Robust Pareto-Optimal Radar Receive Filter Design for Noise and Sidelobe Suppression

Costas A. Kokke[‡], Mario Coutiño[*], Richard Heusdens[†], Geert Leus[‡], Laura Anitori[*]
[*]Radar Technology, Netherlands Organisation for Applied Scientific Research, The Hague, The Netherlands
[†]Netherlands Defence Academy, Den Helder, The Netherlands
[‡]Signal Processing Systems, Delft University of Technology, Delft, The Netherlands

***Abstract*—Integrated sidelobe level is a useful measure to quantify robustness of a waveform-filter pair to unknown range clutter and multiple closely located targets. Sidelobe suppression on receive will incur a loss in the signal to noise ratio after pulse compression. We derive a pulse compression filter that has the greatest integrated sidelobe suppression possible for a given acceptable signal to noise ratio loss. The solution is given in a closed form, which can be adjusted using a single parameter to chose between greater sidelobe or interference and noise suppression. We verify the derived filter using simulations, comparing it to other proposed mismatched filter designs. To expand the robustness of the filter, we additionally investigate noise uncertainty robustness. We derive two robustness measures for noise uncertainty and analyze the performance through simulation.**

***Index Terms*—robust pulse compression, radar signal processing, filter optimization, integrated sidelobe ratio**

I. INTRODUCTION

When a reflected waveform is received at a radar system, it is practically always accompanied by nuisances such as interference. For succesful operation of a radar system, these reflected waveforms must be sufficiently isolated from the nuisances, both for detection and estimation purposes. As such, the signal-to-interference-and-noise-ratio (SINR) is an important metric when evaluating receive filter performance.

To realize high SINR for high performance after the receive filter, pulse compression is traditionally used. Using (generalized) matched filters (GMFs) to compress a pulse into a mainlobe, while the nuisances remain temporally uncompressed creates a sampling instance with optimal SINR. This sampling instance also results in optimal detectability [1]. However, GMFs do not consider sidelobes.

These sidelobes can cause some performance issues, especially in detectability of reflected waveforms. The sidelobes of strong reflected waveforms could mask weaker reflections, resulting in missed detections. Even if the weaker reflection is not masked, it still suffers from the presence of strong sidelobes in the estimation performance. Similarly, strong sidelobes can cause false alarms to occur during detection. When considering multiple-input-multiple-output (MIMO) channels and systems using multiple transmitted waveforms, the sidelobe issues described earlier get worse. While the waveforms in a set used in a MIMO radar system are nearly orthogonal to each other, there will inevitably be additional sidelobes due to crosscorrelations between the different waveforms. Luckily, single-output channel filter design methods are useful in MIMO filter design as well [2]. We can categorize methods to mitigate sidelobes as follows.

Firstly, a waveform can be designed that has an autocorrelation function that is very similar to the Dirac delta function. Such waveforms have been researched extensively [3]–[6]. Such waveforms are typically assumed to be received by a GMF. While the GMF is optimal in terms of SINR as discussed before, it does not have any guarantees on sidelobe performance, although including known signal-dependent clutter is commonly done. Because of the additional constraints/objectives in waveform design, such as peak-to-average power ratio minimization and signal dependent clutter suppression, performing the optimization might be challenging to do in a real-time system.

Secondly, a mismatched filter (MMF) can be designed [2], [6], [7]. An MMF is typically designed by taking a GMF and changing it through tapering using window functions or through an optimization procedure that reduces sidelobes at the output of the filter at the cost of the mainlobe width. This increase in mainlobe width can be seen as an SINR loss, and some MMFs are therefore designed with a maximum SINR loss constraint.

Finally, the two mentioned methods can be combined by designing waveform-filter pairs [8]. Designing waveform-filter pairs instead of a waveform and filter separately allows for greater degrees of freedom in the optimization procedure. However, such methods can be of high computational complexity, making them more challenging to use in adaptive systems.

In this work, we propose a method of obtaining an MMF for a known waveform. The resulting MMF will be Pareto optimal in terms of both SINR and weighted integrated sidelobe ratio (WISR). We pose no constraints on the type of waveform, making the method broadly applicable. We introduce our signal model and problem formulation in Section II. The filter design method will be derived in Section III. Recognizing sidelobe suppression as a robustness measure against unknown clutter, we look at further robustness to nuisances in Section IV. In Section V, we show the performance of the methods through simulations. We close with conclusions and

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suggestions for future work in Section VI.

II. Signal Model and Problem Statement

Consider a length-L target waveform as it arrives at a radar receiver, including nuisances. The received signal frame when the target reflected waveform arrives can be described by

$$\boldsymbol{y} = \underbrace{\boldsymbol{D}(\nu)\boldsymbol{x}}_{\boldsymbol{s}} + \boldsymbol{n} \in \mathbb{C}^L ,$$

where $\boldsymbol{D}(\nu)$ is a diagonal matrix which applies a Doppler shift ν to the transmit waveform $\boldsymbol{x} \in \mathbb{C}^L$, and the nuisance vector $\boldsymbol{n}$ can be comprised of different types of nuisances, mainly interference and self-noise. We will consider $\boldsymbol{n} \sim \mathcal{CN}(\boldsymbol{c}, \boldsymbol{R})$, where $\boldsymbol{c}$ is a vector collecting the clutter and $\boldsymbol{R} \in \mathbb{R}^{L\times L}$. We do not explicitly include the complex value scaling of the target's radar crosssection in our model, since, as we will see in Section III, a phase shift does not change the filter, and we can freely adjust the SINR in simulation by changing $\boldsymbol{R}$. For the remainder, we assume $\boldsymbol{s}$ to be known. In reality, one would design a filter bank for multiple Doppler shifts or use prior information, for example.

First, we would like to design a non-zero receive filter $\boldsymbol{w} \in \mathbb{C}^L$ which, when applied to $\boldsymbol{y}$, minimizes both SINR and WISR.[1] Second, we want to investigate the effect and mitigation of noise uncertainty.

We define the SINR and WISR as

$$\mathrm{SINR} = \frac{\left|\boldsymbol{w}^{\mathrm{H}}\boldsymbol{s}\right|^2}{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{R}\boldsymbol{w}} , \qquad \mathrm{WISR} = \frac{\mathrm{WISL}}{\left|\boldsymbol{w}^{\mathrm{H}}\boldsymbol{s}\right|^2} , \tag{1}$$

where $|\cdot|$ is the absolute value function and the weighted integrated sidelobe level (WISL) is given by

$$\mathrm{WISL} = \sum_{\substack{l=1-L \\ l\neq 0}}^{L-1} k_l |r_{ws}(l)|^2 , \tag{2}$$

where $r_{ws}(l)$ is the cross correlation between the receive filter and transmit waveform at discrete lag index l and k_l is the associated weight. We can reformulate the WISL to make further derivations more convenient:

$$\mathrm{WISL} = \boldsymbol{w}^{\mathrm{H}}\tilde{\boldsymbol{S}}_c^{\mathrm{H}}\boldsymbol{W}\tilde{\boldsymbol{S}}_c\boldsymbol{w} , \tag{3}$$

where $\boldsymbol{W}$ is a diagonal matrix containing all the weights k_l, and $\tilde{\boldsymbol{S}}_c$ is a convolution matrix based on $\boldsymbol{s}$ with its middle row removed such that (2) and (3) are equal.

When the weights are chosen according to the clutter powers, the matrix $\tilde{\boldsymbol{S}}_c^{\mathrm{H}}\boldsymbol{W}\tilde{\boldsymbol{S}}_c$ can be seen as a clutter sample-covariance matrix for clutter with the same Doppler shift as the target. This fact can be used to do targeted clutter suppression using the mismatched filter that we are about to propose, in addition to suppressing unknown clutters and sidelobes in general by adding a constant term to all diagonal elements of $\boldsymbol{W}$ together with the known clutter responses. Alternatively, the constant diagonal terms can instead be chosen to be variable based on known clutter positions, to account for possibly migrating clutter. One can additionally generate variants of the matrix $\tilde{\boldsymbol{S}}_c^{\mathrm{H}}\boldsymbol{W}\tilde{\boldsymbol{S}}_c$ for multiple Doppler shifts, which allows minimization of sidelobes for different Doppler shifts. For brevity we will only show results for $\tilde{\boldsymbol{S}}_c^{\mathrm{H}}\boldsymbol{W}\tilde{\boldsymbol{S}}_c$.

Our first problem is to find a filter $\boldsymbol{w}$ such that it maximizes SINR and minimizes WISR:

$$\begin{cases} \min\limits_{\boldsymbol{w}} \quad f_1(\boldsymbol{w}) = \dfrac{\boldsymbol{w}^{\mathrm{H}}\tilde{\boldsymbol{S}}_c^{\mathrm{H}}\boldsymbol{W}\tilde{\boldsymbol{S}}_c\boldsymbol{w}}{\left|\boldsymbol{w}^{\mathrm{H}}\boldsymbol{s}\right|^2} \\ \max\limits_{\boldsymbol{w}} \quad f_2(\boldsymbol{w}) = \dfrac{\left|\boldsymbol{w}^{\mathrm{H}}\boldsymbol{s}\right|^2}{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{R}\boldsymbol{w}} . \end{cases} \tag{4}$$

III. Closed-Form Pareto-Optimal Filter

The problem formulated in (4) contains two objectives that are conflicting. As such, there are an infinite number of Pareto-optimal solutions, which we cannot order in terms of overall optimality. To deal with this, (4) can be scalarized to obtain a single objective function for which we can find an optimal value. While many scalarizing functions exist, in our case it suffices to consider a straightforward weighting function. That it indeed suffices will be shown in Section V.

Before we apply our scalarization, we rewrite $f_2(\boldsymbol{w})$ to be a minimization by using $f_2^{-1}(\boldsymbol{w})$ as our new function, which is permissible since neither nominator nor denominator should reach zero[2]. This results in

$$\begin{cases} \min\limits_{\boldsymbol{w}} \quad \dfrac{\boldsymbol{w}^{\mathrm{H}}\tilde{\boldsymbol{S}}_c^{\mathrm{H}}\boldsymbol{W}\tilde{\boldsymbol{S}}_c\boldsymbol{w}}{\left|\boldsymbol{w}^{\mathrm{H}}\boldsymbol{s}\right|^2} \\ \min\limits_{\boldsymbol{w}} \quad \dfrac{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{R}\boldsymbol{w}}{\left|\boldsymbol{w}^{\mathrm{H}}\boldsymbol{s}\right|^2} . \end{cases} \tag{5}$$

Recognizing that the two numerators in (5) are equal, we scalarize the cost function to

$$\min_{\boldsymbol{w}} \quad \frac{\boldsymbol{w}^{\mathrm{H}}\left(\alpha\tilde{\boldsymbol{S}}_c^{\mathrm{H}}\boldsymbol{W}\tilde{\boldsymbol{S}}_c + (1-\alpha)\boldsymbol{R}\right)\boldsymbol{w}}{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{s}\boldsymbol{s}^{\mathrm{H}}\boldsymbol{w}} , \tag{6}$$

where $0 \leq \alpha \leq 1$ is a parameter to decide the importance of SINR versus WISR. For different α values, (6) can give us all Pareto solutions of (4) and (5). The setting $\alpha = 0$ will result in the GMF, while $\alpha = 1$ gives a filter with the greatest WISL suppression. Setting $\alpha = \frac{1}{2}$ and chosing weights according to the clutter powers results in a filter optimally suppressing known clutter and noise, equivalent to the filters used in [9].

The expression in (6) can be further reformulated by normalizing the output signal power. Defining

$$\boldsymbol{B} = \alpha\tilde{\boldsymbol{S}}_c^{\mathrm{H}}\boldsymbol{W}\tilde{\boldsymbol{S}}_c + (1-\alpha)\boldsymbol{R} , \tag{7}$$

we can rewrite (6) as

$$\begin{aligned} \min_{\boldsymbol{w}} \quad & \boldsymbol{w}^{\mathrm{H}}\boldsymbol{B}\boldsymbol{w} \\ \text{s.t.} \quad & \boldsymbol{w}^{\mathrm{H}}\boldsymbol{s} = 1 . \end{aligned}$$

This has a closed form solution, which is given by

$$\boldsymbol{w} = \left(\boldsymbol{s}^{\mathrm{H}}\boldsymbol{B}^{-1}\boldsymbol{s}\right)^{-1}\boldsymbol{B}^{-1}\boldsymbol{s} . \tag{8}$$

[1] Note that $\boldsymbol{s}$ can be zero-padded to allow the design of a longer filter.

[2] There will always be at least some self-noise present, and we should never be interested in $\boldsymbol{w}^{\mathrm{H}}\boldsymbol{s} = 0$.

IV. Noise Covariance Uncertainty

In the previous sections, we have assumed that we have access to $\boldsymbol{R}$, the noise covariance matrix. In practice, this will not be true and only an estimate of the true covariance is known, which we will call $\hat{\boldsymbol{R}}$. This estimate will change over time, due to the variance of the estimation method, which means we should be prepared to deal with uncertainty when designing our filters. In the next few sections, we will go through a few measures of robustness to handle the noise covariance uncertainty. For simplicity, we focus in this section only on noise suppression and neglect sidelobe suppression. In both of the following sections, we aim to find a matrix to replace $\boldsymbol{R}$ in (7) to add noise uncertainty robustness to the filter.

A. Worst-case Optimization

To design a robust filter, we can choose to use a minimax approach and find the worst case covariance matrix, $\boldsymbol{R}_l$. This minimax problem is described by

$$\begin{aligned} \min_{\boldsymbol{w}} \max_{\boldsymbol{R}_v \in \mathcal{V}} \quad & \boldsymbol{w}^{\mathrm{H}} \boldsymbol{R}_v \boldsymbol{w} \\ \text{s.t.} \quad & \boldsymbol{w}^{\mathrm{H}} \boldsymbol{s} = 1 \,, \end{aligned}$$

where $\mathcal{V}$, which should be a subset of the set of $L \times L$ positive definite (PD) matrices, is the set in which we expect the true $\boldsymbol{R}$ to lie. According to the minimax theorem there exists a saddle point for this problem when the sets in which $\boldsymbol{w}$ and $\boldsymbol{R}_v$ lie are compact convex sets, and the cost function is convex-concave in $\boldsymbol{w}$ and $\boldsymbol{R}_v$. Alternative conditions for the existence of a saddle point solution for the SINR maximin problem can be found in [10]. Since we can choose $\mathcal{V}$ to be compact and closed, these conditions hold and we can exchange the optimization order, and use the closed form solution of $\boldsymbol{w}$ given $\boldsymbol{R}_v$ to eliminate that variable from the optimization, which results in

$$\min_{\boldsymbol{R}_v \in \mathcal{V}} \quad \boldsymbol{s}^{\mathrm{H}} \boldsymbol{R}_v^{-1} \boldsymbol{s} \,,$$

which is a convex optimization problem [11, Example 3.4].

Solving this problem using off-the-shelf convex solvers, we can find the worst-case $\boldsymbol{R}_v$, $\boldsymbol{R}_l$. When we use $\boldsymbol{R}_l$ to make our filter $\boldsymbol{w}_l = \left(\boldsymbol{s}^{\mathrm{H}} \boldsymbol{R}_l^{-1} \boldsymbol{s}\right)^{-1} \boldsymbol{R}_l^{-1} \boldsymbol{s}$, we obtain the following relations due to the saddle point property:

$$\mathrm{SINR}(\boldsymbol{w}, \boldsymbol{R}_l) \leq \mathrm{SINR}(\boldsymbol{w}_l, \boldsymbol{R}_l) \leq \mathrm{SINR}(\boldsymbol{w}_l, \boldsymbol{R}_v) \,.$$

Now that we have a method of finding the filter that optimizes the worst-case scenario, we need to design an appropriate $\mathcal{V}$. By the distribution of $\hat{\boldsymbol{R}}$, we can define confidence intervals with respect to where we may expect $\boldsymbol{R}$ to lie. If we assume that the noise-plus-interference is described by a multivariate circularly-symmetric complex Gaussian, then a sliding window estimate of its covariance matrix is Wishart distributed if $N \geq L$, where N is the size of the sliding window. The Wishart distribution allows us to find the variance of the elements of $\hat{\boldsymbol{R}}$, which we can use to define our confidence intervals. Given those intervals for each element of the covariance matrix, we formulate

$$\begin{aligned} \min_{\boldsymbol{R}_v \in \mathbb{S}_{++}^L} \quad & \boldsymbol{s}^{\mathrm{H}} \boldsymbol{R}_v^{-1} \boldsymbol{s} \\ \text{s.t.} \quad & \boldsymbol{R}_{\min} \leq \boldsymbol{R}_v \leq \boldsymbol{R}_{\max} \,, \end{aligned} \tag{9}$$

where $\mathbb{S}_{++}^L$ is the set of real $L \times L$ PD matrices, the matrix inequalities are element-wise, and $\boldsymbol{R}_{\min}$ and $\boldsymbol{R}_{\max}$ are the lower and upper bound of our chosen confidence interval, respectively.

A norm constraint can also be used to define $\mathcal{V}$ [12], [13]:

$$\begin{aligned} \min_{\boldsymbol{R}_v \in \mathbb{S}_{++}^L} \quad & \boldsymbol{s}^{\mathrm{H}} \boldsymbol{R}_v^{-1} \boldsymbol{s} \\ \text{s.t.} \quad & \left\| \hat{\boldsymbol{R}} - \boldsymbol{R}_v \right\|_2^2 \leq c \,, \end{aligned} \tag{10}$$

where c is a positive parameter to determine the allowed distance between the covariance estimate and the worst-case covariance. This formulation has the immediate benefit of having a closed-form solution [10], [12], namely $\boldsymbol{R}_l = \hat{\boldsymbol{R}} + c\boldsymbol{I}$. We can use the same confidence intervals we used earlier to find an appropriate value for c, so we can compare the two approaches fairly.

B. Minimizing Probability of High Output Noise Power

Another way of designing a robust filter, instead of optimizing worst-case performance, would be to minimize the probability of false alarm due to noise. We may minimize the probability of the output noise power exceeding a threshold β and find the $\boldsymbol{w}$ that solves

$$\begin{aligned} \max_{\boldsymbol{w}} \quad & \mathrm{P}\Big(\boldsymbol{w}^{\mathrm{H}} \hat{\boldsymbol{R}} \boldsymbol{w} \leq \beta\Big) \\ \text{s.t.} \quad & \boldsymbol{w}^{\mathrm{H}} \boldsymbol{s} = 1 \,. \end{aligned} \tag{11}$$

If we assume, like before, that $\hat{\boldsymbol{R}}$ is Wishart distributed, then $\boldsymbol{w}^{\mathrm{H}} \hat{\boldsymbol{R}} \boldsymbol{w} \left(\boldsymbol{w}^{\mathrm{H}} \boldsymbol{V}_R \boldsymbol{w}\right)^{-1}$ is chi-squared distributed for any non-zero $\boldsymbol{w}$, where $\boldsymbol{V}_R$ is the scale matrix of $\hat{\boldsymbol{R}}$ [14]. We can now start deriving our cost function, the CDF of $\boldsymbol{w}^{\mathrm{H}} \hat{\boldsymbol{R}} \boldsymbol{w}$ which will be Gamma distributed. The random variable $\boldsymbol{w}^{\mathrm{H}} \hat{\boldsymbol{R}} \boldsymbol{w} \sim \Gamma\big(\frac{1}{2}N, 2\boldsymbol{w}^{\mathrm{H}} \boldsymbol{V}_R \boldsymbol{w}\big)$ has a CDF given by

$$\mathrm{P}\Big(\boldsymbol{w}^{\mathrm{H}} \hat{\boldsymbol{R}} \boldsymbol{w} \leq \beta\Big) = \frac{1}{\Gamma\big(\frac{N}{2}\big)} \gamma\bigg(\frac{N}{2}, \frac{\beta}{2\boldsymbol{w}^{\mathrm{H}} \boldsymbol{V}_R \boldsymbol{w}}\bigg) \,,$$

where

$$\gamma\bigg(\frac{N}{2}, \frac{\beta}{2\boldsymbol{w}^{\mathrm{H}} \boldsymbol{V}_R \boldsymbol{w}}\bigg) = \int_0^{\frac{\beta}{2\boldsymbol{w}^{\mathrm{H}} \boldsymbol{V}_R \boldsymbol{w}}} t^{\frac{N}{2}-1} \mathrm{e}^{-t} \, \mathrm{d}t \,.$$

Now that we have obtained the CDF of our random variable, we can rewrite (11) equivalently as

$$\begin{aligned} \max_{\boldsymbol{w}} \quad & \int_0^{\frac{\beta}{2\boldsymbol{w}^{\mathrm{H}} \boldsymbol{V}_R \boldsymbol{w}}} t^{\frac{N}{2}-1} \mathrm{e}^{-t} \, \mathrm{d}t \\ \text{s.t.} \quad & \boldsymbol{w}^{\mathrm{H}} \boldsymbol{s} = 1 \,. \end{aligned} \tag{12}$$

Since $t^{\frac{N}{2}-1} \mathrm{e}^{-t}$ is non-negative, the maximization of the cost in (12) boils down to making the upper bound as large as possible. Since we assume $\boldsymbol{V}_R$ to be positive-definite, we may finally rewrite (12) to

$$\begin{aligned} \min_{\boldsymbol{w}} \quad & \boldsymbol{w}^{\mathrm{H}} \boldsymbol{V}_R \boldsymbol{w} \\ \text{s.t.} \quad & \boldsymbol{w}^{\mathrm{H}} \boldsymbol{s} = 1\,. \end{aligned}$$

Since $\boldsymbol{V}_R$ is given by $N\boldsymbol{V}_R = \mathrm{E}\left\{\hat{\boldsymbol{R}}\right\} = \boldsymbol{R}$, this leads to the conclusion that the GMF is optimal in minimizing the probability of output noise power larger than β, for any positive β.

V. Numerical Results

Here we present a number of simulations verifying the performance of the proposed filter and comparisons to other filters from literature. We first discuss some of the other filter methods, then we give an overview of the simulation parameters before we finally present the numerical results.

A. Benchmark Filter

As a benchmark, we implement the filter described in [2]. This method computes the least-squares estimate of the desired filter output, $\boldsymbol{d}$, which describes a discrete Dirac delta function. The filter that produces this least-squares solution is given by $\boldsymbol{w} = \left(\boldsymbol{S}_c^H \boldsymbol{S}_c\right)^{-1} \boldsymbol{S}_c^H \boldsymbol{d}$, where $\boldsymbol{S}_c$ is a convolution matrix built from $\boldsymbol{s}$. Note that this filter is optimal in terms of integrated sidelobe ratio (ISR). To obtain filters that balance sidelobe and noise suppression, [2] proposes instead to perform a gradient descent method, initialized by the GMF and with iterations given by

$$\boldsymbol{w}^{(n+1)} = \boldsymbol{w}^{(n)} - \mu\left(\boldsymbol{S}_c^H \boldsymbol{S}_c \boldsymbol{w}^{(n)} - \boldsymbol{S}_c^H \boldsymbol{d}\right),$$

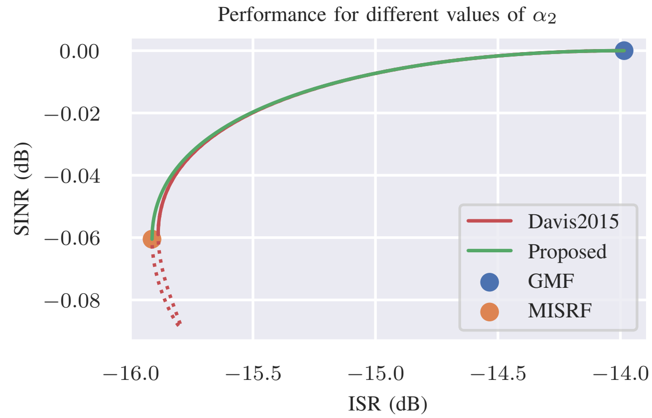
where μ is the stepsize. It is claimed that intermediate iterations of this gradient descent procedure are filters that perform a balance between sidelobe and noise suppression, if μ is sufficiently small.

B. Simulation Parameters

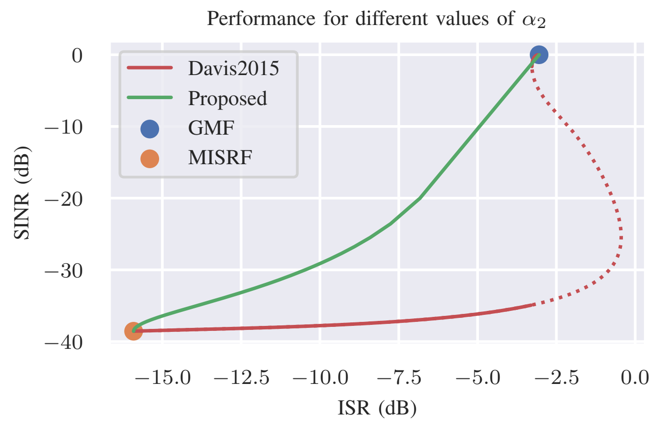
The parameters that are used in the simulations, unless stated otherwise, are: $0 \leq \alpha \leq 1$, $\boldsymbol{W} = \boldsymbol{I}$, $\mathrm{SINR} = 0\,\mathrm{dB}$ at the input of the filter, a critically sampled linear frequency modulation waveform, and $L = 255$. Lastly, for the realization of $\boldsymbol{n}$ we use white noise and colored noise. When colored noise is used, the colored to white noise ratio is $40\,\mathrm{dB}$ and the interference is an auto-regressive function given by the transfer function $H(z) = \left(1 - 1.5z^{-1} + 0.7z^{-2}\right)^{-4}$ [15, Chapter 13].

C. Results

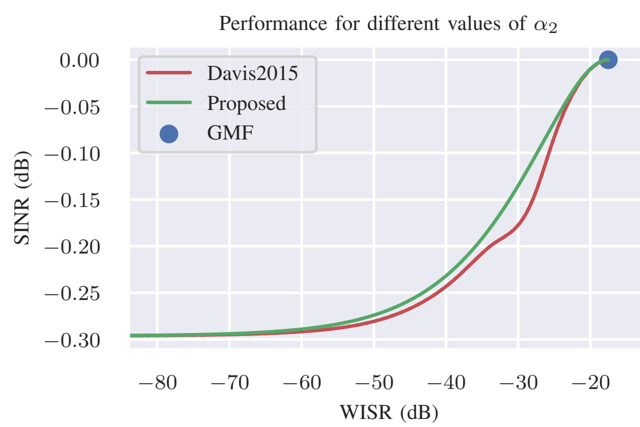
In Fig. 1, we show the Pareto figure of the cost function in (6) and the method described in Section V-A. In Fig. 1a we can see the our method always performs at least as well as the one proposed by [2]. The dotted segments indicate non-Pareto optimal points in the optimization. The winding path of the convergence of [2]'s method in Figs. 1a and 1b would not necessarily be an issue in practice, since a log of the path can be kept to choose a desirable operating point. This is unnecessary for our proposed method.



(a) Filter performance in the presence of exclusively white noise and all sidelobes weighted equally.



(b) Filter performance in the presence of colored noise and all sidelobes weighted equally.



(c) Filter performance in the presence of white noise and sidelobes beyond 42 not weighted.

Fig. 1. Performance of the possible optimal solutions and comparison filters in the presence of two types of noise. Indicated SINR is relative to the output SINR of a GMF. Dotted lines indicate intermediate results which are worse in both metrics compared to the other intermediate results.

We see in Fig. 1b that our proposed method achieves lower SINR losses for given ISRs compared to [2]'s method. We can guarantee that these SINR losses are the lowest possible for different ISRs, because we generate all Pareto optimal

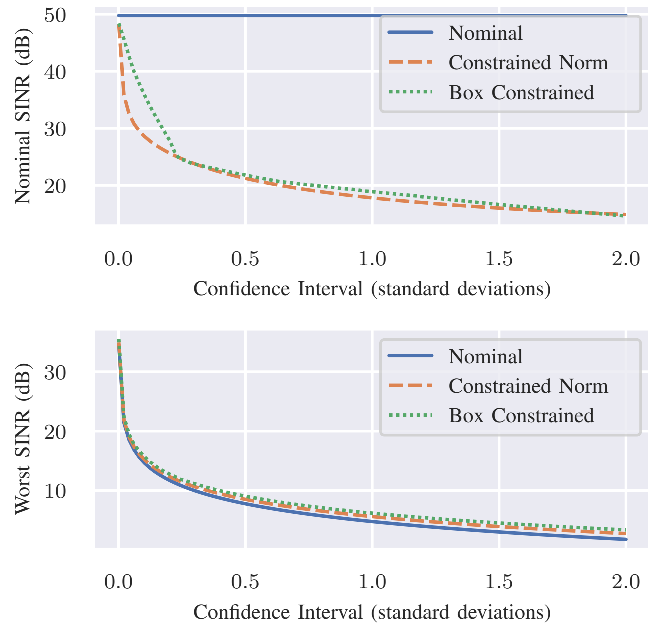


Fig. 2. Filter output SINR for the three different filters. Plotted are the SINR to the best-case, true, and worst-case noise covariance matrix. $L = 16$

solutions of (4).

Figure 1c shows the performance when only a limited number of sidelobes are suppressed, instead of all. The proposed method again performs better or equivalently to the benchmark. While it may seem that more sidelobe suppression can be attained for no more SINR loss, one should recall that this may lead to aggregiously high sidelobe levels outside of the weighted range. In practice, the sidelobes outside of that range will have small weights assigned to them in order to prevent the sidelobes to become too excessive.

In Fig. 2, we see the performance differences of a GMF and the two robust filters described in Section IV-A, for $L = 16$. The GMF is labelled 'Nominal', the filter obtained through (9) is labelled 'Box Constrained' and we labelled the filter from (10) as 'Norm Constrained'. We see that even though the worst-case performance of the constrained norm and box constrained filters are better, the difference is small. On the other hand though, the performance related to the nominal covariance matrix is heavily affected. In both cases though, (9) slightly outperforms (10), which is to be expected, given the smaller feasibility region based on the assumed statistics.

VI. Conclusions

We have proposed a method to obtain all Pareto optimal radar receive filters with maximum SINR and minimum WISR. The solution exists in closed form and all Pareto optimal solutions can be found by sweeping a single bounded parameter. Furthermore, the method allows arbitrary weighting of the sidelobes, which enables it to handle known clutter.

We have investigated robustness not only to unknown clutter, but also to noise process uncertainty. Our findings however, summarised by Fig. 2, show that the improvements to robustness come at a great cost to the expected performance. We expect that this cost may be too high, and the gain negligible, for many applications. Furthermore, in Section IV-B we showed that minimizing the probability of high noise output was equivalent to the GMF.

By jointly performing pulse compression and processing the angle of departure, as described in [2], the receive filter can be employed, without requiring any changes, in MIMO radar for optimal suppression of WISL, given an acceptable SINR loss. Additionally, it can be used to do Pareto-optimal noise and sidelobe suppression on oversampled signals such as in [7].

As future work remains the application of the closed form filter solutions in adaptive settings and considering robustness to interference that is not similar to a circularly-symmetric Gaussian distribution. Furthermore, clutter mitigation will be more successful when considering waveform optimization, and as such, it should be interesting to consider the application of robust filters in joint waveform-filter design methods such as [16].

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