Power Allocation in MIMO Collaborative Co-existing Radar and Communications Systems: A Non-cooperative Game Approach

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***Abstract*—This study investigates power allocation for a MIMO collaborative co-existing radar and communications (CERC) system. Utilizing the received signal models developed for the collaborative CERC system, we propose to formulate a non-cooperative game to allocate power to each antenna in the MIMO radar and MIMO communications subsystems, treating the two subsystems as players. The radar target detection probability and communications mutual information are derived, the dominant terms of which are used as utility functions associated with the two players respectively. We introduce the Nash equilibrium (NE) for the formulated game problem and prove the existence and uniqueness of the NE. Numerical results are presented to demonstrate the efficacy of the proposed game strategy.**

***Index Terms*—Collaborative co-existing radar and communications (CERC), multiple-input and multiple-output (MIMO), Nash equilibrium (NE), power allocation, non-cooperative game.**

I. Introduction

With the increasing demands for spectrum and strain on limited resources, the research on co-existing radar and communications (CERC) system has become one of the major trends. The CERC system comprises a radar subsystem and a communications subsystem. Although initially the radar and communications subsystems are viewed as interference producers for each other, in the recently proposed collaborative CERC system, the two subsystems are regarded as helpers for each other, where they share available signals and positions, etc. with each other, such that the radar subsystem can make use of communications signals to extract target information to enhance the detection [1] and estimation performance [2], [3], etc., and the communications performance can also be improved if the radar findings are shared with the communications side [4], [5]. This paper investigates a collaborative CERC system that incorporates multiple-input and multiple-output (MIMO) radar and communications antennas.

In the study of CERC system, power allocation is one of the important problems. In [6], the authors proposed two designs for spectrum sharing problem by maximizing the output signal-to-interference-plus-noise ratio (SINR) while maintaining certain communication throughput and power constraints. The work in [7] takes the total power consumption of radar system as the objective function to solve the power distribution problem under certain constraints. The authors in [8] proposed a sequential optimization algorithm to solve a joint maximization problem. In this work, we formulate two separate objective functions for the radar and communications subsystems in the collaborative CERC systems to address a multi-objective optimization problem under the constraint of total power.

Increasing power can improve the performance of both the radar and communications parts. Thus, the total power constraints leads to a tradeoff between power allocated to the radar and communications subsystems. In this case, we consider game theory for solving the power allocation problem. Game theory [9], [10] has been applied to engineering applications such as biology [11], computer science [12], radar and communications [13]–[17], etc. In [13], the authors analyzed Nash equilibrium for power allocation schemes in a MIMO radar network. The work in [14] proposed a joint beamforming and power allocation algorithm in a MIMO radar-jammer system based on game theory. Power allocation based on Stackelberg game for the [15] and the non-cooperative game [17] has been studied for the non-collaborative CERC system. In this paper, we apply non-cooperative game theory to address the power allocation problem in the collaborative CERC system. We present the Nash equilibrium (NE), prove the existence and uniqueness of the NE, and provide a power allocation algorithm for the MIMO collaborative CERC system.

II. Problem Formulation

Consider a MIMO collaborative CERC system where the MIMO radar subsystem comprises M_R transmitters and N_R receivers located at $(x_m^t, y_m^t), m = 1, ..., M_R$ and $(x_n^r, y_n^r), n = 1, ..., N_R$, respectively. The emitted signal at the mth transmitter is $\sqrt{E_{R,m}} s_{R,m}(kT_s)$, where $E_{R,m}$ denotes the transmit power, T_s is the sampling period, and k ($k = 1, ..., K$) is an index that runs over samples at different times. The MIMO communications subsystem has M_C transmitters and N_C receivers located at $(x_{m'}^t, y_{m'}^t), m' = 1, ..., M_C$ and $(x_{n'}^r, y_{n'}^r), n' = 1, ..., N_C$, respectively. The transmitted signal at the m'th transmitter is $\sqrt{E_{C,m'}} s_{C,m'}(kT_s)$. The target, if present, is located at $\boldsymbol{\theta} = (x, y)$. The signal received at kT_s

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at the nth ($n=1,...,N_R$) radar receiver can be written as

$$\begin{aligned} r_{R,n}[k] &= \sum\nolimits_{m=1}^{M_R} \zeta_{Rt,nm}\sqrt{E_{R,m}}s_{R,m}\left(kT_s-\tau_{Rt,nm}\right) \\ &+ \sum\nolimits_{m=1}^{M_R} \zeta_{R,nm}\sqrt{E_{R,m}}s_{R,m}\left(kT_s-\tau_{R,nm}\right) \\ &+ \sum\nolimits_{m'=1}^{M_C} \zeta_{Ct,nm'}\sqrt{E_{C,m'}}s_{C,m'}\left(kT_s-\tau_{Ct,nm'}\right) \\ &+ \sum\nolimits_{m'=1}^{M_C} \zeta_{C,nm'}\sqrt{E_{C,m'}}s_{C,m'}\left(kT_s-\tau_{C,nm'}\right) + w_{R,n}[k] \end{aligned} \tag{1}$$

where the target reflection coefficient $\zeta_{Rt,nm} = \delta_{Rt,nm}/(d_m d_n)$, the fading coefficient $\zeta_{R,nm} = \delta_{R,nm}/d_{mn}$, and the $\zeta_{Ct,nm'}$ and $\zeta_{C,nm'}$ are defined similarly. The $d_m = \sqrt{(x_m^t - x)^2 + (y_m^t - y)^2}$ denotes the distance between the target and the mth radar transmitter, and $d_n = \sqrt{(x_n^r - x)^2 + (y_n^r - y)^2}$ the distance between the target and the nth radar receiver. The distance between the radar transmitter and the radar receiver is denoted by $d_{mn} = \sqrt{(x_m^t - x_n^r)^2 + (y_m^t - y_n^r)^2}$. The terms $d_{m'}, d_{n'}$ and $d_{m'n'}$ are similarly defined. The corresponding time delays are represented by $\tau_{Rt,nm}$, $\tau_{Ct,nm'}$, $\tau_{R,nm}$ and $\tau_{C,nm'}$, and $w_{R,n}[k]$ denotes the white Gaussian noise. Define the observation vector of the nth radar receiver as $\mathbf{r}_{R,n} = (r_{R,n}[1],\ldots,r_{R,n}[K])^\dagger$, where $(\cdot)^\dagger$ means transpose. Then, the radar received signal vector can be expressed as

$$\begin{aligned} \mathbf{r}_R &= \left[\mathbf{r}_{R,1}^\dagger,\ldots,\mathbf{r}_{R,N_R}^\dagger\right]^\dagger \\ &= \mathbf{U}_{Rt}\mathbf{s}_{Rt} + \mathbf{U}_R\mathbf{s}_R + \mathbf{U}_{Ct}\mathbf{s}_{Ct} + \mathbf{U}_C\mathbf{s}_C + \mathbf{w}_R, \end{aligned} \tag{2}$$

where $\mathbf{U}_{Rt} = \text{Diag}\left\{\mathbf{u}_{Rt,1}^\dagger[1],\ldots,\mathbf{u}_{Rt,N_R}^\dagger[K]\right\}$, the operator $\text{Diag}\{\cdot\}$ denotes the block diagonal, $\mathbf{u}_{Rt,n}[k] = (u_{Rt,n1}[k],\ldots,u_{Rt,nM_R}[k])^\dagger$, $u_{Rt,nm}[k] = \zeta_{Rt,nm}\sqrt{E_{R,m}}$, $\mathbf{s}_{Rt} = \left(\mathbf{s}_{Rt,1}[1]^\dagger,\ldots,\mathbf{s}_{Rt,N_R}[K]^\dagger\right)^\dagger$, and $\mathbf{s}_{Rt,n}[k] = [s_{R,1}(kT_s - \tau_{Rt,n1}),\ldots,s_{R,M_R}(kT_s - \tau_{Rt,nM_R})]^T$. The $\mathbf{U}_R$, $\mathbf{U}_{Ct}$, $\mathbf{U}_C$, $\mathbf{s}_R$, $\mathbf{s}_{Ct}$, $\mathbf{s}_C$ are defined similarly. The noise vector $\mathbf{w}_R = [\mathbf{w}_{R,1}^\dagger,\ldots,\mathbf{w}_{R,N_R}^\dagger]^\dagger$, and $\mathbf{w}_{R,n} = (\mathbf{w}_{R,n}[1],\ldots,\mathbf{w}_{R,n}[K])^\dagger$, where $\mathbf{w}_R$ is assumed Gaussian distributed with zero mean and covariance matrix $\mathbf{Q}_R$.

The signal received at the communications receiver can be defined similar to (1). Using $\widetilde{\zeta}_{Ct,n'm'}$, $\widetilde{\zeta}_{Rt,n'm}$, $\widetilde{\zeta}_{R,n'm}$ and $\widetilde{\zeta}_{C,n'm'}$ to denote the corresponding coefficients, and $\widetilde{\tau}_{Rt,nm}$, $\widetilde{\tau}_{Ct,nm'}$, $\widetilde{\tau}_{R,nm}$ and $\widetilde{\tau}_{C,nm'}$ to denote the time delays, defining $\mathbf{r}_{C,n'} = (r_{C,n'}[1],\ldots,r_{C,n'}[K])^\dagger$, the communications received signal vector can be written as

$$\begin{aligned} \mathbf{r}_C &= \left[\mathbf{r}_C^\dagger,\ldots,\mathbf{r}_{C,N_C}^\dagger\right]^\dagger \\ &= \widetilde{\mathbf{U}}_{Ct}\tilde{\mathbf{s}}_{Ct} + \widetilde{\mathbf{U}}_C\tilde{\mathbf{s}}_C + \widetilde{\mathbf{U}}_{Rt}\tilde{\mathbf{s}}_{Rt} + \widetilde{\mathbf{U}}_R\tilde{\mathbf{s}}_R + \mathbf{w}_C, \end{aligned} \tag{3}$$

where $\widetilde{\mathbf{U}}_{Ct} = \text{Diag}\left\{\tilde{\mathbf{u}}_{Ct,1}^\dagger[1],\tilde{\mathbf{u}}_{Ct,1}^\dagger[2],\ldots,\tilde{\mathbf{u}}_{Ct,N_C}^\dagger[K]\right\}$, $\tilde{\mathbf{u}}_{Ct,n'}[k] = (\tilde{u}_{Ct,n'1}[k],\ldots,\tilde{u}_{Ct,n'M_C}[k])^\dagger$, $\tilde{u}_{Ct,n'm}[k] = \widetilde{\zeta}_{Ct,n'm}\sqrt{E_{C,m}}$, and $\widetilde{\mathbf{s}}_{Ct} = \left(\widetilde{\mathbf{s}}_{Ct,1}[1]^\dagger,\ldots,\widetilde{\mathbf{s}}_{Ct,N_C}[K]^\dagger\right)^\dagger$. The terms $\widetilde{\mathbf{U}}_R$, $\widetilde{\mathbf{U}}_{Ct}$, $\widetilde{\mathbf{U}}_C$, $\widetilde{\mathbf{S}}_C$, $\widetilde{\mathbf{S}}_{Rt}$, $\widetilde{\mathbf{S}}_R$ are defined similarly. The noise vector $\mathbf{w}_C = \left[\mathbf{w}_{C,1}^\dagger,\ldots,\mathbf{w}_{C,N_C}^\dagger\right]^\dagger$, where $\mathbf{w}_{C,n'} = (\mathbf{w}_{C,n'}[1],\ldots,\mathbf{w}_{C,n'}[K])^\dagger$, and $\mathbf{w}_C$ is assumed Gaussian distributed with zero mean and covariance matrix $\mathbf{Q}_C$.

A. Radar Detection Probability

When the task of the radar subsystem is to detect the existence of a target in the cell-under-test (CUT), employing the Neyman-Pearson (NP) criterion, the radar target detection problem can be described as

$$\begin{aligned} &\mathcal{H}_0 : \mathbf{r}_R = \mathbf{U}_R\mathbf{s}_R + \mathbf{U}_C\mathbf{s}_C + \mathbf{w}_R \\ &\mathcal{H}_1 : \mathbf{r}_R = \mathbf{U}_{Rt}\mathbf{s}_{Rt} + \mathbf{U}_R\mathbf{s}_R + \mathbf{U}_{Ct}\mathbf{s}_{Ct} + \mathbf{U}_C\mathbf{s}_C + \mathbf{w}_R. \end{aligned} \tag{4}$$

The logarithmic likelihood ratio can be computed

$$\ln\frac{p(\mathbf{r}_R|\mathcal{H}_1)}{p(\mathbf{r}_R|\mathcal{H}_0)} = \mathbf{x}_{R,\mathcal{H}_0}^H\mathbf{Q_R}^{-1}\mathbf{x}_{R,\mathcal{H}_0} - \mathbf{x}_{R,\mathcal{H}_1}^H\mathbf{Q_R}^{-1}\mathbf{x}_{R,\mathcal{H}_1} \tag{5}$$

where $\mathbf{x}_{R,\mathcal{H}_1} = \mathbf{r}_R - \mathbf{U}_{Rt}\mathbf{s}_{Rt} - \mathbf{U}_R\mathbf{s}_R - \mathbf{U}_{Ct}\mathbf{s}_{Ct} - \mathbf{U}_C\mathbf{s}_C$, $\mathbf{x}_{R,\mathcal{H}_0} = \mathbf{r}_R - \mathbf{U}_R\mathbf{s}_R - \mathbf{U}_C\mathbf{s}_C$. Ignoring the influence of terms unrelated to $\mathbf{r}_R$ in (5), the detection statistic is

$$T_R = \mathbf{r}_R^H\mathbf{Q}_R^{-1}\left(\mathbf{U}_{Rt}\mathbf{s}_{Rt} + \mathbf{U}_{Ct}\mathbf{s}_{Ct}\right) + \left(\mathbf{U}_{Rt}\mathbf{s}_{Rt} + \mathbf{U}_{Ct}\mathbf{s}_{Ct}\right)^H\mathbf{Q}_R^{-1}\mathbf{r}_R.$$

The distribution of T_R under $\mathcal{H}_0$ and $\mathcal{H}_1$ follows $T_R|\mathcal{H}_0 \sim \mathcal{N}(\mu_0,\sigma^2)$ and $T_R|\mathcal{H}_1 \sim \mathcal{N}(\mu_1,\sigma^2)$, where

$$\begin{aligned} &\mu_0 = 2\Re\left\{\left(\mathbf{U}_R\mathbf{s}_R + \mathbf{U}_C\mathbf{s}_C\right)^H\mathbf{Q}_R^{-1}\left(\mathbf{U}_{Rt}\mathbf{s}_{Rt} + \mathbf{U}_{Ct}\mathbf{s}_{Ct}\right)\right\}, \\ &\mu_1 = 2\Re\left\{\left(\mathbf{U}_R\mathbf{s}_R + \mathbf{U}_C\mathbf{s}_C + \mathbf{U}_{Rt}\mathbf{s}_{Rt} + \mathbf{U}_{Ct}\mathbf{s}_{Ct}\right)^H\mathbf{Q}_R^{-1}\left(\mathbf{U}_{Rt}\mathbf{s}_{Rt} + \mathbf{U}_{Ct}\mathbf{s}_{Ct}\right)\right\}, \\ &\sigma^2 = 2(\mathbf{U}_{Rt}\mathbf{s}_{Rt} + \mathbf{U}_{Ct}\mathbf{s}_{Ct})^H\mathbf{Q}_R^{-1}\left(\mathbf{U}_{Rt}\mathbf{s}_{Rt} + \mathbf{U}_{Ct}\mathbf{s}_{Ct}\right), \end{aligned}$$

in which $\Re\{\cdot\}$ produces the real part of a complex number.

It can be derived that the false alarm probability $P_{\text{FA}} = P\left(T_R > \beta \mid H_0\right) = Q\left(\left(\beta - \mu_0\right)/\sigma\right)$, where $Q(\cdot)$ represents the complementary distribution function of the standard Gaussian distribution, and β denotes the detection threshold, and the detection probability is

$$P_D|\boldsymbol{\theta} = P\left(T_R \geq \beta \mid H_1\right) = Q\left(Q^{-1}\left(P_{\text{FA}}\right) + \left(\mu_0 - \mu_1\right)/\sigma\right). \tag{6}$$

B. Communications Mutual Information

In the collaborative CERC system, the transmitted signals and antenna positions can be shared between the two subsystems, which can help the communications receiver to estimate the target position $\boldsymbol{\theta}$. Target returns due to radar transmission can be eliminated by using the estimation results [4]. Assume that the communications transmitting signals are Gaussian. The ML estimate of $\boldsymbol{\theta}$ can be computed as

$$\begin{aligned} \hat{\boldsymbol{\theta}}_{ML} = \arg\max_{\boldsymbol{\theta}} \log\Big\{1/\left(\pi^{KN_C}\det(\mathbf{A})\right) \\ \times \exp\left[-\left(\mathbf{r}_C - \widetilde{\mathbf{U}}_R\tilde{\mathbf{s}}_R - \widetilde{\mathbf{U}}_{Rt}\tilde{\mathbf{s}}_{Rt}\right)^H \mathbf{A}^{-1}\left(\mathbf{r}_C - \widetilde{\mathbf{U}}_R\tilde{\mathbf{s}}_R - \widetilde{\mathbf{U}}_{Rt}\tilde{\mathbf{s}}_{Rt}\right)\right]\Big\}, \end{aligned}$$

where $\widetilde{\mathbf{S}}_C = \mathbb{E}\left\{\widetilde{\mathbf{s}}_C\widetilde{\mathbf{s}}_C^H\right\}$, $\widetilde{\mathbf{S}}_{Ct} = \mathbb{E}\left\{\widetilde{\mathbf{s}}_C\widetilde{\mathbf{s}}_{Ct}^H\right\}$, $\widetilde{\mathbf{S}}_{Ctt} = \mathbb{E}\left\{\widetilde{\mathbf{s}}_{Ct}\widetilde{\mathbf{s}}_{Ct}^H\right\}$ and $\mathbb{E}\{\cdot\}$ represents mathematical expectation, $\mathbf{A}$ denotes the covariance matrix of $\mathbf{r}_C$. According to [4], we can obtain the estimated time delays utilizing the estimated target position. The estimated time delays due to target reflection can be written as $\hat{\tau}_{C_t,n'm'} = \tilde{\tau}_{C_t,n'm'} + n_{C_t,n'm'}, \hat{\tau}_{R_t,n'm} = \tilde{\tau}_{R_t,n'm} + n_{R_t,n'm}$, where $n_{C_t,n'm'}$ and $n_{R_t,n'm}$ are the estimation errors. Replacing $\tilde{\tau}_{R_t,n'm}$ with $\hat{\tau}_{R_t,n'm}$, the communications received signal vector in (3) after eliminating the terms contributed by radar can be written as

$$\mathbf{r}_C' = \widetilde{\mathbf{U}}_{Ct}\tilde{\mathbf{S}}_{Ct} + \widetilde{\mathbf{U}}_C\tilde{\mathbf{s}}_C + \widetilde{\mathbf{V}}_{Rt}\tilde{\mathbf{n}}_{Rt} + \mathbf{w}_C,$$

where $\widetilde{\mathbf{V}}_{Rt} = \text{Diag}\left\{\widetilde{\mathbf{V}}_{Rt,1},\ldots,\widetilde{\mathbf{V}}_{Rt,N_C}\right\}$, $\widetilde{\mathbf{V}}_{Rt,n'} = \left(\tilde{\mathbf{v}}_{Rt,n'}[1],\ldots,\tilde{\mathbf{v}}_{Rt,n'M_R}[K]\right)^\dagger$, $\tilde{\mathbf{v}}_{Rt,n'}[k] = (\tilde{v}_{Rt,n'1}[k],\ldots,\tilde{v}_{Rt,n'M_R[k]})^\dagger$, $\tilde{v}_{Rt,n'm}[k] =$

$\sqrt{E_{R,m}}\tilde{\zeta}_{Rt,n'm}s^{(1)}_{R,m}\left(kT_s-\tilde{\tau}_{Rt,n',m}\right)$, in which $s^{(1)}_{R,m}(t)$ denotes the derivative of $s_{R,m}(t)$ with respect to t. The mutual information can be computed [4]

$$\begin{aligned}\mathrm{MI}|\boldsymbol{\theta} &= \log\det\left\{\mathbf{I}+\left(\widetilde{\mathbf{V}}_{Rt}\mathbf{Q}_{Rt}\widetilde{\mathbf{V}}_{Rt}^{H}+\mathbf{Q}_{C}\right)^{-1}\left(\widetilde{\mathbf{U}}_{C}\tilde{\mathbf{S}}_{C}\widetilde{\mathbf{U}}_{C}^{H}\right.\right.\\ &\quad\left.\left.+\widetilde{\mathbf{U}}_{C}\tilde{\mathbf{S}}_{Ct}\widetilde{\mathbf{U}}_{Ct}^{H}+\widetilde{\mathbf{U}}_{Ct}\widetilde{\mathbf{S}}_{Ct}^{H}\widetilde{\mathbf{U}}_{C}+\widetilde{\mathbf{U}}_{Ct}\widetilde{\mathbf{S}}_{Ctt}\widetilde{\mathbf{U}}_{Ct}^{H}\right)\right\}\\ &= \sum_{n'=1}^{N_C}\sum_{k=1}^{K}\log\left(\frac{\sum_{m=1}^{M_C}\sum_{m'=1}^{M_C}\chi_{m,m',n',k}\sqrt{E_{C,m}E_{C,m'}}}{\sigma^2_{Ct,n'}[k]+\sum_{m=1}^{M_R}\psi_{n',m,k}E_{R,m}}+1\right),\end{aligned} \tag{7}$$

where

$$\begin{aligned}\chi_{m,m',n,k} &= \widetilde{\zeta}_{C,n'm}\widetilde{\zeta}_{C,n'm'}\mathbb{E}\left[\widetilde{s}_{C,m}\left(kT_s-\widetilde{\tau}_{c,n'm}\right)\widetilde{s}_{C,m'}\left(kT_s-\widetilde{\tau}_{c,n'm'}\right)\right]\\ &\quad+2\widetilde{\zeta}_{C,n'm}\widetilde{\zeta}_{Ct,n'm'}\mathbb{E}\left[\widetilde{s}_{C,m}\left(kT_s-\widetilde{\tau}_{c,n'm}\right)\widetilde{s}_{C,m'}\left(kT_s-\widetilde{\tau}_{ct,n'm'}\right)\right]\\ &\quad+\widetilde{\zeta}_{Ct,n'm}\widetilde{\zeta}_{Ct,n'm'}\mathbb{E}\left[\widetilde{s}_{C,m}\left(kT_s-\widetilde{\tau}_{ct,n'm}\right)\widetilde{s}_{C,m'}\left(kT_s-\widetilde{\tau}_{ct,n'm'}\right)\right],\\ \psi_{n,m,k} &= \zeta^2_{Rt,n'm}\mathbb{E}\left\{\widetilde{n}_{Rt,n'm}\widetilde{n}^{\dagger}_{Rt,n'm}\right\}\left(s^{(1)}_{R,m}\left(kT_s-\widetilde{\tau}_{Rt,n'm}\right)\right)^2.\end{aligned}$$

III. Non-cooperative game Based Power Allocation

For the MIMO collaborative CERC system, even if the radar and communications subsystems help each other and share useful information, there is a tradeoff between the power allocated to each of the transmitters associated with both the radar and communications subsystems. In this section, we formulate the power allocation problem as a non-cooperative game to address the tradeoff between the power allocated to the radar and communications transmitters under a total power constraint. Defining $\mathfrak{E}$ as the total transmit power of the MIMO collaborative CERC system, and η_R the allocation weights associated with the radar transmitted power, the power allocated to the radar and communications subsystems are $\mathfrak{E}_R=\eta_R\mathfrak{E}$ and $\mathfrak{E}_C=(1-\eta_R)\mathfrak{E}$. The power allocation vector indicating the power allocated to different radar transmitters is $\boldsymbol{E}_R=\{E_{R,1},\cdots,E_{R,M_R}\}$, where $E_{R,m}\in[0,\mathfrak{E}_R]$. The power allocation vector for the communications subsystem is $\boldsymbol{E}_C=\{E_{C,1},\cdots,E_{C,M_C}\}$ with $E_{C,m'}\in[0,\mathfrak{E}_C]$. Use the detection probability P_D in (6) and the mutual information MI in (7) as the performance metric for the radar and communications subsystem, respectively. The power allocation problem can be formulated as an optimization problems with two objective functions as below

$$\begin{aligned}&(\mathcal{P}.1)\quad \max_{\boldsymbol{E}_R,\boldsymbol{E}_C}\ P_D\left(\boldsymbol{E}_R,\boldsymbol{E}_C\right)\\ &(\mathcal{P}.2)\quad \max_{\boldsymbol{E}_R,\boldsymbol{E}_C}\ \mathrm{MI}\left(\boldsymbol{E}_R,\boldsymbol{E}_C\right)\\ &s.t.\quad \textstyle\sum_{m=1}^{M_R}E_{R,m}\leq\mathfrak{E}_R\leq\mathfrak{E},\\ &\qquad\ \textstyle\sum_{m'=1}^{M_C}E_{C,m'}\leq\mathfrak{E}_C\leq\mathfrak{E}.\end{aligned} \tag{8}$$

Noted that the previous analysis about the radar detection task of the MIMO collaborative CERC system considers a given cell-under-test (CUT). Expansion from a given CUT to an area under monitoring can be implemented by replacing the objective functions in (8) to

$$\begin{aligned}&(\mathcal{P}.3)\quad \max_{\boldsymbol{E}_R,\boldsymbol{E}_C}\ \overline{P_D}=\mathbb{E}_{\boldsymbol{\theta}}\left\{P_D|\boldsymbol{\theta}\right\}\\ &(\mathcal{P}.4)\quad \max_{\boldsymbol{E}_R,\boldsymbol{E}_C}\ \overline{\mathrm{MI}}=\mathbb{E}_{\boldsymbol{\theta}}\left\{\mathrm{MI}|\boldsymbol{\theta}\right\}\end{aligned} \tag{9}$$

where the average detection probability $\overline{P_D}$ and the average mutual information $\overline{\mathrm{MI}}$ are computed by taking average over all possible CUTs in the area of interest. For simplicity, we stick to (8) for the sequential analytical discussions, and the general case (9) are considered in the numerical examples.

A. Non-cooperative Game Model

Define the player set $\mathcal{P}$ as

$$\mathcal{P}=\{\text{Radar subsystem},\text{Communications subsystem}\},$$

the strategy set $\mathcal{E}=\{\boldsymbol{E}_R,\boldsymbol{E}_C\}$ and utility function set $\mathcal{U}=\{u_R,u_C\}$, where the radar utility function u_R and the communications utility function u_C are determined by P_D and MI in (8). Since P_D is a monotone function with respect to

$$\begin{aligned}\frac{\mu_0-\mu_1}{\sigma} &= -\left\{2\left(\sum_{m=1}^{M_R}\sum_{m'=1}^{M_R}\alpha_{m,m'}\sqrt{E_{R,m}E_{R,m'}}\right.\right.\\ &\quad\left.\left.+2\sum_{m=1}^{M_R}\sum_{m'=1}^{M_C}\beta_{m,m'}\sqrt{E_{R,m}E_{C,m'}}+\sum_{m=1}^{M_C}\sum_{m'=1}^{M_C}\gamma_{m,m'}\sqrt{E_{C,m}E_{C,m'}}\right)\right\}^{\frac{1}{2}},\end{aligned}$$

$$\begin{aligned}\alpha_{m,m'} &= \sum_{n=1}^{N_R}\sum_{k=1}^{K}\Re\left\{\zeta_{Rt,nm}\zeta_{Rt,nm'}s_{R,m}\left(kT_s-\tau_{Rt,nm}\right)s_{R,m'}\left(kT_s-\tau_{Rt,nm'}\right)\right\},\\ \beta_{m,m'} &= \sum_{n=1}^{N_R}\sum_{k=1}^{K}\Re\left\{\zeta_{Rt,nm}\zeta_{Ct,nm'}s_{R,m}\left(kT_s-\tau_{Rt,nm}\right)s_{C,m'}\left(kT_s-\tau_{Ct,nm'}\right)\right\},\\ \gamma_{m,m'} &= \sum_{n=1}^{N_R}\sum_{k=1}^{K}\Re\left\{\zeta_{Ct,nm}\zeta_{Ct,nm'}s_{C,m}\left(kT_s-\tau_{Ct,nm}\right)s_{C,m'}\left(kT_s-\tau_{Ct,nm'}\right)\right\},\end{aligned}$$

the radar utility function is defined as

$$\begin{aligned}u_R\left(\boldsymbol{E}_R,\boldsymbol{E}_C\right) &= \sum_{m=1}^{M_R}\sum_{m'=1}^{M_R}\alpha_{m,m'}\sqrt{E_{R,m}E_{R,m'}}\\ &\quad+2\sum_{m=1}^{M_R}\sum_{m'=1}^{M_C}\beta_{m,m'}\sqrt{E_{R,m}E_{C,m'}}+\sum_{m=1}^{M_C}\sum_{m'=1}^{M_C}\gamma_{m,m'}\sqrt{E_{C,m}E_{C,m'}}.\end{aligned}$$

The communications utility function is defined as

$$u_C\left(\boldsymbol{E}_R,\boldsymbol{E}_C\right)=\sum_{n'=1}^{N_C}\sum_{k=1}^{K}\log\left(\frac{\sum\limits_{m=1}^{M_C}\sum\limits_{m'=1}^{M_C}\chi_{m,m',n',k}\sqrt{E_{C,m}E_{C,m'}}}{\sigma^2_{Ct,n'}[k]+\sum\limits_{m=1}^{M_R}\psi_{n',m,k}E_{R,m}}+1\right),$$

which is monotone with respect to MI. Thus, the non-cooperative game associated with (8) can be summarized as

$$\mathcal{G}=\langle\mathcal{P},\mathcal{E},\mathcal{U}\rangle. \tag{10}$$

B. Nash Equilibrium of Power Allocation Game

In the game $\mathcal{G}$, each player wants to maximize their own utility by choosing a proper power strategy. Each player iteratively changes their optimal strategies until reaching an equilibrium point, which is called NE in the game. For the non-cooperative game, considering that each player has the correct expectation about the other players' action and chooses strategies rationally [10], the optimal strategy of each iteration process can be expressed as

$$\begin{aligned}&(\mathcal{P}.5)\quad \max_{\boldsymbol{E}_R}\ u_R\left(\boldsymbol{E}_R,\boldsymbol{E}_C\right)\\ &(\mathcal{P}.6)\quad \max_{\boldsymbol{E}_C}\ u_C\left(\boldsymbol{E}_R,\boldsymbol{E}_C\right)\\ &s.t.\quad \textstyle\sum_{m=1}^{M_R}E_{R,m}\leq\mathfrak{E}_R\leq\mathfrak{E},\\ &\qquad\ \textstyle\sum_{m'=1}^{M_C}E_{C,m'}\leq\mathfrak{E}_C\leq\mathfrak{E}.\end{aligned} \tag{11}$$

Given the initial power of each transmitter in the radar and communications subsystems, the solutions of $(\mathcal{P}.5)$ and $(\mathcal{P}.6)$ are selected as the respective strategies in the first iteration process. Introducing Lagrange multiplier λ, $(\mathcal{P}.5)$ can be expressed as

$$\mathcal{L} = \sum_{m=1}^{M_R} \sum_{m'=1}^{M_R} \alpha_{m,m'} \sqrt{E_{R,m} E_{R,m'}} + 2 \sum_{m=1}^{M_R} \sum_{m'=1}^{M_C} \beta_{m,m'} \sqrt{E_{R,m} E_{C,m'}} + \sum_{m=1}^{M_C} \sum_{m'=1}^{M_C} \gamma_{m,m'} \sqrt{E_{C,m} E_{C,m'}} + \lambda \left(\mathfrak{E}_R - \sum_{m=1}^{M_R} E_{R,m} \right)$$

Taking the derivative of $\mathcal{L}$ with respect to $E_{R,m}$ and setting the result to be zero, the optimal strategy of the power allocated to the mth radar transmitter can be obtained by

$$E_{R,m}^{*} = \left(\left(\sum_{m'=1}^{M_R} \alpha_{m,m'} \sqrt{E_{R,m'}} + 2 \sum_{m'=1}^{M_C} \beta_{m,m'} \sqrt{E_{C,m'}} \right) / \lambda \right)^2, \quad (12)$$

Define i as the iteration times, the strategy obtained in the ith iteration process is known to the $(i+1)$th iteration process. The NE can be obtained by carrying out the above process until convergence. We can obtain the results of the $(i+1)$th iteration process as

$$E_{R,m}^{(i+1)} = \left(\left(\sum_{m'=1}^{M_R} \alpha_{m,m'} \sqrt{E_{R,m'}^{(i)}} + 2 \sum_{m'=1}^{M_C} \beta_{m,m'} \sqrt{E_{C,m'}} \right) / \lambda^{(i)} \right)^2 \quad (13)$$

Updating λ by letting $\lambda^{(i+1)} = \left[\lambda^{(i)} + s_r \left(\mathfrak{E}_R - \sum_{m=1}^{M_R} E_{R,m} \right) \right]_0^{\varsigma}$, where $[x]_a^b = \max\{\min(x,b), a\}$, ς represents a positive number in the right-neighborhood of 0 and s_r is a step size. $(\mathcal{P}.6)$ can be solved similarly to obtain the $(i+1)$th iteration result of the m'th communications transmitter

$$E_{C,m'}^{(i+1)} = \left(\frac{1}{\kappa^{(i)}} \sum_{n'=1}^{N_C} \sum_{k=1}^{K} \frac{\sum\limits_{m=1}^{M_C} \chi_{m',m,n',k} \sqrt{E_{C,m}^{(i)}}}{\sum\limits_{m=1}^{M_C} \sum\limits_{m'=1}^{M_C} \chi_{m',m,n',k} \sqrt{E_{C,m}^{(i)}} + \sigma_{Ct,n'}^2 [k] + \sum\limits_{m=1}^{M_R} E_{R,m}^{(i)} \psi_{n',m,k}} \right)^2, \quad (14)$$

where $\kappa^{(i+1)} = \left[\kappa^{(i)} + s_c \left(\mathfrak{E}_C - \sum_{m'=1}^{M_C} E_{C,m'} \right) \right]_0^{\varsigma}$, in which s_c is a step size.

Based on (13) and (14), the NE of the game $\mathcal{G}$ described in the optimization problem in (11) can be obtained in an iterative way, where the detailed steps are described in Algorithm 1.

Algorithm 1 Iterative NE Algorithm

Initialize: $\boldsymbol{E}_R^{(0)}, \boldsymbol{E}_R^{*(0)}, \boldsymbol{E}_C^{(0)}, \boldsymbol{E}_C^{*(0)}, \lambda^{(0)}, \kappa^{(0)}, s_r, s_c, i=0, i'=0, \epsilon > 0$.
repeat
    repeat until $\left| E_{R,m}^{(i')} - E_{R,m}^{(i'-1)} \right| < \varepsilon$
        for $m = 1, 2, ..., M_R$ **do**
            Calculate $E_{R,m}^{(i'+1)}$ by (12) and $\lambda^{(i'+1)}$;
        end for
        set $i' \leftarrow i' + 1$;
    set $\boldsymbol{E}_{R,m}^{*(i+1)} \leftarrow \boldsymbol{E}_{R,m}^{(i')}, i' = 1$.
    repeat until $\left| E_{C,m'}^{(i')} - E_{C,m'}^{(i'-1)} \right| < \varepsilon$
        for $m' = 1, 2, ..., M_C$ **do**
            Calculate $E_{C,m'}^{(i'+1)}$ by (14) and $\kappa^{(i'+1)}$;
        end for
        set $i' \leftarrow i' + 1$;
    set $\boldsymbol{E}_{C,m'}^{*(i+1)} \leftarrow \boldsymbol{E}_{C,m'}^{(i')}, i' = 1$.
    set $i \leftarrow i + 1$.
until $\left| E_{R,m}^{*(i)} - E_{R,m}^{*(i-1)} \right| < \varepsilon$ and $\left| E_{C,m'}^{*(i)} - E_{C,m'}^{*(i-1)} \right| < \varepsilon$
Output $E_{R,m}^{*} \leftarrow E_{R,m}^{*(i)}, \forall m$ and $E_{C,m'}^{*} \leftarrow E_{C,m'}^{*(i)}, \forall m'$.

Next, we prove the existence and uniqueness of the NE.

Theorem 1 (Existence). The proposed game $\mathcal{G}$ in (10) for power allocation has at least one NE.

Proof: At least one NE exists in the game (10) if the following two conditions are satisfied [17].

(1) The strategy set $\mathcal{E}$ is a non-empty, convex and compact subset of Euclidean space; (2) The utility functions $u_R(\boldsymbol{E}_R, \boldsymbol{E}_C)$ and $u_C(\boldsymbol{E}_R, \boldsymbol{E}_C)$ are continuous and quasi-concave in definition domain.

It is apparent that game $\mathcal{G}$ satisfies the first condition. To prove the second condition, as the utility function $u_R(\boldsymbol{E}_R, \boldsymbol{E}_C)$ are continuous with respect to $E_{R,m}$ and it is easy to obtain that $\frac{\partial u_R(\boldsymbol{E}_R, \boldsymbol{E}_C)}{\partial E_{R,m}} > 0$. Define the Hessian matrix of u_R as $\mathbf{H}(u_R) = \nabla^2 (u_R(\boldsymbol{E}_R, \boldsymbol{E}_C))$. We can prove that all the leading principle minors of $\mathbf{H}(u_R)$ are less than or equal to zero, i.e. the Hessian matrix is negative semi-definite, which proves that $u_R(\boldsymbol{E}_R, \boldsymbol{E}_C)$ are quasi-concave. Similarly, we can prove that the utility function $u_C(\boldsymbol{E}_R, \boldsymbol{E}_C)$ are quasi-concave. All the utility functions are continuous and quasi-concave, which proves the existence of NE in game $\mathcal{G}$. □

Theorem 2 (Uniqueness). The NE of the proposed game $\mathcal{G}$ in (10) for power allocation is unique.

Proof: To prove the uniqueness of the NE, one need to prove that the best response strategy functions are standard functions [13]. From (12), the radar's best response strategy function is

$$f(E_{R,m}) = \left(\left(\sum_{m'=1}^{M_R} \alpha_{m,m'} \sqrt{E_{R,m'}} + 2 \sum_{m'=1}^{M_C} \beta_{m,m'} \sqrt{E_{C,m'}} \right) / \lambda \right)^2 \quad (15)$$

It is easy to show that (15) satisfies the following conditions:

(a)*Positivity:* For $m = 1, 2, ..., M_R, f(E_{R,m}) > 0$.

(b)*Monotonicity:* If $E_{R,m}^1 > E_{R,m}^2, f(E_{R,m}^1) > f(E_{R,m}^2)$.

(c)*Scalability:* For all $a > 1$, $af(E_{R,m}) > f(aE_{R,m})$.

Therefore, the best response function of radar subsystem is a standard function [13]. Similarly, we can prove that the best response function of communications subsystem is a standard function. Thus, the NE is unique in this game model. □

IV. SIMULATION RESULTS

In this section, we present numerical results to illustrate the enhanced performance achieved through the proposed power allocation strategy. In the collaborative CERC system, we assume that the radar subsystem has $M_R = 2$ transmitting antennas and $N_R = 3$ receiving antennas, the communications subsystem has $M_C = 2$ transmitting antennas and $N_C = 3$ receiving antennas. Suppose that all the radar and communications stations are located 70 km away from the origin of the coordinate system. The radar subsystem transmitted signals are $s_{R,m}(t) = \left(\frac{2}{T^2}\right)^{\frac{1}{4}} \exp(\frac{-\pi t^2}{T^2}) e^{j2\pi m f_\Delta t}$, where f_Δ denotes the frequency offset between adjacent radar transmit signals and T the pulsewidth. OFDM signals are adopted in communications transmission $s_{C,m'}(t) = \sum_{i=-\infty}^{\infty} s_{m',i}(t - iT')$, in which $s_{m',i}(t) = \sum_{n=-N_f/2}^{N_f/2-1} a_{m',i}[n] e^{j2\pi n \Delta f t} p_{T'}(t)$, where $a_{m',i}[n]$ are data symbols, $p_{T'}(t)$ is a rectangular pulse with unit amplitude and width T', Δf is the frequency spacing between two adjacent subcarriers, and N_f the number of subcarriers. Set T' = $0.01s$, $\Delta f = 125$Hz, and N_f = 6. Define the signal to clutter-plus-noise as $\mathrm{SCNR} = 10 \log_{10} \left(E/\sigma_w^2 \right)$, σ_w^2 denotes the noise variance. Set $\mathfrak{E}$ = 10KW, $\mathrm{SCNR} = -10$dB.

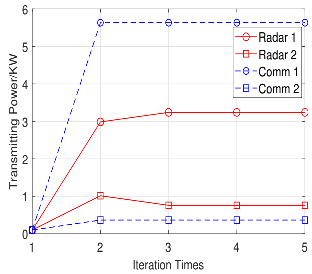

Fig. 1. Power allocation results , $\eta_R = 0.4$, Sc.1

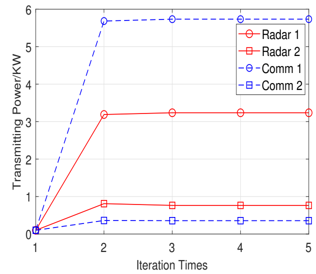

Fig. 2. Power allocation results, $\eta_R = 0.4$, Sc.2

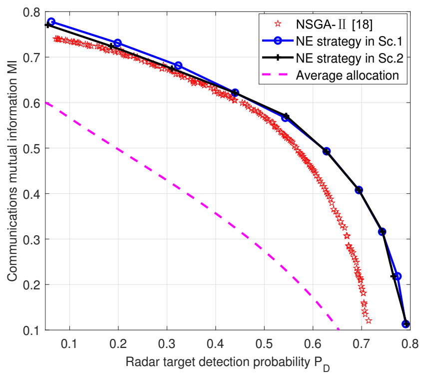

Fig. 3. MI versus P_D for the MIMO collaborative CERC system.

Set the power allocation weight $\eta_R = 0.4$. We apply Algorithm 1 to find NE for the game $\mathcal{G}$. Fig.1 plots the power allocated to each transmitters after several iteration times when the detection task of the collaborative CERC system considers a given CUT (Sc.1), and similar curves are plotted in Fig.2 for the case where the CUT is extended to an area under monitoring (Sc.2). It can be seen that the proposed algorithm can reach the optimal power allocation strategy after about 3 iteration times, which illustrates the convergence of Algorithm 1 and implies the achievement of the NE for both scenarios.

To evaluate the effectiveness of the proposed power allocation strategy, we compare its performance against the average allocation strategy [5] and that obtained using non-dominated sorting genetic algorithm-II (NSGA-II) [18]. Fig.3 plots the MI versus P_D obtained using different methods. It is seen that the proposed NE strategy brings performance improvement for both the radar and communications subsystems compared with the other methods. The P_D and MI obtained in Sc.2 approach closely to those in Sc.1, which shows that the proposed method works well for both scenarios.

V. Conclusion

This paper investigated a power allocation problem in the MIMO collaborative CERC system based on the non-cooperative game. We derived the radar target detection probability P_D and communications mutual information MI, which are regarded as the performance metrics for the radar and communications subsystems. To optimize the P_D and MI when the total power is constrained, we solved the power allocation problem based on the non-cooperative game. We proved the existence and the uniqueness of the NE, and provided an iterative algorithm to find it. Simulation results demonstrated that the NE-based power allocation strategy can enhance the performance of both subsystems in different scenarios.

References

[1] M. Bica, K.-W. Huang, U. Mitra, and V. Koivunen, "Opportunistic radar waveform design in joint radar and cellular communication systems," in *2015 IEEE Global Communications Conference (GLOBECOM)*. IEEE, 2015, pp. 1–7.

[2] M. Bica and V. Koivunen, "Delay estimation method for coexisting radar and wireless communication systems," in *2017 IEEE Radar Conference (RadarConf)*, 2017, pp. 1557–1561.

[3] M. Bică and V. Koivunen, "Radar waveform optimization for target parameter estimation in cooperative radar-communications systems," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 55, no. 5, pp. 2314–2326, 2019.

[4] Q. He, Z. Wang, J. Hu, and R. S. Blum, "Performance gains from cooperative MIMO radar and MIMO communication systems," *IEEE Signal Processing Letters*, vol. 26, no. 1, pp. 194–198, 2018.

[5] Z. Wang, Q. He, and R. S. Blum, "Target localization and mutual information improvement for cooperative MIMO radar and mimo communication systems," in *2019 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pp. 4315–4319.

[6] F. Wang and H. Li, "Power allocation for coexisting multicarrier radar and communication systems in cluttered environments," *IEEE Transactions on Signal Processing*, vol. 69, pp. 1603–1613, 2021.

[7] Y. Wang, C. Shi, F. Wang, and J. Zhou, "LPI-based optimal radar power allocation for target time delay estimation in joint radar and communications system," in *2020 IEEE 11th Sensor Array and Multichannel Signal Processing Workshop (SAM)*, pp. 1–4.

[8] T. Tian, T. Zhang, G. Li, and T. Zhou, "Mutual information-based power allocation and co-design for multicarrier radar and communication systems in coexistence," *IEEE Access*, pp. 159 300–159 312, 2019.

[9] S. Abapour, M. Nazari-Heris, B. Mohammadi-Ivatloo, and M. Tarafdar Hagh, "Game theory approaches for the solution of power system problems: A comprehensive review," *Archives of Computational Methods in Engineering*, vol. 27, pp. 81–103.

[10] T. S. Ferguson, *A course in game theory*. World Scientific, 2020.

[11] A. Lamiable, F. Quessette, S. Vial, D. Barth, and A. Denise, "An algorithmic game-theory approach for coarse-grain prediction of RNA 3D structure," *IEEE/ACM Transactions on Computational Biology and Bioinformatics*, vol. 10, no. 1, pp. 193–199, 2013.

[12] X. Huang, S. Garg, J. Nie, W. Y. B. Lim, Y. Qi, Y. Zhang, and M. S. Hossain, "Toward efficient data trading in AI enabled reconfigurable wireless sensor network using contract and game theories," *IEEE Transactions on Network Science and Engineering*, vol. 9, no. 1, pp. 98–108, 2022.

[13] A. Deligiannis, A. Panoui, S. Lambotharan, and J. A. Chambers, "Game-theoretic power allocation and the Nash equilibrium analysis for a multistatic mimo radar network," *IEEE Transactions on Signal Processing*, vol. 65, no. 24, pp. 6397–6408, 2017.

[14] B. He and H. Su, "Game theoretic countermeasure analysis for multistatic radars and multiple jammers," *Radio Science*, pp. 1–14, 2021.

[15] C. Shi, F. Wang, S. Salous, and J. Zhou, "Distributed power allocation for spectral coexisting multistatic radar and communication systems based on stackelberg game," in *2019 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pp. 4265–4269.

[16] A. Garnaev, A. Petropulu, W. Trappe, and H. V. Poor, "A power control problem for a dual communication-radar system facing a jamming threat," in *2020 IEEE Radar Conference (RadarConf)*, pp. 1–6.

[17] C. Shi, F. Wang, M. Sellathurai, and J. Zhou, "Game theoretic power allocation for coexisting multistatic radar and communication systems," in *2018 14th IEEE International Conference on Signal Processing (ICSP)*, 2018, pp. 872–877.

[18] E. Yousif, F. Khan, T. Ratnarajah, and M. Sellathurai, "On the spectral coexistence of colocated MIMO radars and wireless communications systems," in *2016 IEEE 17th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*. IEEE, 2016, pp. 1–5.