Compressive Sampling Patterns for Sparse Recovery via Discrete Cosine Transform Type-I Even

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***Abstract*—In this work, the problem of recovery of sparse signals by means of compressed sensing in the Discrete Cosine Transform (DCT) domain is addressed. We design a new sampling pattern in the transform domain which is a generalization of arithmetic sequences of difference d. For the DCT Type-I even (DCT1e), we provide a rigorous characterization of the differences d that ensure perfect recovery of sparse signals from their corresponding arithmetic sampling sequences in the transform domain. They constitute new universal sampling patterns, which guarantee the reconstruction of sparse signals from a minimal amount of measurements in the DCT1e domain. Simulations illustrate the good behavior of traditional compressed sensing solvers with this novel compressive sampling scheme, outperforming the sparse recovery rate of the existing solution for the DCT1e.**

***Index Terms*—Compressed sensing, sparse signals, universal sampling pattern, spark, DCT.**

I. INTRODUCTION

Discrete cosine transforms (DCTs) have become an alternative to Discrete Fourier Transform (DFT) in some signal processing applications. For instance, they are widely used for signal and image compression, due to their property of compactation of the information [1]. But they are not limited to these applications: DCTs also present very good behaviour with respect to carrier frequency offset (CFO), outperforming the DFT in some scenarios [2], so they are constitute a good alternative in telecommunications [3].

Among the four types of even DCTs, the Discrete Cosine Transform Type-I even (DCT1e) is the unique that presents two additional properties [4]: On one hand, DCT1e transforms convolution of symmetric signals into a pointwise product of their DCT1e in the transform domain. On the other hand, DCT1e equals its inverse (up to a constant factor), speeding up the implementation of the corresponding algorithms. Based on these properties, recent works have shown the effectiveness of DCT1e both for signal reconstruction and channel estimation in multicarrier communications [5], [6].

In this paper, we address the problem of reconstruction of sparse signals from a small number of its DCT1e coefficients. From compressed sensing (CS) theory [7], it is possible to recover s-sparse signals by means of a set of $p \geq 2s$ measurements in a transform domain, whenever the corresponding transform matrix has maximum spark. Recall that a matrix with p rows has *maximum spark* if any set of p out of its columns are linearly independent. For a given transform matrix, proving that it presents maximum spark is a very difficult issue. Nevertheless, for the DCTs, in [8] it was mathematically shown that all the even-type DCTs present maximum spark in its first rows. This important result guaranteed that any $s-$sparse signal could be perfectly reconstructed by just measuring its first $2s$ DCT coefficients, regardless the signal length. However, that method cannot be applied if any of the first DCT samples is missing.

For this reason, here we present a more general sampling pattern, based on a set of samplers that keep only 1 out of each d coefficients of the DCT vector. This arithmetic sequence is a generalization of the previous solution [8], which corresponds to $d = 1$, since it considered consecutive samples. The advantage of this general method is the flexibility for choosing the difference d and the number p of samples.

In order to prove the validity of the proposed solution, it is necessary to study the maximum spark issue. To this aim, in this paper we develop thorough mathematical results which are valid for the DCT1e, and yield an important characterization of the differences d which guarantee maximum spark of the corresponding DCT1e measurement matrix. Thus, we provide universal sampling patterns for perfect sparse recovery in the DCT1e domain.

The paper is organized as follows: Section II presents the new sampling scheme. Section III provides the main contributions of this work, say, the theorems that guarantee the validity of the proposed solution when using the DCT1e. Section IV illustrates some numerical simulations of the performance of the proposed method. Finally, Section V summarizes the conclusions of this work. Throughout the paper, upper boldface letters denote matrices, vectors are denoted by lowercase boldface letters, and the superscript ^{T} stands for transposition.

II. DESIGN OF THE NOVEL SAMPLING PATTERN

In this section we present a new sampling method from the components of a vector. For the sake of simplicity, we will assume that the vector length is $(N + 1)$. Let us fix an integer $1 \leq d \leq N$, and consider the samples indexed as

$$d_m = dm, \quad m = 0, \ldots, p - 1. \tag{1}$$

As the difference d between consecutive samples is constant, they constitute an arithmetic sequence. We may consider that it is the output of a sampler that works at a fixed rate, as in [9]. However, this would imply that $(p-1)\,d \leq N$: hence, for high values of the sampling difference d, a very small amount of measurements p could be applied, and only sparse signals of small order $s = p/2$ would be reconstructed.

In order to overcome this problem, we propose an extension of the initial sampling scheme, which is also valid for any amount $p \leq (N+1)$, regardless the difference d of the arithmetic progression. Let us explain this method:

1) We start with the first coefficient (indexed as $m = 0$), and continue by taking 1 out of d consecutive samples, as in Eq. (1).
2) In case there is an integer $m_0 \leq p-1$ such that
$$N < dm_0 < 2N,$$
then we compute the number
$$m_0' = 2N - dm_0, \quad 0 < m_0' < N \qquad (2)$$
and keep a new sample indexed as m_0' instead of dm_0. Note that $m_0' = -dm_0 \mod (2N)$.
3) Next, we continue the arithmetic sequence, considering the samples indexed by numbers $d(m_0+k)$. As
$$m_0' - dk \quad = \quad -d\,(m_0+k) \mod (2N),$$
we substitute these numbers by the indices $m_0' - dk$, which also constitute a (descending) arithmetic progression of difference $-d$.
4) Moreover, if any other $m_1 \leq p$ satisfies
$$2N \leq dm_1 \leq 3N$$
then we compute the number
$$m_1' = dm_1 - 2N, \quad 0 \leq m_1' \leq N \qquad (3)$$
and keep the sample indexed as $m_1' = dm_1 \mod (2N)$.
5) The next samples are indexed as $m_1' + dk = d\,(m_1+k) \mod (2N)$, which constitute an (increasing) arithmetic progression of difference d.
6) This procedure is applied until we complete the desired amount p of samples.

In summary, for any number $dm > N$ of the arithmetic sequence, we write it as $dm = 2Nk \pm m'$ for some integer k, and $0 \leq m' \leq N$. Then it suffices to substitute the index dm by the sample index m' defined as
$$m' = |2Nk - dm|\,, \quad 0 \leq m' \leq N. \qquad (4)$$

Note that the particular cases of Eqs. (2) and (3) are derived from the expression given by Eq. (4). Hence, Eq. (4) yields the general procedure for our novel sampling pattern, which can be applied to any $m = 0\ldots,p-1$. This way, we keep the desired number p of samples.

Remark: The proposed sampling pattern provides a family of indices which form arithmetic sequences of the same difference d, with eventually different initial values. Thus, they can be considered as the output of a set of asynchronous samplers, all of them with the same fixed rate. In this sense, this approach is similar (but not equal) to the one proposed in [10] for the DFT.

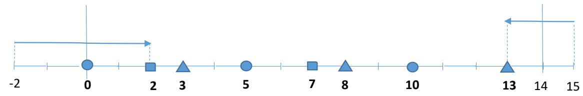

Fig. 1. Example of the proposed sampling pattern for $N = 14$, $p = 8$ and $d = 5$, as explained in Section II.A.

A. Example

For the case $N = 14$, let us choose $p = 8$ numbers with difference $d = 5$ through our approach. The first eight multiples of d are $\{0, 5, 10, 15, 20, 25, 30, 35\}$. With our method, we keep the ones in $[0, N]$, say, $\{0, 5, 10\}$. Secondly, the numbers in the interval $(N, 2N) = (14, 28)$, say, $\{15, 20, 25\}$, following Eq. (2) should be substituted by the respective indices $\{13, 8, 3\}$ (which indeed form an arithmetic progression of difference $-d = -5$). Next, the numbers greater than $2N = 28$ are simply replaced by their remainders modulo $(2N = 28)$ as in Eq. (3). Hence, 30 and 35 are replaced by 2 and 7 (with difference $d = 5$, as expected).

Fig. 1 shows a diagram for this example: notice that this procedure is equivalent to folding the numbers on the right of $[0, N]$ with respect to $N = 14$ (marked with triangles), and folding the samples on the left with respect to 0 (marked with squares). Finally, we have obtained the pattern
$$\{0, 5, 10\} \cup \{3, 8, 13\} \cup \{2, 7\}$$
which corresponds to three arithmetic sequences of the same difference $d = 5$. In other words, it equals the output of three samplers at the same rate.

III. Perfect recovery from the DCT1e domain

Once the sampling scheme has been introduced, now we study its validity for perfect sparse reconstruction in the DCT1e domain. First, let us recall that the DCT1e matrix of order $N+1$, $\mathbf{C}_{1e}$, is defined in [4] as
$$[\mathbf{C}_{1e}]_{k,n} = \alpha_n \cos\left(\frac{kn\pi}{N}\right), \; 0 \leq k, n \leq N,$$
where $\alpha_n = 1$ if $n \in \{0, N\}$, and $\alpha_n = 2$ if $0 < n < N$. Notice that the factor α_n only multiplies each column, so it does not affect the spark of its submatrices. Therefore, we will consider $\alpha_n = 2$ from now on.

Let us analyze the submatrices of $\mathbf{C}_{1e}$ built by p rows indexed as multiples of an integer d, say, $k = dm$, $m = 0, ..., p-1$. If $\mathbf{A}$ denotes such submatrix, its entries are
$$a_{m,n} = 2\cos\left(\frac{\pi mnd}{N}\right) \;\; m = 0, ..., p-1, \;\; n = 0, \ldots, N. \quad (5)$$

Our aim is the characterization of d which yield maximum spark of $\mathbf{A}$. Our first result constitutes a necessary condition:

Theorem 1: *If d and $2N$ are not coprime, then* $\mathbf{A}$ *does not have maximum spark.*

Proof: It suffices to show that if d is even or d is not coprime to N, then $\mathbf{A}$ has not maximum spark.

Notice that the first column of $\mathbf{A}$ $(n = 0)$ equals $2\left(1, 1, ..., 1\right)^T$, whereas its last column $(n = N)$ is:

$$2\left(1, (-1)^d, 1, ..., (-1)^{d(p-1)}\right)^T.$$

Thus, if d is even, it is straightforward that $\mathbf{A}$ has two identical columns, which are linearly dependent, so its spark is not maximum. Here we had assumed that $d\left(p-1\right) \leq N$; if $dm > N$, the proposed sampling scheme will substitute the even number dm by its remainder modulo $2N$ which is also even. Besides, in case $dm = Nk + r$ with odd k, then

$$dm + (N - r) = N\left(k + 1\right)$$

and the last number is even, so in case dm is even, so is $(N - r)$. We conclude that all the corresponding rows are even-indexed, so this scheme will also provide a submatrix whose first and last columns are equal; therefore, the spark will not be maximum either.

Let us now suppose that d is odd and d,N are not coprime, there exist integers $1 < k < N$ and $1 \leq L < d$ such that $kd = NL$. Hence the column of index $k < N$ has entries

$$2\cos\left(\frac{\pi mkd}{N}\right) = 2\cos\left(\pi mL\right) = 2\left(-1\right)^{mL}.$$

If L is even, this column has entries $(2, 2, ...2)^T$ proportional to the first column, and if L is odd this column is

$$2\left(1, (-1)^L, 1, ..., (-1)^{L(p-1)}\right)^T = 2\left(1, -1, 1, ..., (-1)^{(p-1)}\right)^T$$

which is proportional to the last column, since

$$\left(1, (-1)^d, 1, ..., (-1)^{d(p-1)}\right)^T = \left(1, -1, 1, ..., (-1)^{(p-1)}\right)^T$$

because d is odd. Thus, in this case the spark of $\mathbf{A}$ would not be maximum, and the claim holds. □

The previous result assures that for maximum spark, d and $2N$ must be coprime, so this is a necessary condition for perfect recovery. Conversely, the following theorem states that this condition is not only necessary, but sufficient. This constitutes one of the main contributions of this work:

Theorem 2: For any integers $1 \leq d, p \leq N$, the $p \times (N+1)$ matrix $\mathbf{A}$ defined in Eq.(5) has maximum spark (p) if and only if d and $2N$ are coprime.

Proof: Theorem 1 guarantees the first part of the proof. Let us prove that if d is coprime to $2N$, then $\mathbf{A}$ has maximum spark, say, any set of p of its columns are linearly independent. Let us consider any p columns with indices $0 \leq n_1 < n_2 < ... < n_p \leq N$. We build the $p \times p$ square submatrix $\mathbf{B}$ formed by these p generic columns; its entries are

$$b_{m,n} = 2\cos\left(\frac{\pi mnd}{N}\right),\ 0 \leq m \leq p-1,\ n \in \{n_1, n_2, ..., n_p\}.$$

In order to prove that $\mathbf{B}$ is invertible, it suffices to demonstrate that the unique row vector $\mathbf{a} = (a_0, a_1, ..., a_{p-1})$ such that $\mathbf{aB} = \mathbf{0}$ is $\mathbf{a} = \mathbf{0}$. Let us rewrite the condition $\mathbf{aB} = \mathbf{0}$ as

$$\sum_{m=0}^{p-1} 2a_m \cos\left(\frac{\pi m n_k d}{N}\right) = 0, \quad k = 1, ..., p. \tag{6}$$

By using the complex unitary numbers

$$w_k = \exp\left(\frac{\pi n_k d}{N} j\right) \quad k = 1, \ldots, p \tag{7}$$

then Eq. (6) is rewritten as

$$\sum_{m=0}^{p-1} a_m \left(w_k^m + w_k^{-m}\right) = 0, \quad k = 1, ..., p.$$

Now we multiply the latter expression by w_k^{p-1}:

$$\sum_{m=0}^{p-1} a_m \left(w_k^{m+p-1} + w_k^{p-1-m}\right) = 0, \quad k = 1, ..., p.$$

This way, each w_k of Eq. (7) is a root of the polynomial

$$q\left(z\right) = \sum_{m=0}^{p-1} a_m \left(z^{m+p-1} + z^{p-1-m}\right) \tag{8}$$

of degree $2p - 2$. But q is a self-reciprocal polynomial (its coefficients are symmetric with respect to the central one) so w_k^{-1} is also root of q :

$$w_k^{-1} = \exp\left(-\frac{\pi n_k d}{N} j\right) \quad k = 1, ..., p.$$

Let us count how many different roots of q there are: it suffices to see that their arguments do not differ in an integer multiplied by 2π. If there exist $0 < n_k < n_{k'} < N$ such that

$$\frac{\pi n_k d}{N} \pm \frac{\pi n_{k'} d}{N} = 2\pi m \iff \left(n_k \pm n_{k'}\right) d = 2mN$$

then d would not be coprime to $2N$, and it is impossible. This implies that q has at least $2(p-1)$ different roots. Notice that if $n_1 > 0$ and $n_p < N$, then q would have $2p$ different roots, more than its degree, so q should be the null polynomial, and the claim follows.

Let us analyze the cases $n_1 = 0$ or $n_p = N$: for $n_1 = 0$, the corresponding root is $w_1 = 1 = w_1^{-1}$, which should be counted only once; the same occurs for $n_p = N$ with the root $w_p = (-1)^d = -1 = w_p^{-1}$. Thus, if $n_1 = 0$ or $n_p = N$ (not simultaneously), then the amount of different roots of q is at least $2p - 1$, there would be more roots than its degree, and the claim follows in the same way.

It remains to study the case when $n_1 = 0$ and $n_p = N$: both $w_1 = 1$ and $w_p = -1$ are roots of q, and the others are pairs of conjugate roots $w_k,\ w_k^{-1},\ k = 2, \ldots, p-1$, so there are $2p - 2$ different roots. Let us now show that this is impossible: as the degree of q defined in Eq. (8) is $2p - 2$ (because $a_{p-1} \neq 0$), it is known that the product of all its roots z_k is equal to the quantity

$$\prod_{k=1}^{2p-2} z_k = (-1)^{2p-2} \frac{a_{p-1}}{a_{p-1}} = 1$$

but in this case the product is

$$\prod_{k=1}^{2p-2} z_k = w_1 w_p \left(\prod_{k=2}^{p-1} w_k\right)\left(\prod_{k=2}^{p-1} w_k^{-1}\right) = 1 \cdot (-1) = -1.$$

Thus, we get a contradiction, that comes from the assumption that $a_{p-1} \neq 0$ so we derive that $a_{p-1} = 0$. But this implies that the degree of q is $\leq 2p-3$ and q has $2p-2$ different roots, more than its degree. Therefore, the only chance is that q is the null polynomial, and the claim follows. □

Remark: Fortunately, the assumption $d(p-1) \leq N$ is not necessary for the validity of this result, since it has not been required for its proof. In other words, if $d(p-1) > N$, then the corresponding matrix $\mathbf{A}$ of Eq. (5) also has maximum spark if and only if d is coprime to $2N$. But notice that its rows are the same as the rows of the DCT1e matrix indexed by our sampling pattern: in effect, for each row index m, by writing $dm = 2Nk \pm m'$ as in Eq. (4), its entries are:

$$\cos\left(\frac{(dm)\,n\pi}{N}\right) = \cos\left(\frac{(2Nk \pm m')\,n\pi}{N}\right) = \cos\left(\frac{m' n\pi}{N}\right)$$

which correspond to the entries of the m'-indexed row of the DCT1e matrix. Thus, we have shown that the maximum spark result is valid for the novel sampling pattern presented in Section II.

Finally, note that the example given in Section II.A verifies that d and $2N$ are coprime. Thus, it is guaranteed that the corresponding novel sampling pattern yields maximum spark.

IV. Experimental results

In this section we show some simulations where traditional compressed sensing techniques have been applied to the proposed sampling pattern in the DCT1e domain. In each experiment, first we set the parameters N (the length of the signal minus 1), p (number of measurements in the transform domain), and the difference parameter d (coprime to $2N$). Then, for each sparsity value s, $1 \leq s \leq p$, a s-sparse signal $\mathbf{x}$ of length $N+1$ is built: its s nonzero locations are drawn at random, and its respective nonzero values are drawn from a normal Gaussian distribution $\mathcal{N}(0,1)$. We compute its DCT1e transform vector $\mathbf{b} = \mathbf{C}_{1e} \cdot \mathbf{x}$, and extract its p components indexed by the arithmetic sequence of difference d.

Secondly, from these p measurements we apply traditional CS solvers; basis pursuit (BP), smoothed ℓ_0 algorithm (SL0) [11], and Orthogonal Matching Pursuit in its modified version (OMP1) [12]. Each of these algorithms compute an estimation of the sparse signal $\mathbf{x}$. Finally, the experiment is repeated 100 times for each sparsity value s, and the empirical recovery rate of each algorithm is computed for each sparsity value s.

Let us show some results obtained for $N = 14$, $p = 8$: with this setting, we can choose any value of difference d coprime to $2N = 28$. Here we consider two cases, $d = 1$ and $d = 5$:

- $d = 1$ always yields the sampling pattern $\{0, 1, ..., p-1\}$, which keeps the first $p = 8$ samples of the transformed vector $\mathbf{b}$. This corresponds to the existing solution [8].
- Difference $d = 5$ provides the new sampling pattern in $\mathbf{b}$ designed by our proposed method. In this case, it has already been obtained in Section II.A, and depicted in Fig. 1: it corresponds to the samples $\{0, 2, 3, 5, 7, 8, 10, 13\}$.

Fig. 2 shows the corresponding recovery rate of the CS solvers (SL0, BP, OMP1) along the sparsity values s, for the former solution $d = 1$ (top) and our new approach with $d = 5$ (bottom). As expected, high recovery rates are obtained through these algorithms for sparsity values $s \leq p/2 = 4$, both for $d = 1$ and $d = 5$. Recall that CS theory never ensures recovery for sparsity values $s > p/2 = 4$. Hence, in these simulations, both arithmetic pattern schemes present similar good behaviour.

Nonetheless, their results differ if we consider random s-sparse signals with concentrated support, say, whose support is an (unknown) interval of length s. This is a very common scenario in some applications, for instance in cognitive radio, where the support of each signal is an interval whose location is unknown. Fig. 3 shows the respective results for the former solution [8] with $d = 1$ (top) and our new approach with $d = 5$ (bottom): in this case, the recovery rate for $d = 1$ drops quickly for $s > 2$, whereas the novel sampling pattern with $d = 5$ provides as good results as for the general sparse signals.

Simulations have also been done for higher values of N. By setting $N = 64$ and $p = 32$, Fig. 4 compares the recovery rates of the BP solver for $d = 1$ and $d = 13$ (coprime to $2N$) for sparse signals of concentrated support: the choice of $d = 13$ outperforms greatly the recovery rate of $d = 1$, which decreases dramatically. Hence, the proposed sampling pattern presents better behaviour than the existing solution for DCT1e given in [8].

V. Conclusions

In this work, a novel compressive sampling pattern has been designed in the DCT1e domain. It consists of a simple generalization of arithmetic sequences, which can be obtained by sampling the transform vector at a fixed rate. The main contributions of this work are the theorems that provide the necessary and sufficient condition that guarantees that the corresponding measurement matrix has maximum spark, so recovery of sparse signals can be ensured. In summary, this is possible if and only if the difference d of the arithmetic sequence is odd and coprime to the length of the signal minus 1. Thus, s-sparse signals can be perfectly recovered by a small amount of DCT1e coefficients: the $2s$ ones given by the proposed universal sampling scheme with difference d. Simulations illustrate the good behaviour of this technique, outperforming the previous solution in case the support of the sparse signal is localized. Future research will address the problem of finding the optimal values of d for numerical issues. Further theoretical results should also be developed in order to show the validity of the proposed method for the rest of the DCTs.

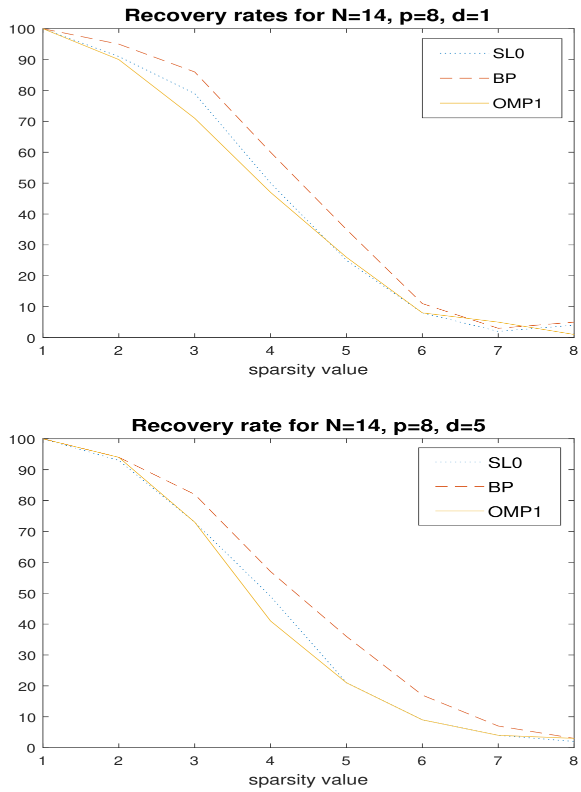

Fig. 2. Recovery rate versus sparsity value s for the CS solvers SL0, BP and OMP1 for the DCT1e. In all cases $N = 14$, and $p = 8$ samples are selected from the DCT1e vector, following the arithmetic sequence of difference $d = 1$ (top), or the proposed arithmetic sequence with $d = 5$ (bottom).

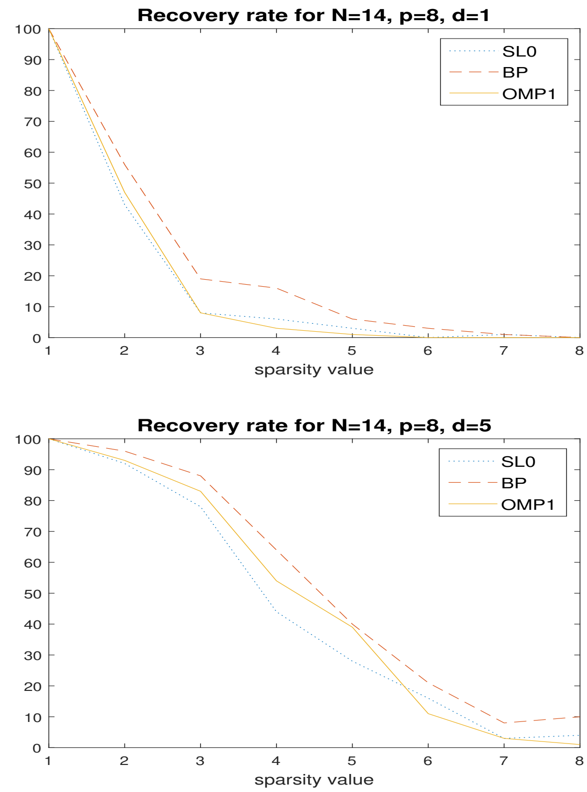

Fig. 3. Recovery rate of sparse signals with concentrated support, versus sparsity value s for the same CS solvers in the DCT1e domain. In all cases $N = 14$, and $p = 8$ samples are selected from arithmetic sequences of difference $d = 1$ (top) or the proposed approach with $d = 5$ (bottom).

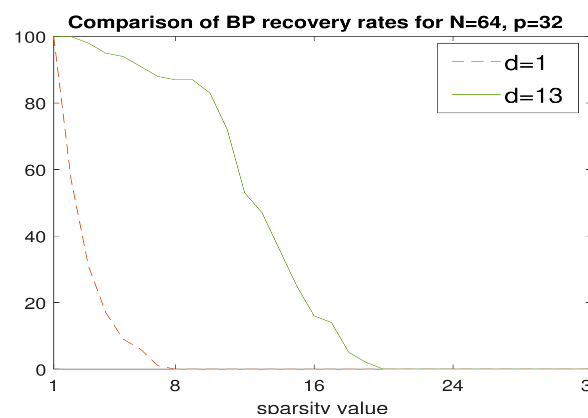

Fig. 4. Comparison of recovery rates of sparse signals with concentrated support via BP solver for $N = 64$, $p = 32$, obtained by $d = 1$ and $d = 13$.

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