Robust Beamforming for Reconfigurable Intelligent Surfaces-Aided Dynamic TDD Systems

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***Abstract*—We propose a robust beamforming design for a reconfigurable intelligent surface (RIS) aided dynamic time-division-duplexing system. The main focus is to design the optimal transmit beamforming vectors and the passive RIS reflection vector to minimize the total transmit power of the downlink cells in the presence of channel imperfections. We consider a conventional worst-case formulation that has deterministic upper bounds on the norms of the channel imperfection. We adopt a semidefinite relaxation (SDR) technique and an S-procedure to reformulate the problem into a semidefinite programming (SDP) form with linear matrix inequality (LMI) constraints. Then, we adopt the alternating optimization approach to update the active transmit beamforming vectors and the passive RIS reflection vector sequentially. Numerical results are presented showing the effectiveness of the proposed method as compared to the non-robust design.**

***Index Terms*—Dynamic TDD, MIMO communications, small cells, reconfigurable intelligent surface.**

I. Introduction

Small cells using low-power nodes are meant to be deployed in hot spots, where the number of users varies strongly with time and between adjacent cells [1]. As a result, small cells are expected to have burst-like traffic, which makes the static time division duplex (TDD) frame configuration strategy, where a common TDD pattern is selected for the whole network, not able to meet the users' requirements and the traffic fluctuations. This inadvertently leads to a high drop out rate for the small cells. Dynamic time division duplex (DTDD) has been proposed as a solution to satisfy the asymmetric and dynamic traffic demand of small cells [2]. In DTDD, each cell is allowed to dynamically reconfigure its TDD pattern based on its instantaneous traffic demand and/or interference status. In [3], the DTDD system performance was evaluated with different performance metrics, and it was found that the DTDD system provides a significant improvement in throughput as compared to the static TDD system.

The main challenge brought by DTDD is the cross-link interference issue, because adjacent cells may use different TDD frame configurations according to traffic needs at a given time, thereby giving rise to opposite transmission directions among neighboring cells. There are two kinds of cross-link interference: base station-to-base station (BS-to-BS) and user equipment-to-user equipment (UE-to-UE) interference, which may degrade the system performance significantly. Among the two, the BS-to-BS interference is extremely detrimental due to the large transmit power and line-of-sight (LOS) propagation characteristics.

Recently, reconfigurable intelligent surfaces (RISs) have gained significant attention as a cost-effective solution to improve the current and future wireless networks [4]–[6]. In a passive RIS-aided communication system, where the RIS has no radio-frequency chains, the phase shifts of the RIS elements can be adjusted to meet a certain cost function, e.g., the reflected signals add constructively at the intended users and/or destructively at the unintended users [4].

In [7]-[8], an RIS has been exploited to improve the performance of a DTDD system. The performance improvement that can be obtained from the joint design of the active and passive beamforming vectors heavily depends on the accuracy of the channel state information (CSI). However, most of the works on RIS-aided communication systems have assumed the availability of perfect CSI, i.e., there are no CSI estimation errors, e.g, [5]–[8]. In practice, only partial CSI is available at the design center, due to errors in the estimation of the channel vectors. Therefore, robust active and passive beamforming designs taking CSI imperfections into account are desirable to fully realize the potential benefits of RIS-aided communication systems.

However, all the existing contributions [9]–[11] on robust beamforming designs for RIS-aided communication systems have been considered for TDD systems. Against this background, we study robust beamforming for a RIS-aided DTDD communication system. The problem is formulated as a sum-power minimization problem for the downlink cells under individual UE's target signal-to-interference-plus-noise ratio (SINR) constraints, interference temperature constraints, and unit-modulus constraints of the RIS in the presence of CSI errors. We consider a conventional worst-case formulation that has deterministic upper bounds on the norms of the channel imperfection. We adopt a semidefinite relaxation (SDR) technique and S-procedure [12] to reformulate the problem into a semidefinite programming (SDP) form with linear matrix inequality (LMI) constraints. Then, we adopt the alternating optimization (AO) approach to update the active transmit beamforming vectors and the passive RIS reflection vector sequentially. We assume that the directions of the small cells have been optimized *a priori*, e.g., by using the cell reconfiguration method proposed in [13], and are known at the central processing unit (CPU).

II. System Model

In this paper[1], we consider an RIS-aided DTDD system consisting of Q small cells, where each cell has a BS with a uniform linear array (ULA) of N antennas serving a single UE[2] that is equipped with a single-antenna. As shown in Fig. 1, we assume that the communication is aided by an RIS with M passive reflection elements, where the BSs and the RIS are controlled by a CPU via backhaul connections. We focus on a challenging scenario where the *direct links* are not available due to unfavorable propagation conditions.

Denote $\mathcal{Q} \triangleq \{1, \cdots, Q\}$ as the set of BSs (cells). At the considered time instant, we assume that there are $|\mathcal{Q}^{\text{ul}}|$ cells operating in the uplink (UL) direction and $|\mathcal{Q}^{\text{dl}}|$ cells operating in the downlink (DL) direction, such that $|\mathcal{Q}^{\text{ul}}| + |\mathcal{Q}^{\text{dl}}| = Q$ and $\mathcal{Q}^{\text{ul}} \bigcap \mathcal{Q}^{\text{dl}} = \emptyset$. Let $\boldsymbol{H}_q \in \mathbb{C}^{M\times N}$ be the channel matrix from the qth BS to the RIS, and $\boldsymbol{h}_q \in \mathbb{C}^{M}$ be the channel vector from the RIS to the qth UE. Then, the received signal by the UE in the qth DL cell, i.e., $q \in \mathcal{Q}^{\text{dl}}$, can be expressed as

$$y_q^{(\text{dl})} = \sum_{\forall k \in \mathcal{Q}^{\text{dl}}} (\boldsymbol{h}_{q,k}^{\text{BS-UE}})^{\mathsf{H}} \boldsymbol{f}_k x_k + \sum_{\forall r \in \mathcal{Q}^{\text{ul}}} h_{q,r}^{\text{UE-UE}} \sqrt{p_r^{\text{ul}}} x_r + z_q, \quad (1)$$

where $\boldsymbol{h}_{q,k}^{\text{BS-UE}} = (\boldsymbol{h}_q^{\mathsf{H}} \boldsymbol{\Theta} \boldsymbol{H}_k)^{\mathsf{H}}$, $h_{q,r}^{\text{UE-UE}} = \boldsymbol{h}_q^{\mathsf{H}} \boldsymbol{\Theta} \boldsymbol{h}_r$, $\boldsymbol{\Theta} = \text{diag}(\boldsymbol{\theta})$ is the RIS reflection diagonal matrix, $\boldsymbol{\theta} = [e^{j\phi_1}, \cdots, e^{j\phi_M}]^{\mathsf{T}} \in \mathbb{C}^M$ with $\phi_m \in [0, 2\pi]$, $\boldsymbol{f}_k \in \mathbb{C}^N$ is the transmit precoding vector with $\|\boldsymbol{f}_k\|_2^2 = p_k^{\text{dl}}$, x_k is the unit-norm transmit symbol, z_q is additive white Gaussian noise with variance σ_q^2, and p_r^{X} is the transmit power in the $\text{X} \in \{\text{dl}, \text{ul}\}$ direction. Therefore, the SINR of the qth UE, $q \in \mathcal{Q}^{\text{dl}}$, is given as

$$\Gamma_q^{(\text{dl})} = \frac{|(\boldsymbol{h}_{q,q}^{\text{BS-UE}})^{\mathsf{H}} \boldsymbol{f}_q|^2}{\sum\limits_{\substack{\forall k \in \mathcal{Q}^{\text{dl}} \\ k \neq q}} |(\boldsymbol{h}_{q,k}^{\text{BS-UE}})^{\mathsf{H}} \boldsymbol{f}_k|^2 + \sum\limits_{\forall r \in \mathcal{Q}^{\text{ul}}} |h_{q,r}^{\text{UE-UE}}|^2 p_r^{\text{ul}} + \sigma_q^2}. \quad (2)$$

On the other hand, the total received BS-BS interference power at the rth UL BS, i.e., $r \in \mathcal{Q}^{\text{ul}}$, from the DL BSs, can be expressed as

$$\text{IP}_r = \sum_{\forall q \in \mathcal{Q}^{\text{dl}}} \|\boldsymbol{H}_{r,q}^{\text{BS-BS}} \boldsymbol{f}_q\|^2, \quad (3)$$

where $\boldsymbol{H}_{r,q}^{\text{BS-BS}} = \boldsymbol{H}_r^{\mathsf{H}} \boldsymbol{\Theta} \boldsymbol{H}_q$. This interference power should be limited by the interference threshold I_{th} in order to guarantee a certain QoS of the UL cells. Using the following matrix properties; $\text{vec}(\boldsymbol{A}\text{diag}(\boldsymbol{b})\boldsymbol{C}) = (\boldsymbol{C}^{\mathsf{T}} \diamond \boldsymbol{A})\boldsymbol{b}$ and $(\boldsymbol{AC}) \diamond (\boldsymbol{BE}) = (\boldsymbol{A} \otimes \boldsymbol{B})(\boldsymbol{C} \diamond \boldsymbol{E})$ the BS-BS interference power can be written as

$$\text{IP}_r = \sum_{\forall q \in \mathcal{Q}^{\text{dl}}} \|(\boldsymbol{f}_q^{\mathsf{T}} \otimes \boldsymbol{I}_N)(\boldsymbol{G}_{r,q})\boldsymbol{\theta}\|^2 \quad (4)$$

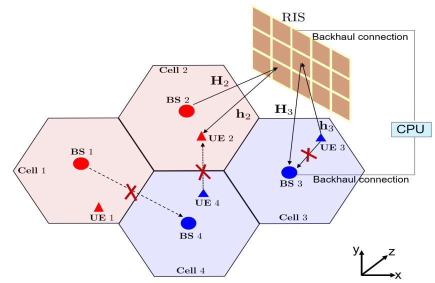


Fig. 1: An RIS-aided DTDD system comprising $Q = 4$ cells.

where $\boldsymbol{G}_{r,q} = (\boldsymbol{H}_q^{\mathsf{T}} \diamond \boldsymbol{H}_r^{\mathsf{H}}) \in \mathbb{C}^{N^2 \times M}$ is the cascaded (BS-RIS-BS) channel from the qth DL BS to the rth UL BS via the RIS. Assume that $\boldsymbol{E}_{q,q} = \text{diag}(\boldsymbol{h}_q^{\mathsf{H}})\boldsymbol{H}_q \in \mathbb{C}^{M\times N}$ denotes the cascaded (BS-RIS-UE) channel from the qth BS through the RIS to the UE in the qth DL cell, and $\boldsymbol{e}_{q,r} = \text{diag}(\boldsymbol{h}_q^{\mathsf{H}})\boldsymbol{h}_r \in \mathbb{C}^M$ the cascaded (UE-RIS-UE) channel from the UE in the rth UL cell through the RIS to the UE in the qth DL cell. In this paper, we consider a scenario where the CSI for the cascaded UE-RIS-UE, BS-RIS-UE, and BS-RIS-BS channels $\{\boldsymbol{e}_{q,r}, \boldsymbol{E}_{q,q}, \boldsymbol{G}_{r,q}\}$ are known with imperfections. Therefore, the imperfect cascaded channels can be represented as

$$\boldsymbol{e}_{q,r} = \hat{\boldsymbol{e}}_{q,r} + \boldsymbol{\delta}_{q,r} \quad \forall q, r \in \mathcal{Q} \quad (5)$$

$$\boldsymbol{E}_{q,q} = \hat{\boldsymbol{E}}_{q,q} + \boldsymbol{\Delta}_{q,q}, \quad \forall q \in \mathcal{Q}^{\text{dl}} \quad (6)$$

$$\boldsymbol{G}_{r,q} = \hat{\boldsymbol{G}}_{r,q} + \boldsymbol{\Delta}_{r,q}, \quad \forall q, r \in \mathcal{Q} \quad (7)$$

where $\hat{\boldsymbol{e}}_{q,r}, \hat{\boldsymbol{E}}_{q,q}$, and $\hat{\boldsymbol{G}}_{r,q}$ are the estimated cascaded CSI known at the BSs through the CPU, while $\boldsymbol{\delta}_{q,r}, \boldsymbol{\Delta}_{q,q}$, and $\boldsymbol{\Delta}_{r,q}$ are the corresponding unknown channel estimation errors. Then, by applying a change of variables, $\Gamma_q^{(\text{dl})}$ and IP_r are equivalently written as

$$\Gamma_q^{(\text{dl})} = \frac{|(\boldsymbol{\theta}^{\mathsf{H}}(\hat{\boldsymbol{E}}_{q,q} + \boldsymbol{\Delta}_{q,q}))\boldsymbol{f}_q|^2}{\sum\limits_{\substack{\forall k \in \mathcal{Q}^{\text{dl}} \\ k \neq q}} |(\boldsymbol{\theta}^{\mathsf{H}}(\hat{\boldsymbol{E}}_{q,k} + \boldsymbol{\Delta}_{q,k}))\boldsymbol{f}_k|^2 + \sum\limits_{\forall r \in \mathcal{Q}^{\text{ul}}} |\boldsymbol{\theta}^{\mathsf{H}}(\hat{\boldsymbol{e}}_{q,r} + \boldsymbol{\delta}_{q,r})|^2 p_r^{\text{ul}} + \sigma_q^2} \quad (8)$$

$$\text{IP}_r = \sum_{\forall q \in \mathcal{Q}^{\text{dl}}} \|(\boldsymbol{f}_q^{\mathsf{T}} \otimes \boldsymbol{I}_N)(\hat{\boldsymbol{G}}_{r,q} + \boldsymbol{\Delta}_{r,q})\boldsymbol{\theta}\|^2 \quad (9)$$

We propose a robust approach to minimize the total transmit power of the DL cells in the network, under the constraints of satisfying the SINR requirements at the individual DL UEs, the interference leakage power at each UL cell, and the unit-modulus constraints of the RIS in the presence of channel estimation errors.

III. Worst-case Robust Beamforming Design

In this section, we consider the worst-case robust beamforming design based on the bounded uncertainty model. We assume that the norm of the channel estimation errors is upper-bounded by some known constants, i.e.,$\|\boldsymbol{\delta}_{q,r}\|_2 \leq \zeta_{q,r}$, $\|\boldsymbol{\Delta}_{q,q}\|_F \leq \zeta_{q,q}$ and $\|\boldsymbol{\Delta}_{r,q}\|_F \leq \zeta_{r,q}$.

Using this error model, the proposed worst-case beamforming approach consists of jointly designing the beamforming vectors $\boldsymbol{f}_q, \forall q \in \mathcal{Q}^{\text{dl}}$ and the passive reflection vector of the RIS $\boldsymbol{\theta}$, to meet the quality of service (QoS) and interference targets at the DL cells and UL cells, respectively, in all possible

[1]Vectors and matrices are written as lowercase and uppercase boldface letters, respectively. The notation $\diamond$ is used to denote the Khatri-Rao product, while $\otimes$ is used to denote the Kronecker product. The transpose and the conjugate transpose (Hermitian) of $\boldsymbol{X}$ are represented by $\boldsymbol{X}^{\mathsf{T}}$ and $\boldsymbol{X}^{\mathsf{H}}$, respectively. $\mathbb{H}^n$ stands for the set of $n \times n$ complex Hermitian matrices. For a matrix $\boldsymbol{A} \in \mathbb{H}^n$, we write $\boldsymbol{A} \succeq \boldsymbol{0}$ and $\boldsymbol{A} \succ \boldsymbol{0}$ to denote that $\boldsymbol{A}$ is positive semidefinite and positive definite, respectively. Furthermore, $\boldsymbol{I}_n$ denotes the $n \times n$ identity matrix, while $\|\cdot\|$, $\|\cdot\|_{\text{F}}$ represent the Euclidean norm and matrix Frobenius norm respectively.

[2]The extension of our proposed solution to multi-user scenarios is straightforward.

error cases described by the error model. The worst-cast robust design problem can be mathematically formulated as

$$\min \sum_{\forall q\in\mathcal{Q}^{\text{dl}}} \|\boldsymbol{f}_q\|^2 \tag{10a}$$
$$\text{s.t}\quad \Gamma_q^{(\text{dl})} \geq \gamma_q\ \forall q\in\mathcal{Q}^{\text{dl}} \tag{10b}$$
$$\text{IP}_r \leq I_{\text{th}},\ \forall r\in\mathcal{Q}^{\text{ul}} \tag{10c}$$
$$\|\boldsymbol{\delta}_{q,r}\|_2 \leq \zeta_{q,r} \tag{10d}$$
$$\|\boldsymbol{\Delta}_{q,q}\|_F \leq \zeta_{q,q} \tag{10e}$$
$$\|\boldsymbol{\Delta}_{r,q}\|_F \leq \zeta_{r,q} \tag{10f}$$
$$|\boldsymbol{\theta}[m]| = 1,\ m=1,\cdots,M \tag{10g}$$

where the optimization variables are $\boldsymbol{f}_q, \boldsymbol{\delta}_{q,r}, \boldsymbol{\Delta}_{q,q}, \boldsymbol{\Delta}_{r,q}$, and $\boldsymbol{\theta}$. Furthermore, γ_q is the QoS constrained SINR threshold, while I_{th} is the uplink interference threshold. Problem (10) is non-convex due to its joint optimization and the constraints in (10g). To obtain a solution, we propose an AO-based algorithm using the S-procedure and SDR techniques.

First, we introduce new slack variables, $t_{q,k},\ \forall k\diagdown q\in\mathcal{Q}^{\text{dl}}$, $I_{r,q},\ \forall\{q,r\}\in\mathcal{Q}$, and $\nu_{q,r},\ \forall\{q,r\}\in\mathcal{Q}$. Since $\boldsymbol{\Delta}_{q,q}$ and $\boldsymbol{\Delta}_{q,k}$ are independent, problem (10) can be recast equivalently as

$$\min \sum_{\forall q\in\mathcal{Q}^{\text{dl}}} \|\boldsymbol{f}_q\|^2 \tag{11a}$$
$$\text{s.t}\quad |(\boldsymbol{\theta}^{\mathsf{H}}(\hat{\boldsymbol{E}}_{q,q}+\boldsymbol{\Delta}_{q,q}))\boldsymbol{f}_q|^2 \geq \gamma_q\Big(\sum_{\substack{\forall k\in\mathcal{Q}^{\text{dl}}\\ k\neq q}} t_{q,k} + \sum_{\forall r\in\mathcal{Q}^{\text{ul}}} \nu_{q,r}p_r^{\text{ul}} + \sigma_q^2\Big) \tag{11b}$$
$$|(\boldsymbol{\theta}^{\mathsf{H}}(\hat{\boldsymbol{E}}_{q,k}+\boldsymbol{\Delta}_{q,k}))\boldsymbol{f}_k|^2 \leq t_{q,k} \tag{11c}$$
$$|\boldsymbol{\theta}^{\mathsf{H}}(\hat{\boldsymbol{e}}_{q,r}+\boldsymbol{\delta}_{q,r})|^2 \leq \nu_{q,r} \tag{11d}$$
$$\sum_{\forall q\in\mathcal{Q}^{\text{dl}}} I_{r,q} \leq I_{\text{th}},\ \forall r\in\mathcal{Q}^{\text{ul}} \tag{11e}$$
$$\|(\boldsymbol{f}_q^{\mathsf{T}}\otimes\boldsymbol{I}_N)(\hat{\boldsymbol{G}}_{r,q}+\boldsymbol{\Delta}_{r,q})\boldsymbol{\theta}\|^2 \leq I_{r,q} \tag{11f}$$
$$\|\boldsymbol{\Delta}_{q,q}\|_F \leq \zeta_{q,q} \tag{11g}$$
$$\|\boldsymbol{\Delta}_{q,k}\|_F \leq \zeta_{q,k} \tag{11h}$$
$$(10\text{d}), (10\text{f}), (10\text{g}) \tag{11i}$$

where the optimization variables are $\boldsymbol{f}_q, \boldsymbol{\delta}_{q,r}, \boldsymbol{\Delta}_{q,q}, \boldsymbol{\Delta}_{r,q}, \boldsymbol{\Delta}_{q,k}, \boldsymbol{\theta}, \nu_{q,r}, t_{q,k}$, and $I_{r,q}$. It can be easily shown (e.g, by contradiction) that the constraints (11b), (11c), and (11d) are tight (i.e., they hold with equality at optimality). Therefore, problem (11) is an equivalent reformulation of problem (10).

Furthermore, we introduce a new set of variables $\boldsymbol{F}_q = \boldsymbol{f}_q\boldsymbol{f}_q^{\mathsf{H}}, \forall q\in\mathcal{Q}^{\text{dl}}$ and $\boldsymbol{\Psi} = \boldsymbol{\theta}\boldsymbol{\theta}^{\mathsf{H}}$, where $\boldsymbol{F}_q \succeq \boldsymbol{0}, \boldsymbol{\Psi}\succeq\boldsymbol{0}, \boldsymbol{F}_q\in\mathbb{H}^{N\times N}, \boldsymbol{\Psi}\in\mathbb{H}^{M\times M}$, and $\boldsymbol{F}_q$ and $\boldsymbol{\Psi}$ are rank-one matrices. Based on the following matrix properties; $\text{Tr}(\boldsymbol{A}^{\mathsf{H}}\boldsymbol{B}) = \text{vec}^{\mathsf{H}}(\boldsymbol{A})\text{vec}(\boldsymbol{B})$ and $\text{vec}(\boldsymbol{ABC}) = (\boldsymbol{C}^{\mathsf{T}}\otimes\boldsymbol{A})\text{vec}(\boldsymbol{B})$, the left hand side (LHS) of constraints (11b), (11c), (11d), and (11f) can be written, respectively as in (12) at the top of the next page, where

$$b_q = \text{vec}^{\mathsf{H}}(\hat{\boldsymbol{E}}_{q,q})(\boldsymbol{F}_q^{\mathsf{T}}\otimes\boldsymbol{\Psi})\text{vec}(\hat{\boldsymbol{E}}_{q,q}) \tag{13a}$$
$$b_k = \text{vec}^{\mathsf{H}}(\hat{\boldsymbol{E}}_{q,k})(\boldsymbol{F}_k^{\mathsf{T}}\otimes\boldsymbol{\Psi})\text{vec}(\hat{\boldsymbol{E}}_{q,k}) \tag{13b}$$
$$u_{r,q} = \text{vec}^{\mathsf{H}}(\hat{\boldsymbol{G}}_{r,q})\boldsymbol{U}_{r,q}\text{vec}(\hat{\boldsymbol{G}}_{r,q}). \tag{13c}$$

For notational convenience, we use the following variables $\boldsymbol{B}_q = \boldsymbol{F}_q^{\mathsf{T}}\otimes\boldsymbol{\Psi}, \boldsymbol{b}_q = \boldsymbol{B}_q\text{vec}(\hat{\boldsymbol{E}}_{q,q}), \boldsymbol{B}_k = \boldsymbol{F}_k^{\mathsf{T}}\otimes\boldsymbol{\Psi}$, and $\boldsymbol{b}_k = \boldsymbol{B}_k\text{vec}(\hat{\boldsymbol{E}}_{q,k})$. Additionally, $\boldsymbol{U}_{r,q} = \boldsymbol{\Psi}^{\mathsf{T}}\otimes(\boldsymbol{F}_q^{\mathsf{T}}\otimes\boldsymbol{I}_N)$ and $\boldsymbol{u}_{r,q} = \boldsymbol{U}_{r,q}\text{vec}(\hat{\boldsymbol{G}}_{r,q})$. Then, we can rewrite problem (11) as given in (14),

$$\min \sum_{\forall q\in\mathcal{Q}^{\text{dl}}} \text{Tr}(\boldsymbol{F}_q) \tag{14a}$$
$$\text{s.t}\quad (10\text{d}), (10\text{f}), (11\text{e}), (11\text{g}), (11\text{h}) \tag{14b}$$
$$\text{vec}^{\mathsf{H}}(\boldsymbol{\Delta}_{q,q})\boldsymbol{B}_q\text{vec}(\boldsymbol{\Delta}_{q,q}) + 2\,\text{Re}\{\text{vec}^{\mathsf{H}}(\boldsymbol{\Delta}_{q,q})\boldsymbol{b}_q\} + b_q - \gamma_q\Big(\sum_{\substack{\forall k\in\mathcal{Q}^{\text{dl}}\\ k\neq q}} t_{q,k} + \sum_{\forall r\in\mathcal{Q}^{\text{ul}}} \nu_{q,r}p_r^{\text{ul}} + \sigma_q^2\Big) \geq 0, \tag{14c}$$
$$t_{q,k} - \text{vec}^{\mathsf{H}}(\boldsymbol{\Delta}_{q,k})\boldsymbol{B}_k\text{vec}(\boldsymbol{\Delta}_{q,k}) - 2\,\text{Re}\{\text{vec}^{\mathsf{H}}(\boldsymbol{\Delta}_{q,k})\boldsymbol{b}_k\} - b_k \geq 0 \tag{14d}$$
$$\nu_{q,r} - \boldsymbol{\delta}_{q,r}^{\mathsf{H}}\boldsymbol{\Psi}\boldsymbol{\delta}_{q,r} - 2\,\text{Re}\{\boldsymbol{\delta}_{q,r}^{\mathsf{H}}\boldsymbol{\Psi}\hat{\boldsymbol{e}}_{q,r}\} - \hat{\boldsymbol{e}}_{q,r}^{\mathsf{H}}\boldsymbol{\Psi}\hat{\boldsymbol{e}}_{q,r} \geq 0 \tag{14e}$$
$$I_{r,q} - \text{vec}^{\mathsf{H}}(\boldsymbol{\Delta}_{r,q})\boldsymbol{U}_{r,q}\text{vec}(\boldsymbol{\Delta}_{r,q}) - 2\,\text{Re}\{\text{vec}^{\mathsf{H}}(\boldsymbol{\Delta}_{r,q})\boldsymbol{u}_{r,q}\} - u_{r,q} \geq 0 \tag{14f}$$
$$\boldsymbol{F}_q\succeq\boldsymbol{0},\ \text{rank}(\boldsymbol{F}_q) = 1, \tag{14g}$$
$$\text{rank}(\boldsymbol{\Psi}) = 1,\ \text{diag}\{\boldsymbol{\Psi}\} = \boldsymbol{1}_M,\ \boldsymbol{\Psi}\succeq\boldsymbol{0}, \tag{14h}$$

where the optimization variables are $\boldsymbol{F}_q, \boldsymbol{\Delta}_{q,q}, \boldsymbol{\Delta}_{r,q}, \boldsymbol{\Psi}, t_{q,k}, \boldsymbol{\Delta}_{q,k}, \boldsymbol{\delta}_{q,r}, \nu_{q,r}$, and $I_{r,q}$. Problem (14) is a semi-infinite optimization problem, i.e., an optimization problem with a finite number of variables and an infinite number of constraints. Therefore, we can leverage on the following lemma to recast these quadratic constraints in such a way that they become linear matrix inequalities (LMIs).

Lemma 1. (The S-procedure [12]) Let

$$f_i(\boldsymbol{x}) = \boldsymbol{x}^{\mathsf{H}}\boldsymbol{A}_i\boldsymbol{x} + 2\,\text{Re}\{\boldsymbol{x}^{\mathsf{H}}\boldsymbol{b}_i\} + c_i,\ i=1,2,$$

where $\boldsymbol{A}_i\in\mathbb{H}^M$, $\boldsymbol{b}_i\in\mathbb{C}^M$, and $c_i\in\mathbb{R}$. Suppose that there exists an $\hat{\boldsymbol{x}}\in\mathbb{C}^M$ such that $f_2(\hat{\boldsymbol{x}})\leq 0$. Then, the conditions $f_1(\boldsymbol{x})\geq 0$ and $f_2(\boldsymbol{x})\leq 0$ are satisfied for all $\boldsymbol{x}\in\mathbb{C}^M$ if there exists a $\lambda\geq 0$ such that

$$\begin{bmatrix}\boldsymbol{A}_1+\lambda\boldsymbol{A}_2 & \boldsymbol{b}_1+\lambda\boldsymbol{b}_2\\ \boldsymbol{b}_1^{\mathsf{H}}+\lambda\boldsymbol{b}_2^{\mathsf{H}} & c_1+\lambda c_2\end{bmatrix}\succeq\boldsymbol{0}.$$

Consider first the constraints (14c) and (11g). According to Lemma 1, the inequality (14c) is satisfied for all channel errors $\text{vec}(\boldsymbol{\Delta}_{q,q})$ that satisfy (11g) if there exists $\lambda_{q,q}\geq 0$ such that the condition

$$\boldsymbol{\Sigma}_{q,q} \triangleq \begin{bmatrix}\boldsymbol{B}_q+\lambda_{q,q}\boldsymbol{I} & \boldsymbol{b}_q\\ \boldsymbol{b}_q^{\mathsf{H}} & k_q\end{bmatrix}\succeq\boldsymbol{0}, \forall q\in\mathcal{Q}^{\text{dl}} \tag{15}$$

is satisfied, where $k_q = b_q - \gamma_q\Big(\sum_{\substack{\forall k\in\mathcal{Q}^{\text{dl}}\\ k\neq q}} t_{q,k} + \sum_{\forall r\in\mathcal{Q}^{\text{ul}}} \nu_{q,r}p_r^{\text{ul}} + \sigma_q^2\Big) - \lambda_{q,q}\zeta_{q,q}^2$. Following the same procedure, it follows that the inequality (14d) is satisfied for all channel errors $\text{vec}(\boldsymbol{\Delta}_{q,k})$, that satisfy (11h) if there exists $\lambda_{q,k}\geq 0$ such that the condition

$$\boldsymbol{\Phi}_{q,k} \triangleq \begin{bmatrix}-\boldsymbol{B}_k+\lambda_{q,k}\boldsymbol{I} & -\boldsymbol{b}_k\\ -\boldsymbol{b}_k^{\mathsf{H}} & k_k\end{bmatrix}\succeq\boldsymbol{0}, \forall k\diagdown q\in\mathcal{Q}^{\text{dl}} \tag{16}$$

is satisfied, where $k_k = -b_k - \lambda_{q,k}\zeta_{q,k}^2 + t_{q,k}$. Equally, the inequality in (14e) is satisfied for all channel errors $\boldsymbol{\delta}_{q,r}$, that

$$|(\boldsymbol{\theta}^{\mathsf{H}}(\hat{\boldsymbol{E}}_{q,q}+\boldsymbol{\Delta}_{q,q}))\boldsymbol{f}_q|^2=\mathrm{vec}^{\mathsf{H}}(\boldsymbol{\Delta}_{q,q})(\boldsymbol{F}_q^{\mathsf{T}}\otimes\boldsymbol{\Psi})\mathrm{vec}(\boldsymbol{\Delta}_{q,q})+2\,\mathrm{Re}\{\mathrm{vec}^{\mathsf{H}}(\boldsymbol{\Delta}_{q,q})(\boldsymbol{F}_q^{\mathsf{T}}\otimes\boldsymbol{\Psi})\mathrm{vec}(\hat{\boldsymbol{E}}_{q,q})\}+b_q \quad (12a)$$

$$|(\boldsymbol{\theta}^{\mathsf{H}}(\hat{\boldsymbol{E}}_{q,k}+\boldsymbol{\Delta}_{q,k}))\boldsymbol{f}_k|^2=\mathrm{vec}^{\mathsf{H}}(\boldsymbol{\Delta}_{q,k})(\boldsymbol{F}_k^{\mathsf{T}}\otimes\boldsymbol{\Psi})\mathrm{vec}(\boldsymbol{\Delta}_{q,k})+2\,\mathrm{Re}\{\mathrm{vec}^{\mathsf{H}}(\boldsymbol{\Delta}_{q,k})(\boldsymbol{F}_k^{\mathsf{T}}\otimes\boldsymbol{\Psi})\mathrm{vec}(\hat{\boldsymbol{E}}_{q,k})\}+b_k \quad (12b)$$

$$|\boldsymbol{\theta}^{\mathsf{H}}(\hat{\boldsymbol{e}}_{q,r}+\boldsymbol{\delta}_{q,r})|^2=\boldsymbol{\delta}_{q,r}^{\mathsf{H}}\boldsymbol{\Psi}\boldsymbol{\delta}_{q,r}+2\,\mathrm{Re}\{\boldsymbol{\delta}_{q,r}^{\mathsf{H}}\boldsymbol{\Psi}\hat{\boldsymbol{e}}_{q,r}\}+\hat{\boldsymbol{e}}_{q,r}^{\mathsf{H}}\boldsymbol{\Psi}\hat{\boldsymbol{e}}_{q,r} \quad (12c)$$

$$\|(\boldsymbol{f}_q^{\mathsf{T}}\otimes\boldsymbol{I}_N)(\hat{\boldsymbol{G}}_{r,q}+\boldsymbol{\Delta}_{r,q})\boldsymbol{\theta}\|^2=\mathrm{vec}^{\mathsf{H}}(\boldsymbol{\Delta}_{r,q})(\boldsymbol{\Psi}^{\mathsf{T}}\otimes(\boldsymbol{F}_q^{\mathsf{T}}\otimes\boldsymbol{I}_N))\mathrm{vec}(\boldsymbol{\Delta}_{r,q})+2\,\mathrm{Re}\{\mathrm{vec}^{\mathsf{H}}(\boldsymbol{\Delta}_{r,q})(\boldsymbol{\Psi}^{\mathsf{T}}\otimes(\boldsymbol{F}_q^{\mathsf{T}}\otimes\boldsymbol{I}_N))\mathrm{vec}(\hat{\boldsymbol{G}}_{r,q})\}+u_{r,q} \quad (12d)$$

satisfy (10d) if there exists $\ell_{q,r}\geq 0$ such that the condition

$$\boldsymbol{\mho}_{q,r}\triangleq\begin{bmatrix}-\boldsymbol{\Psi}+\ell_{q,r}\boldsymbol{I} & -\boldsymbol{\Psi}\hat{\boldsymbol{e}}_{q,r}\\ -(\boldsymbol{\Psi}\hat{\boldsymbol{e}}_{q,r})^{\mathsf{H}} & k_{q,r}\end{bmatrix}\succeq\boldsymbol{0}, \quad (17)$$

$\forall r\in\mathcal{Q}^{\text{ul}},\forall q\in\mathcal{Q}^{\text{dl}}$ is satisfied, where $k_{q,r}=-\hat{\boldsymbol{e}}_{q,r}^{\mathsf{H}}\boldsymbol{\Psi}\hat{\boldsymbol{e}}_{q,r}-\ell_{q,r}\zeta_{q,r}^2+\nu_{q,r}$. Similarly, the inequality (14f) is satisfied for all channel errors $\mathrm{vec}(\boldsymbol{\Delta}_{r,q})$ that satisfy (10f) if there exists $\xi_{r,q}\geq 0$ such that the condition

$$\boldsymbol{\Omega}_{r,q}\triangleq\begin{bmatrix}-\boldsymbol{U}_{r,q}+\xi_{r,q}\boldsymbol{I} & -\boldsymbol{u}_{r,q}\\ -\boldsymbol{u}_{r,q}^{\mathsf{H}} & k_{r,q}\end{bmatrix}\succeq\boldsymbol{0}, \quad (18)$$

$\forall r\in\mathcal{Q}^{\text{ul}},\forall q\in\mathcal{Q}^{\text{dl}}$ is satisfied, where $k_{r,q}=-u_{r,q}-\xi_{r,q}\zeta_{rq}^2+I_{r,q}$. Thus, we can rewrite problem (14) equivalently as follows:

$$\min \sum_{\forall q\in\mathcal{Q}^{\text{dl}}}\mathrm{Tr}(\boldsymbol{F}_q) \quad (19a)$$

$$\text{s.t} \quad (11e),(14g),(14h) \quad (19b)$$

$$\boldsymbol{\Sigma}_{q,q}\succeq\boldsymbol{0},\ \forall q\in\mathcal{Q}^{\text{dl}} \quad (19c)$$

$$\boldsymbol{\Phi}_{q,k}\succeq\boldsymbol{0},\ \lambda_{q,k}\geq 0,\ \forall k\setminus q\in\mathcal{Q}^{\text{dl}} \quad (19d)$$

$$\boldsymbol{\mho}_{q,r}\succeq\boldsymbol{0},\ \boldsymbol{\Omega}_{r,q}\succeq\boldsymbol{0},\forall r\in\mathcal{Q}^{\text{ul}},\forall q\in\mathcal{Q}^{\text{dl}} \quad (19e)$$

$$\ell_{q,r}\geq 0,\ \xi_{r,q}\geq 0,\ \forall q\in\mathcal{Q}^{\text{dl}},\forall r\in\mathcal{Q}^{\text{ul}} \quad (19f)$$

where the optimization variables are $\boldsymbol{F}_q,\lambda_{q,k},\xi_{r,q},\boldsymbol{\Psi},t_{q,k}$, $\nu_{q,r},\ell_{q,r}$ and $I_{r,q}$. Problem (19) is still non-convex and hard to solve due to the rank-one constraints, and the coupling between $\boldsymbol{F}_q$ and $\boldsymbol{\Psi}$ in $\boldsymbol{\Sigma}$, $\boldsymbol{\Phi},\boldsymbol{\mho}$, and $\boldsymbol{\Omega}$. In the following, we adopt the AO approach to design $\boldsymbol{F}_q$ and $\boldsymbol{\Psi}$ successively in an iterative manner. In particular, for a given $\boldsymbol{\Psi}$, we solve the following sub-problem of $\boldsymbol{F}_q$ during the ith iteration

$$\min \sum_{\forall q\in\mathcal{Q}^{\text{dl}}}\mathrm{Tr}(\boldsymbol{F}_q^{(i)}) \quad (20a)$$

$$\text{s.t} \quad (11e),(14g),(19c)-(19f) \quad (20b)$$

where the optimization variables are $\boldsymbol{F}_q,\lambda_{q,k},\xi_{r,q},t_{q,k}$, $\nu_{q,r},\ell_{q,r}$, and $I_{r,q}$. Problem (20) is a rank-constrained SDP. Therefore, we adopt the SDR technique and the resulting convex SDP can be solved via CVX [14]. With the obtained solution of the relaxed problem (20), the sub-problem of $\boldsymbol{\Psi}^{(i)}$ becomes a feasibility-check problem given as

$$\text{find} \quad \boldsymbol{\Psi}^{(i)} \quad (21a)$$

$$\text{s.t} \quad (14h),(19c)-(19f) \quad (21b)$$

To further improve the converged solution in the optimization of $\boldsymbol{\Psi}^{(i)}$, we introduce a slack variable ω_q,which is interpreted as the SINR residual of the users [5]. Therefore, the feasibility-check problem is formulated as

$$\max \sum_{\forall q\in\mathcal{Q}^{\text{dl}}}\omega_q \quad (22a)$$

$$\text{s.t} \quad (14h),(19d)-(19f),\text{modified-(19c)},\ \boldsymbol{\omega}\geq 0. \quad (22b)$$

where the optimization variables are $\boldsymbol{\omega},\boldsymbol{\Psi}^{(i)},\lambda_{q,k},\xi_{r,q},t_{q,k}$, $\nu_{q,r},\ell_{q,r}$, and $I_{r,q}$. The modified-(19c) is obtained by replacing the constant k_q in (15) with

$$\bar{k}_q=b_q-\gamma_q\Big(\sum_{\substack{\forall k\in\mathcal{Q}^{\text{dl}}\\ k\neq q}}t_{q,k}+\sum_{\forall r\in\mathcal{Q}^{\text{ul}}}\nu_{q,r}p_r^{\text{ul}}+\sigma_q^2\Big)-\lambda_{q,q}\zeta_{q,q}^2-\omega_q$$

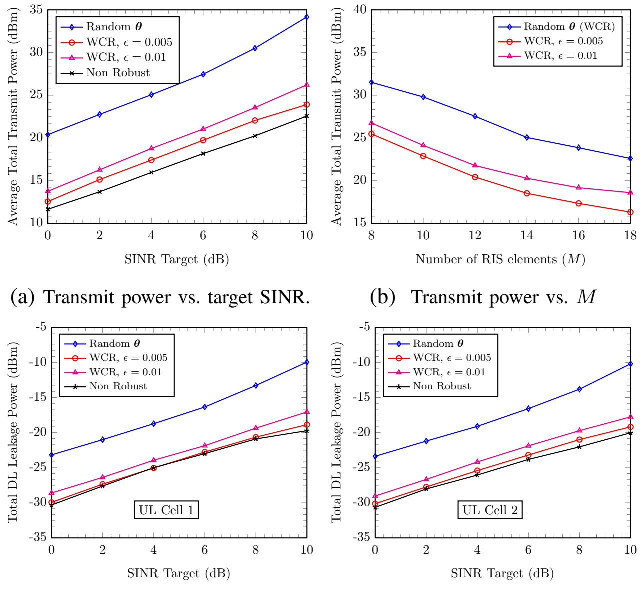
We adopt the SDR technique to transform the rank-constrained problem (22) into a convex SDP which can be solved using CVX tools. It can be seen that the relaxed (20) and (22) are convex problems and their feasible domains are convex sets. Hence, the proposed AO based algorithm will converge to a fixed point solution when the sub-problems (20) and (22) are feasible.

IV. Numerical Results

This section presents some numerical results illustrating the performance of our proposed robust design method. We consider a DTDD system with $Q=4$ cells, as shown in Fig. 1, where $|\mathcal{Q}^{\text{dl}}|=2$ and $|\mathcal{Q}^{\text{ul}}|=2$ and the interference power threshold is $I_{\text{th}}=34\,\text{dBm}$. We assume the same noise variance for DL cell UEs, i.e., $\sigma_q^2=-80\,\text{dBm},\ \forall q\in\mathcal{Q}^{\text{dl}}$. The channel models are assumed to include large-scale fading and small-scale fading. The distance-dependent large-scale fading model is given as $L(d)=C_0(d/D_0)^{-\alpha}$, where $C_0=-30\,\text{dB}$ is the path loss at the reference distance $D_0=1\,\text{m}$, d is the individual link distance, and α denotes the path loss exponent. For the channels $\boldsymbol{H}$ and $\boldsymbol{h}$ we set α to be 2, and 2.2, respectively. The small-scale fading model is assumed to be a Rayleigh fading distribution. We set the radii of the uncertainty regions for the bounded CSI model as $\zeta_{q,q}^2{=}\epsilon^2\|\mathrm{vec}(\hat{\boldsymbol{E}}_{q,q})\|_2^2$, $\zeta_{q,r}^2{=}\epsilon^2\|\hat{\boldsymbol{e}}_{q,r}\|_2^2$, and $\zeta_{r,q}^2=\epsilon^2\|\mathrm{vec}(\hat{\boldsymbol{G}}_{r,q})\|_2^2$, where $\epsilon\in[0,1)$ controls the relative amount of CSI uncertainty. The target minimal SINR is set to be the same for different DL cell users, i.e., $\gamma_q\triangleq\gamma,\ \forall q\in\mathcal{Q}^{\text{dl}}$.

We include results for the following two baseline cases: 1) Random, where the entries of the RIS reflection vector are designed randomly as $[\boldsymbol{\theta}]_{[m]}=e^{j\phi_m},\forall m$, with $\phi_m\in[0,2\pi]$, and 2) Non Robust scheme, the case where the beamforming vectors are obtained based only on the estimated cascaded channels and ignoring the uncertainty, i.e., $\{\hat{\boldsymbol{E}},\hat{\boldsymbol{e}},\hat{\boldsymbol{G}}\}$ are used as if they were perfect CSI.

In Fig. 2a, we show the average total transmit power for the DL cells versus the various SINR targets for the worst-case robust (WCR) transmission strategy, while assuming that the RIS has $M=12$ passive reflecting elements and each BS has $N=6$ antennas. From the figure, it can be observed that the proposed robust strategy and its non robust counterpart performs better than the Random RIS scenario in terms of



(a) Transmit power vs. target SINR. (b) Transmit power vs. M

(c) Total leakage power to UL cell 1 (d) Total leakage power to UL cell 2

Fig. 2: Simulation results

providing a better power efficiency. This is due to the benefit of the beamforming gains obtained from the joint design of active transmit beamforming vectors at the DL BSs and the passive RIS reflection vector. We can clearly see that the average total transmit power increases as we increase the CSI uncertainty variance. This shows that the effects of CSI uncertainty are more difficult to cope with when there is demand for higher SINRs.

Next, we investigate the impact of the number of passive reflective elements of the RIS on the downlink transmit power. The average transmit power versus the number of passive reflective elements in the RIS is shown in Fig. 2b, the number of antennas in each BS is $N = 6$ and the SINR target is fixed as 6 dB. It is seen that the transmit power decreases as the number of the RIS elements M increases. The reason for this significant drop in the transmit power is that the passive beamforming gain of the RIS increases with the number of passive reflective elements, pointing to the benefit of the RIS in power saving.

In Fig. 2c and 2d, we show the total leakage power from the DL cells to the UL cells, while assuming that the RIS has $M = 12$ passive reflecting elements and each BS has $N = 6$ antennas.The results demonstrate how the joint design of the active transmit beamforming vectors at the DL BSs and the passive RIS reflection vector of the considered robust transmission schemes has reduced the sum leakage power from the DL cells to the UL cells (BS-BS interference) below the given threshold of 34 dBm.

V. Conclusions

In this paper, we have considered robust transmission strategies for an RIS-aided DTDD wireless network with imperfect channel state information. We consider a conventional worst-case formulation that has deterministic upper bounds on the norms of the channel imperfection. We adopt a semidefinite relaxation (SDR) technique and an S-procedure to reformulate the problem into a semidefinite programming (SDP) form with linear matrix inequality (LMI) constraints. The optimal beamforming vectors at the downlink BSs and the passive reflecting vector of the RIS are then iteratively computed via an alternating optimization approach.

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