP $m$ is required to meet the average normalized power constraint, i.e., $\mathbb{E}\left\{\left\|\mathbf{x}_{m}[i]\right\|^{2}\right\} \leq \rho_{d}$, which can also be expressed as the following per-AP power constraint

$$
\begin{equation*}
\sum_{k \in \mathcal{K}} \sigma_{\mathrm{RD}, m k}^{2} \vartheta_{m k}^{2} \leq \frac{1}{N}, \forall m \tag{7}
\end{equation*}
$$

Since all the UL and DL APs operate in the same frequency and time, the UL APs receive signals not only from the source nodes but also the interference from the DL APs. Let $\mathbf{H}_{m j} \in$ $\mathbb{C}^{N \times N}$ be the Rayleigh fading channel matrix of the link from AP $m$ to AP $j$, whose elements are i.i.d. $\mathcal{C N}\left(0, \beta_{m j}\right)$ RVs. At the UL AP $m$ (i.e., $b_{m}=1$ ), the received signal is

$$
\begin{align*}
\mathbf{y}_{m}[i]= & \sqrt{\rho_{u}} \sum_{k \in \mathcal{K}} \sqrt{b_{m} \zeta_{k}} \mathbf{g}_{\mathrm{SR}, m k} s_{k}[i] \\
& +\sum_{j \in\left\{j^{\prime} \mid j^{\prime} \in \mathcal{M}, a_{j^{\prime}}=1\right\}} \mathbf{H}_{m j} \mathbf{x}_{j}[i]+\sqrt{b_{m}} \mathbf{w}_{m}[i] \\
\stackrel{(3),(6)}{=} & \sqrt{\rho_{u}} \sum_{k \in \mathcal{K}} \sqrt{b_{m} \zeta_{k}} \mathbf{g}_{\mathrm{SR}, m k} s_{k}[i] \\
& +\sqrt{\rho_{d}} \sum_{j \in \mathcal{M} \backslash\{m\}} \sum_{\ell \in \mathcal{K}} \sqrt{b_{m} \vartheta_{j \ell}^{2}} \mathbf{H}_{m j} \hat{\mathbf{g}}_{\mathrm{RD}, j \ell}^{*} s_{\ell}[i-d] \\
& +\sqrt{b_{m}} \mathbf{w}_{m}[i], \tag{8}
\end{align*}
$$

where $\mathbf{w}_{m}[i] \sim \mathcal{C N}(\mathbf{0}, \mathbf{I})$ is a vector of additive noise. Equation (8) captures the fact that if AP $m$ does not operate in the UL mode, i.e., $b_{m}=0$, it does not receive any signal, i.e, $\mathbf{y}_{m}[i]=\mathbf{0}$. The UL AP $m$ then performs maximum-ratio combining (MRC) processing (i.e., matched filter) to combine the received signals. Specifically, it computes $\hat{\mathbf{g}}_{\mathrm{SR}, m k}^{H} \mathbf{y}_{m}[i]$ and then forwards this signal to the CPU for signal detection. In order to improve the SE, the CPU performs LSFD to obtain aggregated signal for detecting $s_{k}[i]$ as follows [10]:

$$
\begin{equation*}
\widetilde{\mathbf{y}}_{k}[i]=\sum_{m \in \mathcal{M}} \alpha_{m k} \hat{\mathbf{g}}_{\mathrm{SR}, m k}^{H} \mathbf{y}_{m}[i], \tag{9}
\end{equation*}
$$

where $\alpha_{m k}, \forall m, k$, are LSFD weights and

$$
\begin{equation*}
\left|\alpha_{m k}\right|^{2} \leq 1, \forall m, k \tag{10}
\end{equation*}
$$

Then, by using the use-and-then-forget capacity-bounding technique [2], a closed-form expression for the UL achievable rate of $S_{k}$ is

$$
\begin{equation*}
R_{\mathrm{SR}, k}(\mathbf{b}, \boldsymbol{\zeta}, \boldsymbol{\vartheta}, \boldsymbol{\alpha})=\frac{\tau_{c}-\tau_{p}}{\tau_{c}} \log _{2}\left(1+\Gamma_{\mathrm{SR}, k}\right), \tag{11}
\end{equation*}
$$

where $\Gamma_{\mathrm{SR}, k}$ is shown at the top of the next page, $\mathbf{b} \triangleq$ $\left\{b_{m}\right\}, \boldsymbol{\zeta} \triangleq\left\{\zeta_{m k}\right\}, \boldsymbol{\vartheta} \triangleq\left\{\vartheta_{m k}\right\}, \boldsymbol{\alpha} \triangleq\left\{\alpha_{m k}\right\}, \forall m, k$.
At the destination node $\mathrm{D}_{k}$, the received signal is $r_{k}[i]=\sum_{m \in \mathcal{M}} \mathbf{g}_{\mathrm{RD}, m k}^{T} \mathbf{x}_{m}[i-d]+n_{k}[i]$
$\Gamma_{\mathrm{SR}, k}=\frac{N \rho_{u}\left(\sum_{m \in \mathcal{M}} \sqrt{b_{m} \zeta_{k}} \alpha_{m k} \sigma_{\mathrm{SR}, m k}^{2}\right)^{2}}{\rho_{u} \sum_{m \in \mathcal{M}} \sum_{\ell \in \mathcal{K}} b_{m} \zeta_{\ell} \alpha_{m k}^{2} \beta_{\mathrm{SR}, m k} \sigma_{\mathrm{SR}, m \ell}^{2}+\rho_{d} N \sum_{m \in \mathcal{M}} \sum_{j \in \mathcal{M}} \sum_{\ell \in \mathcal{K}} b_{m} \alpha_{m k}^{2} \vartheta_{j \ell}^{2} \sigma_{\mathrm{SR}, m k}^{2} \beta_{m j} \sigma_{\mathrm{RD}, j \ell}^{2}+\sum_{m \in \mathcal{M}} b_{m} \alpha_{m k}^{2} \sigma_{\mathrm{SR}, m k}^{2}}$

$$
\begin{align*}
= & \sqrt{\rho_{d}} \sum_{m \in \mathcal{M}} \vartheta_{m k} \mathbf{g}_{\mathrm{RD}, m k}^{T} \hat{\mathbf{g}}_{\mathrm{RD}, m k}^{*} s_{k}[i-d]+n_{k}[i] \\
& +\sqrt{\rho_{d}} \sum_{m \in \mathcal{M}} \sum_{\ell \in \mathcal{K} \backslash\{k\}} \vartheta_{m \ell} \mathbf{g}_{\mathrm{RD}, m k}^{T} \hat{\mathbf{g}}_{\mathrm{RD}, m \ell}^{*} s_{\ell}[i-d] \tag{12}
\end{align*}
$$

where $n_{k} \sim \mathcal{C N}(0,1)$ is the additive noise. Then, by again using the use-and-then-forget capacity-bounding technique [2], we obtain the achievable rate of the transmission link from the DL APs to $\mathrm{D}_{k}$ as

$$
\begin{equation*}
R_{\mathrm{RD}, k}(\mathbf{a}, \boldsymbol{\vartheta})=\frac{\tau_{c}-\tau_{p}}{\tau_{c}} \log _{2}\left(1+\Gamma_{\mathrm{RD}, k}\right) \tag{13}
\end{equation*}
$$

where $\Gamma_{\mathrm{RD}, k}=\frac{N^{2} \rho_{d}\left(\sum_{m \in \mathcal{M}} \vartheta_{m k} \sigma_{\mathrm{RD}, m k}^{2}\right)^{2}}{\rho_{d} N \sum_{m \in \mathcal{M}} \sum_{\ell \in \mathcal{K}} \vartheta_{m \ell}^{2} \beta_{\mathrm{RD}, m k} \sigma_{\mathrm{RD}, m \ell}^{2}+1}$, and $\mathbf{a} \triangleq$ $\left\{a_{m}\right\}, \forall m$.

Thus, the end-to-end SE of the transmission pair $k$ from $\mathrm{S}_{k}$ to the APs and then to $\mathrm{D}_{k}$ is [3], i.e.,

$$
\begin{equation*}
R_{k}(\mathbf{a}, \mathbf{b}, \boldsymbol{\zeta}, \boldsymbol{\vartheta}, \boldsymbol{\alpha})=\min \left\{R_{\mathrm{SR}, k}, R_{\mathrm{RD}, k}\right\} \tag{14}
\end{equation*}
$$

where $R_{\mathrm{SR}, k}$ and $R_{\mathrm{RD}, k}$ are given by (11) and (13), respectively.

## III. Problem Formulation and Solution

1) Problem Formulation: In this section, we design the UL and DL mode assignment (a, b), allocating DL and UL power control coefficients ( $\boldsymbol{\zeta}, \boldsymbol{\eta}$ ), and choosing LSFD weights $\boldsymbol{\alpha}$ to maximize the sum SE of transmission pairs, under the constraints on transmit power at each AP, transmit power at each source node, and per-pair SE requirements. The optimization problem is formulated as

$$
\begin{align*}
\max _{\mathbf{x}} & \sum_{k \in \mathcal{K}} R_{k}(\mathbf{x})  \tag{15a}\\
\text { s.t. } & (1)-(3),(4),(6),(7),(10) \\
& R_{k}(\mathbf{x}) \geq R_{\mathrm{Qos}}, \forall k, \tag{15b}
\end{align*}
$$

where $\mathbf{x} \triangleq\{\mathbf{a}, \mathbf{b}, \boldsymbol{\zeta}, \boldsymbol{\alpha}, \boldsymbol{\vartheta}\}$, and $R_{\mathrm{QoS}}$ is the minimum rate required for quality of service. Problem (15) is equivalent to

$$
\begin{align*}
\min _{\mathbf{x}, t} & -\sum_{k \in \mathcal{K}} t_{k}  \tag{16a}\\
\text { s.t. } & (1)-(3),(4),(6),(7),(10) \\
t_{k} & \leq R_{\mathrm{SR}, k}(\mathbf{a}, \mathbf{b}, \boldsymbol{\mu}, \boldsymbol{\vartheta}, \boldsymbol{\alpha}), \forall k  \tag{16b}\\
t_{k} & \leq R_{\mathrm{RD}, k}(\mathbf{a}, \boldsymbol{\vartheta}), \forall k  \tag{16c}\\
t_{k} & \geq R_{\mathrm{QoS}}, \forall k \tag{16d}
\end{align*}
$$

where $\mathbf{t} \triangleq\left\{t_{k}\right\}$ are an additional variable. (16) is a mixedinteger optimization problem with strong coupling among variables and highly nonconvex constraints. Therefore, we transform it into a more tractable form and use successive convex approximation techniques to find its solution.
2) Proposed Solution: First, we make constraint (16b) more tractable by introducing new non-negative variables $\boldsymbol{\mu} \triangleq$ $\left\{\mu_{m k}\right\}, \overline{\boldsymbol{\mu}} \triangleq\left\{\bar{\mu}_{m k}\right\}, \tilde{\boldsymbol{\mu}} \triangleq\left\{\tilde{\mu}_{m k}\right\}, \hat{\boldsymbol{\mu}} \triangleq\left\{\hat{\mu}_{m \ell k}\right\}, \tilde{\boldsymbol{\alpha}} \triangleq$ $\left\{\tilde{\alpha}_{m k}\right\}, \hat{\boldsymbol{\alpha}} \triangleq\left\{\hat{\alpha}_{m k}\right\}, \overline{\boldsymbol{\eta}} \triangleq\left\{\bar{\eta}_{j \ell}\right\}, \hat{\boldsymbol{\eta}} \triangleq\left\{\hat{\eta}_{m j k \ell}\right\}$, where

$$
\begin{align*}
& \mu_{m k}^{2} \leq b_{m} \zeta_{k}, \forall m, k  \tag{17}\\
& \bar{\mu}_{m \ell}^{2} \geq b_{m} \zeta_{\ell}, \forall m, \ell  \tag{18}\\
& \mu_{m k} \alpha_{m k} \geq \tilde{\mu}_{m k}, \forall m, k  \tag{19}\\
& \bar{\mu}_{m \ell} \alpha_{m k} \leq \hat{\mu}_{m \ell k}, \forall m, \ell, k \tag{20}
\end{align*}
$$

$$
\begin{align*}
& \alpha_{m k}^{2} \leq \tilde{\alpha}_{m k}, \forall m, k  \tag{21}\\
& b_{m} \tilde{\alpha}_{m k} \leq \hat{\alpha}_{m k}, \forall m, k  \tag{22}\\
& \vartheta_{j \ell}^{2} \leq \bar{\eta}_{j \ell}, \forall j, \ell  \tag{23}\\
& \hat{\alpha}_{m k} \bar{\eta}_{j \ell} \leq \hat{\eta}_{m j k \ell}, \forall m, j, k, \ell . \tag{24}
\end{align*}
$$

Here, (17)-(24) imply

$$
\begin{align*}
& \sqrt{b_{m} \zeta_{k}} \alpha_{m k} \geq \tilde{\mu}_{m k}, \forall m, k  \tag{25}\\
& b_{m} \zeta_{\ell} \alpha_{m k}^{2} \leq \hat{\mu}_{m \ell k}^{2}, \forall m, \ell, k  \tag{26}\\
& b_{m} \alpha_{m k}^{2} \leq \hat{\alpha}_{m k}, \forall m, k  \tag{27}\\
& b_{m} \alpha_{m k}^{2} \vartheta_{j \ell}^{2} \leq \hat{\eta}_{m j k \ell}, \forall m, j, k, \ell . \tag{28}
\end{align*}
$$

Then, from (11), (25)-(28), we have

$$
\begin{equation*}
\tilde{R}_{\mathrm{SR}, k}(\tilde{\boldsymbol{\mu}}, \hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\eta}})=\frac{\tau_{c}-\tau_{p}}{\tau_{c}} \log _{2}\left(1+\frac{U_{k}^{2}}{V_{k}}\right) \leq R_{\mathrm{SR}, k}, \tag{29}
\end{equation*}
$$

where $U_{k}(\tilde{\boldsymbol{\mu}})=\sqrt{N \rho_{u}} \sum_{m \in \mathcal{M}_{2}} \tilde{\mu}_{m k} \sigma_{\mathrm{SR}, m k}^{2}$ and $V_{k}(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\eta}})=$ $\rho_{u} \sum_{m \in \mathcal{M}} \sum_{\ell \in \mathcal{K}} \hat{\mu}_{m \ell k}^{2} \beta_{\mathrm{SR}, m k} \sigma_{\mathrm{SR}, m \ell}^{2}+\sum_{m \in \mathcal{M}} \hat{\mathcal{M}}_{m k} \sigma_{\mathrm{SR}, m k}^{2}+$ $\rho_{d} N \sum_{m \in \mathcal{M}} \sum_{j \in \mathcal{M}} \sum_{\ell \in \mathcal{K}} \hat{\eta}_{m j k \ell} \sigma_{\mathrm{SR}, m k}^{2} \beta_{m j} \sigma_{\mathrm{RD}, j \ell}^{2}$. Therefore, constraint (16b) can be replaced by

$$
\begin{equation*}
t_{k} \leq \tilde{R}_{\mathrm{SR}, k}(\tilde{\boldsymbol{\mu}}, \hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\eta}}), \forall k \tag{30}
\end{equation*}
$$

In the light of (7), we replace constraint (6) by

$$
\begin{equation*}
N \sigma_{\mathrm{RD}, m k}^{2} \vartheta_{m k}^{2} \leq a_{m}, \forall m, k \tag{31}
\end{equation*}
$$

Regarding binary constraints (1) and (2), we see that $x \in$ $\{0,1\} \Leftrightarrow x \in[0,1] \& x-x^{2} \leq 0$. Therefore, (1) and (2) can be replaced by

$$
\begin{align*}
& Q(\mathbf{a}, \mathbf{b}) \triangleq \sum_{m \in \mathcal{M}}\left(a_{m}-a_{m}^{2}\right)+\sum_{m \in \mathcal{M}}\left(b_{m}-b_{m}^{2}\right) \leq 0  \tag{32}\\
& 0 \leq a_{m} \leq 1,0 \leq b_{m} \leq 1, \forall m \tag{33}
\end{align*}
$$

From the discussions above, problem (16) can be written in a more tractable epigraph form as

$$
\begin{equation*}
\min _{\hat{\mathbf{x}} \in \mathcal{F}}-\sum_{k \in \mathcal{K}} t_{k} \tag{34}
\end{equation*}
$$

where $\mathcal{F} \triangleq\{(3),(4),(7),(10),(16 b)-(16 \mathrm{~d}),(17)-(24),(30)-$ (33) $\}$ and $\hat{\mathbf{x}}=\{\mathbf{x}, \mathbf{t}, \boldsymbol{\mu}, \overline{\boldsymbol{\mu}}, \tilde{\boldsymbol{\mu}}, \hat{\boldsymbol{\mu}}, \tilde{\boldsymbol{\alpha}}, \hat{\boldsymbol{\alpha}}, \overline{\boldsymbol{\eta}}, \hat{\boldsymbol{\eta}}\}$. Then, we consider the following problem

$$
\begin{equation*}
\min _{\hat{\mathbf{x}} \in \widehat{\mathcal{F}}} \mathcal{L}(\hat{\mathbf{x}}) \tag{35}
\end{equation*}
$$

where $\mathcal{L}(\hat{\mathbf{x}}) \triangleq-\sum_{k \in \mathcal{K}} t_{k}+\lambda Q(\mathbf{a}, \mathbf{b})$ is the Lagrangian of (34), $\lambda$ is the Lagrangian multiplier corresponding to constraint (32), and $\widehat{\mathcal{F}} \triangleq \mathcal{F} \backslash\{(32)\}$.

Proposition 1. The values $Q_{\lambda}$ of $Q$ at the solution of (35) corresponding to $\lambda$ converge to 0 as $\lambda \rightarrow+\infty$. Also, problem (34) has strong duality, i.e.,

$$
\begin{equation*}
\min _{\hat{\mathbf{x}} \in \mathcal{F}}-\sum_{k \in \mathcal{K}} t_{k}=\sup _{\lambda \geq 0} \min _{\hat{\mathbf{x}} \in \widehat{\mathcal{F}}} \mathcal{L}(\hat{\mathbf{x}}) \tag{36}
\end{equation*}
$$

Then, (35) is equivalent to (34) at the optimal solution $\lambda^{*} \geq 0$ of the sup-min problem in (36).

Proof. The proof follows [11], and hence, omitted due to lack of space.
Theoretically, $Q_{\lambda}$ must be zero to obtain the optimal solution to (34). According to Proposition 1, the optimal solution to (34) can be obtained as $\lambda \rightarrow+\infty$. For practical implementation, it is acceptable for $Q_{\lambda}$ to be sufficiently small with a sufficiently large value of $\lambda$. In our numerical experiments, $\lambda=1$ is enough
to ensure that $Q_{\lambda} \leq \varepsilon$ with $\varepsilon=10^{-3}$. Note that this approach of selecting $\lambda$ has been widely used in the literature (see [11] and references therein).

Problem (35) is now solved by successive convex approximation techniques as follows. To deal with the constraints (16c), following [12, Eq. (40)], we see that $R_{\mathrm{RD}, k}$ has a concave lower bound $\hat{R}_{\mathrm{RD}, k}$ which is given by

$$
\begin{aligned}
\hat{R}_{\mathrm{RD}, k}(\mathbf{a}, \boldsymbol{\vartheta})= & \frac{\tau_{c}-\tau_{p}}{\tau_{c} \log 2}\left[\log \left(1+\frac{\left(A_{k}^{(n)}\right)^{2}}{B_{k}^{(n)}}\right)-\frac{\left(A_{k}^{(n)}\right)^{2}}{B_{k}^{(n)}}\right. \\
& \left.+2 \frac{A_{k}^{(n)} A_{k}}{B_{k}^{(n)}}-\frac{\left(A_{k}^{(n)}\right)^{2}\left(A_{k}^{2}+B_{k}\right)}{B_{k}^{(n)}\left(\left(A_{k}^{(n)}\right)^{2}+B_{k}^{(n)}\right)}\right]
\end{aligned}
$$

where $A_{k}=N \sqrt{\rho_{d}} \sum_{m \in \mathcal{M}} \vartheta_{m k} \sigma_{\mathrm{RD}, m k}^{2}$ and $B_{k}=$ $\rho_{u} \sum_{m \in \mathcal{M}} \sum_{\ell \in \mathcal{K}} \vartheta_{m \ell}^{2} \beta_{m k} \sigma_{\mathrm{RD}, m \ell}^{2}+1$. Then, constraint (16c) is approximated by the following convex constraint:

$$
\begin{equation*}
t_{k} \leq \hat{R}_{\mathrm{RD}, k}(\mathbf{a}, \boldsymbol{\vartheta}), \forall k \tag{37}
\end{equation*}
$$

Similarly, we see that the concave lower bound $\hat{R}_{\mathrm{SR}, k}$ of $\tilde{R}_{\mathrm{SR}, k}$ is given by [12, Eq. (40)]

$$
\begin{aligned}
\hat{R}_{\mathrm{RD}, k}(\tilde{\boldsymbol{\mu}}, \hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\eta}})= & \frac{\tau_{c}-\tau_{p}}{\tau_{c} \log 2}\left[\log \left(1+\frac{\left(U_{k}^{(n)}\right)^{2}}{V_{k}^{(n)}}\right)-\frac{\left(U_{k}^{(n)}\right)^{2}}{V_{k}^{(n)}}\right. \\
& \left.+2 \frac{U_{k}^{(n)} U_{k}}{V_{k}^{(n)}}-\frac{\left(U_{k}^{(n)}\right)^{2}\left(U_{k}^{2}+V_{k}\right)}{V_{k}^{(n)}\left(\left(U_{k}^{(n)}\right)^{2}+V_{k}^{(n)}\right)}\right]
\end{aligned}
$$

Therefore, the constraint (30) (i.e., (16b)) is approximated by the following convex constraint

$$
\begin{equation*}
t_{k} \leq \hat{R}_{\mathrm{SR}, k}(\tilde{\boldsymbol{\mu}}, \hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\eta}}), \forall k \tag{38}
\end{equation*}
$$

We also observe that $x y \leq 0.25\left[(x+y)^{2}-2\left(x^{(n)}-y^{(n)}\right)(x-\right.$ $\left.y)+\left(x^{(n)}-y^{(n)}\right)^{2}\right]$ and $-x y \leq 0.25\left[(x-y)^{2}-2\left(x^{(n)}+\right.\right.$ $\left.\left.y^{(n)}\right)(x+y)+\left(x^{(n)}+y^{(n)}\right)^{2}\right], \forall x \geq 0, y \geq 0$ [12]. Thus, $Q$ has the convex upper bounds which are given by

$$
\begin{align*}
\widehat{Q}(\mathbf{a}, \mathbf{b}) \triangleq & \sum_{m \in \mathcal{M}}\left(a_{m}-2 a_{m}^{(n)} a_{m}+\left(a_{m}^{(n)}\right)^{2}\right) \\
& +\sum_{m \in \mathcal{M}}\left(b_{m}-2 b_{m}^{(n)} b_{m}+\left(b_{m}^{(n)}\right)^{2}\right) \tag{39}
\end{align*}
$$

Similarly, constraints (17)-(20), (22), and (24) can be approximated by the following convex constraints

$$
\begin{align*}
& \mu_{m k}^{2}+0.25\left[\left(b_{m}-\zeta_{k}\right)^{2}-2\left(b_{m}^{(n)}+\zeta_{k}^{(n)}\right)\left(b_{m}+\zeta_{k}\right)\right. \\
&\left.+\left(b_{m}^{(n)}+\zeta_{k}^{(n)}\right)^{2}\right] \leq 0, \forall m, k  \tag{40}\\
& 0.25\left[\left(b_{m}+\zeta_{\ell}\right)^{2}-2\left(b_{m}^{(n)}-\zeta_{\ell}^{(n)}\right)\left(b_{m}-\zeta_{\ell}\right)+\left(b_{m}^{(n)}-\zeta_{\ell}^{(n)}\right)^{2}\right] \\
& \quad-2 \bar{\mu}_{m \ell}^{(n)} \bar{\mu}_{m \ell}+\left(\bar{\mu}_{m \ell}^{(n)}\right)^{2} \leq 0, \forall m, \ell  \tag{41}\\
& \tilde{\mu}_{m k}+0.25\left[\left(\mu_{m k}-\alpha_{m k}\right)^{2}-2\left(\mu_{m k}^{(n)}+\alpha_{m k}^{(n)}\right)\left(\mu_{m k}+\alpha_{m k}\right)\right. \\
&\left.+\left(\mu_{m k}^{(n)}+\alpha_{m k}^{(n)}\right)^{2}\right] \leq 0, \forall m, k  \tag{42}\\
& 0.25\left[\left(\bar{\mu}_{m \ell}+\alpha_{m k}\right)^{2}-2\left(\bar{\mu}_{m \ell}^{(n)}-\alpha_{m k}^{(n)}\right)\left(\bar{\mu}_{m \ell}-\alpha_{m k}\right)\right. \\
&\left.\quad+\left(\bar{\mu}_{m \ell}^{(n)}-\alpha_{m k}^{(n)}\right)^{2}\right]-\hat{\mu}_{m \ell k} \leq 0, \forall m, \ell, k  \tag{43}\\
& 0.25\left[\left(b_{m}+\tilde{\alpha}_{m k}\right)^{2}-2\left(b_{m}^{(n)}-\tilde{\alpha}_{m k}^{(n)}\right)\left(b_{m}-\tilde{\alpha}_{m k}\right)\right. \\
&\left.\quad+\left(b_{m}^{(n)}-\tilde{\alpha}_{m k}^{(n)}\right)^{2}\right]-\hat{\alpha}_{m k} \leq 0, \forall m, k  \tag{44}\\
& 0.25\left[\left(\hat{\alpha}_{m k}+\bar{\eta}_{j \ell)^{2}-2\left(\hat{\alpha}_{m k}^{(n)}-\bar{\eta}_{j \ell}^{(n)}\right)\left(\hat{\alpha}_{m k}-\bar{\eta}_{j \ell}\right)} \quad+\left(\hat{\alpha}_{m k}^{(n)}-\bar{\eta}_{j \ell}^{(n)}\right)^{2}\right]-\hat{\eta}_{m j k \ell} \leq 0, \forall m, j, k, \ell .\right.
\end{align*}
$$

At iteration $(n+1)$, for given point $\hat{\mathbf{x}}^{(n)}$, problem (35) can be approximated by the following convex problem

$$
\begin{equation*}
\min _{\hat{\mathbf{x}} \in \widetilde{\mathcal{F}}} \widehat{\mathcal{L}}(\hat{\mathbf{x}}) \tag{46}
\end{equation*}
$$

```
Algorithm 1 Solving problem (35)
    Initialize: \(n=0\) and a random point \(\hat{\mathbf{x}}^{(0)} \in \widehat{\mathcal{F}}\).
    repeat
        Update \(n=n+1\)
        Solve (46) to obtain its optimal solution \(\hat{\mathbf{x}}^{*}\)
        Update \(\hat{\mathbf{x}}^{(n)}=\hat{\mathbf{x}}^{*}\)
    until convergence
```

where $\widehat{\mathcal{L}}(\hat{\mathbf{x}}) \triangleq-\sum_{k \in \mathcal{K}} t_{k}+\lambda \widehat{Q}(\mathbf{a}, \mathbf{b}), \widetilde{\mathcal{F}} \triangleq\{\widehat{\mathcal{F}},(37),(38),(42)$ $-(40)\} \backslash\{(16 \mathrm{c}),(17)-(20),(22),(24),(30)\}$ is a convex feasible set. We outline the main steps to solve problem (34) in Algorithm 1. Starting from a random point $\hat{\mathbf{x}} \in \widehat{\mathcal{F}}$, we solve problem (46) to obtain its optimal $\hat{\mathbf{x}}^{*}$ and use this solution as an initial point to the next iteration. Algorithm 1 terminates when a certain accuracy threshold is obtained.
3) Convergence: Algorithm 1 will converge to a stationary point, i.e., a Fritz John solution, of problem (34) (hence (16) or (15)). The proof of this fact is rather standard, and follows from [11, Proposition 2].

## IV. Numerical Examples

1) Network Setup and Parameter Settings: We consider a cell-free massive MIMO network, where APs and UEs are randomly distributed in a square of $0.5 \times 0.5 \mathrm{~km}^{2}$, whose edges are wrapped around to avoid boundary effects. The distances between adjacent APs are at least 50 m . We set $R_{\mathrm{Qos}}=0.2$ bits $/ \mathrm{s} / \mathrm{Hz}, N=2, K=4$ transmission pairs, $\tau_{c}=200$, $\tau_{p}=2 K$ and $\varepsilon=10^{-3}$, bandwidth $B=50 \mathrm{MHz}$, and noise figure $F=9 \mathrm{~dB}$. Thus, the noise power $\sigma_{n}^{2}=k_{B} T_{0} B F$, where $k_{B}=1.381 \times 10^{-23} \mathrm{~J} /{ }^{o} \mathrm{~K}$ is the Boltzmann constant, while $T_{0}=290^{\circ} \mathrm{K}$ is the noise temperature. Let $\tilde{\rho}_{d}=1$ $\mathrm{W}, \tilde{\rho}_{u}=0.1 \mathrm{~W}$ and $\tilde{\rho}_{t}=0.1 \mathrm{~W}$ be the maximum transmit power of the APs, UL UEs and UL training pilot sequences, respectively. The normalized maximum transmit powers $\rho_{d}, \rho_{u}$, and $\rho_{t}$ are calculated by dividing these powers by the noise power. The large-scale fading coefficients $\beta_{m k}$ are modelled in the same manner as [13, Eqs. (37), (38)]. The following simulation results are averaged over 200 channel realizations. In each channel realization, if the optimization problem of a scheme is infeasible, we set the SE achieved at all the nodes of that scheme to zero.
2) Results and Discussions: To evaluate the effectiveness of our proposed NAFD approach, we consider the following baseline schemes:

- Half-duplex (HD): The data transmission phase is divided into two equally long parts, of length $\left(\tau_{c}-\tau_{p}\right) / 2$. Each part is used for the transmission from source nodes to the APs or from the APs to the destination nodes. The HD scheme does not experience interference from DL APs to UL APs. All APs operate in the UL mode to receive signals from source nodes and then switch to the DL mode to transmit to destination nodes. We then apply our algorithm with some modifications to optimize the power control coefficients $(\boldsymbol{\zeta}, \boldsymbol{\eta})$, and LSFD weights $\boldsymbol{\alpha}$, under the same transmit power constraints for source nodes and DL APs.
- Heuristic (HEU): The AP modes $\mathbf{a}$ and $\mathbf{b}$ are assigned randomly. Specifically, $M_{d}$ APs are assigned to operate in


Fig. 2. Comparisons among considered schemes.

DL mode, and the remaining $\left(M-M_{d}\right)$ APs operate in UL mode. Here, $M_{d}$ is a random integer number in $\left[\frac{M}{6}, \frac{5 M}{6}\right]$. Here, power control coefficients ( $\boldsymbol{\zeta}, \boldsymbol{\eta}$ ), and LSFD weights $\boldsymbol{\alpha}$ are also optimized similarly as HD.
Note that [6] considers the total power constraint for all the APs. To make a fair comparison between the two-way DF CFmMIMO relaying system in [6] and our one-way DF NAFD CFmMIMO relaying system, the proposed power control algorithm in [6] must be modified with the per-AP power constraints (7). However, modifying this algorithm is not straightforward. Therefore, we leave a comprehensive comparison between the two-way DF CFmMIMO relaying system in [6] and our oneway DF NAFD CFmMIMO relaying system for our future work.

Fig. 2 compares the spectral efficiencies of all the considered schemes. As seen, in terms of $90 \%$-likely performance of sum and per-pair spectral efficiencies, our proposed scheme NAFD significantly outperforms the baseline schemes. In particular, NAFD substantially increases the $90 \%$-likely sum SE, compared with that of HD, e.g., by more than $63 \%$ with $M=30$ and $60.26 \%$ with $M=60$. This is reasonable because NAFD can serve the source and destination nodes at the same time, and hence, does not have the pre-log factor of $1 / 2$ in the SE as the HD scheme does. Moreover, the AP-to-AP inferences in the system are well managed by optimizing AP mode assignment, power control, and LSFD weights.

On the other hand, the HEU scheme is infeasible for nearly $50 \%$ of all channel realizations. In the channel realizations that the HEU scheme is feasible, the SE of HEU and HD are close. These results show the significant advantage of the optimized AP mode assignment to improve the SE of multipair CFmMIMO relaying systems, compared with the heuristic AP mode assignment. Finally, Fig. 2c demonstrates the advantage of massive MIMO in improving the SE of transmission pairs. The average SE gaps between NAFD and HD increase from $54 \%$ to $68 \%$ when the number of APs increases from 30 to 60 .

## V. Conclusion

We proposed a joint optimization approach to design AP mode assignment, power control and LSFD weights for multipair decode-and-forward cell-free massive MIMO relaying systems. We formulated a mixed-integer nonconvex optimization
problem to maximize the sum SE of transmission pairs, under per-pair SE requirements, and constraints on the per-sourcenode and per-AP maximum transmit powers. Utilizing successive convex approximation techniques, we proposed a novel algorithm to solve the formulated problem. Numerical results showed that our proposed NAFD approach can significantly increase the SE compared with the traditional HD and heuristic approaches.

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