ON OPTIMUM PILOT DESIGN FOR OFDM SYSTEMS WITH VIRTUAL CARRIERS

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ABSTRACT

Pilot design significantly affects the performance of channel estimation. Optimum pilot design for fully loaded OFDM systems has been well studied. In the practical case of OFDM with virtual carriers, the pilot design issue has so far not been properly addressed. Here, we fill this gap. In designing the pilots, we consider pilot placement as well as power allocation. Towards this objective, we use the MMSE estimation and a lower bound on the ergodic capacity.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has become the standard of choice for wireless LAN's such as HIPERLAN/2 and IEEE 802.11a; it has been adopted in Europe for Digital Audio Broadcasting (DAB) and Digital Video Broadcasting (DVB), MMAC in Japan, and fixed wireless; and is being considered for several IEEE 802.11 and 802.16 standards, including wideband Metropolitan Area Networks (MAN) [1]. The popularity of OFDM stems from its ability to transform a wideband frequency selective channel to a set of parallel flat-fading narrowband channels, which substantially simplifies the channel equalization problem. Because of the time-frequency granularity that it offers, OFDM appears to be a natural solution when the available spectrum is not contiguous, for overlay systems, and to cope with issues such as narrowband jamming. In the multi-user context, this granularity also accommodates variable quality-of-service (QoS) requirements and bursty data.

When the channel is fast-varying, pilots must be inserted in each OFDM block in order to track channel variations. Optimal insertion of pilots for OFDM was investigated in [2] [5] [7] [8]. It was shown that OFDM with equispaced and equipowered pilot tones is optimal in the sense of minimizing the mean square error of channel estimation. It was shown in [8] that choosing the number of pilot tones to be equal to the number of channel taps maximizes a lower bound on the ergodic capacity. Also, using the latter, the optimal power allocation between the pilot and data carriers was derived. However, in all these findings, the OFDM system was assumed fully loaded, i.e. all the carriers are modulated. Practical OFDM systems have some of the carriers nulled (or deactivated). Indeed, in order to avoid aliasing/interference with adjacent systems, some carriers at the edges of the spectrum are nulled. The number of these virtual carriers is dictated by system design and is around 10% of the total number of carriers. Further, some other carriers might be nulled if their frequencies are known to experience strong jamming/inteference. In this paper, we address the problem of optimal pilot design for such OFDM systems.

Notation: $E\{\}$ will denote the statistical expectation. $\mathbf{F}$ will denote the $N \times N$ DFT matrix, i.e.

$$\mathbf{F} = (1/\sqrt{N})\{\exp(-j2\pi nk/N)\}_{n,k=0}^{N-1}.$$

$\mathbf{W} = \sqrt{N}\mathbf{F}$. If $\mathcal{P}$ denotes an index set consisting of N_p elements from $\{0, ..., N-1\}$, then $\mathbf{W}_\mathcal{P}$ will denote the $(N_p \times N)$ submatrix obtained from the $n \in \mathcal{P}$ rows of $\mathbf{W}$. $\mathbf{D}_{\boldsymbol{z}}$ will denote a diagonal matrix whose diagonal is $\boldsymbol{z}$. Superscripts *, ^{T} and † denote Hermitian, transpose and pseudo-inverse operators. The trace, rank and statistical expectation are denoted by $\mathrm{trace}\{\cdot\}$, $\mathrm{rank}\{\cdot\}$ and $E\{\cdot\}$. The $(N \times N)$ identity matrix will be denoted by $\mathbf{I}$.

2. SIGNAL MODEL

We model the frequency-selective channel as an FIR filter with channel impulse response (CIR) $\boldsymbol{h} = [h_0, ..., h_{L-1}]^T$ where L is the length of the CIR. We assume $\boldsymbol{h}$ to be Gaussian with zero mean and covariance matrix $\mathbf{R}_h = \mathrm{diag}\left(\sigma^2_{h_1}, \cdots, \sigma^2_{h_{L-1}}\right)$ where $\sigma^2_{h_\ell} = E\left\{|h_\ell|^2\right\}$.

Let N denote the total number of carriers. We assume that each OFDM block is preceded by a CP whose length is $L-1$, so that IBI can be eliminated at the receiver, without affecting the orthogonality of the sub-carriers. We assume that the CIR is time-invariant over each block but is allowed to vary from block to block. Since we focus on block-by-block processing, we omit the block index in what follows. Let $\mathcal{N} = \{0, ..., N-1\}$ denote the entire set of carriers, and let $\mathcal{A}$ denote the subset of activated carriers. Also let $\mathcal{P}$ (resp. $\mathcal{D}$) denote the subset of $\mathcal{A}$ that contains the N_p (resp. N_d) pilot (resp. data) carriers. Note that $\mathcal{A} = \mathcal{P} \cup \mathcal{D}$. If the system is fully loaded, then $\mathcal{A} = \mathcal{N}$.

Let i_m denote the mth element of $\mathcal{P}$, i.e. $\mathcal{P} = \{i_1, \cdots, i_{N_p}\}$. After removing the CP and performing FFT, the baseband discrete-time received pilot and data carriers can be modeled as

$$x_n = H_n s_n + v_n \qquad n \in \mathcal{D} \tag{1}$$

$$x_{i_m} = H_{i_m} c_m + v_{i_m}, \qquad m = 1, \cdots, N_p \tag{2}$$

where $\{s_n\}$ are the data symbols, $\{c_m\}$ are known non-zero pilot symbols, v_n is an AWGN with variance σ^2_v and

$$H_n = \sum_{\ell=0}^{L-1} h_\ell e^{-j2\pi\ell n/N}$$

In vector form, the received pilot signal can be expressed as

$$\boldsymbol{x}_\mathcal{P} = \mathbf{D}_{\boldsymbol{c}} \mathbf{W}_\mathcal{P} \boldsymbol{h} + \boldsymbol{v}_\mathcal{P} \tag{3}$$

where $\boldsymbol{c} = [c_1, \cdots, c_{N_p}]^T$.

3. CHANNEL ESTIMATION

Using eq. (3), the MMSE estimator of $\boldsymbol{h}$ is given by

$$\hat{\boldsymbol{h}} = \left(\sigma^2_v \mathbf{R}_h^{-1} + \mathbf{W}^H_\mathcal{P} \mathbf{D}_{\boldsymbol{\rho}} \mathbf{W}_\mathcal{P}\right)^{-1} \mathbf{W}^H_\mathcal{P} \mathbf{D}^H_{\boldsymbol{c}} \boldsymbol{x}_\mathcal{P}$$

where $\boldsymbol{\rho} = [|c_1|^2, \cdots, |c_{N_p}|^2]$. The least squares (LS) estimator is obtained by setting $\mathbf{R}_h^{-1} = 0$ in the above estimator. The MMSE estimate of H_n is

$$\hat{H}_n = \boldsymbol{w}_n^H \hat{\boldsymbol{h}}$$

where $\boldsymbol{w}_n^H := \mathbf{W}(n, :)$.

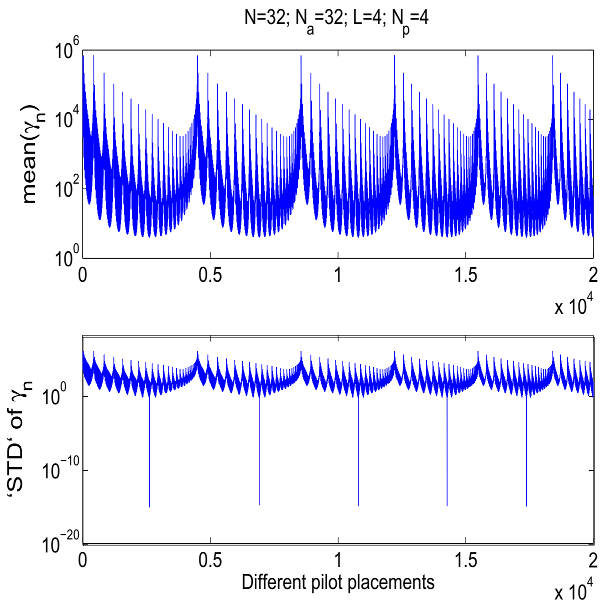

Fig. 1. Mean and 'std' of γ_n for different pilot placements in the case of fully loaded systems.

The channel identifiability condition is

$$\operatorname{rank}\{\mathbf{D}_{\boldsymbol{c}}\mathbf{W}_{\mathcal{P}}\} = L$$

which is equivalent to

$$N_p \geq L$$

since $c_m \neq 0$. Choosing $P = L$ maximizes the bandwidth efficiency but minimizes the signal-to-noise ratio (SNR) per data carrier. However, in terms of overall capacity, bandwidth efficiency is preferred to SNR (see e.g. [8].) Hence, we set $P = L$ in what follows.

The MSE of $\hat{\boldsymbol{h}}$ is given by

$$\boldsymbol{\Sigma}_{\hat{\boldsymbol{h}}} := E\left\{(\hat{\boldsymbol{h}}-\boldsymbol{h})(\hat{\boldsymbol{h}}-\boldsymbol{h})^H\right\} = \left(\mathbf{R}_h^{-1} + \frac{1}{\sigma_v^2}\mathbf{W}_{\mathcal{P}}^H\mathbf{D}_{\boldsymbol{\rho}}\mathbf{W}_{\mathcal{P}}\right)^{-1} \tag{4}$$

The MSE of $\hat{H}_n$ is

$$\gamma_n := E\left\{|\hat{H}_n|^2\right\} = \boldsymbol{w}_n^H\boldsymbol{\Sigma}_{\hat{\boldsymbol{h}}}\boldsymbol{w}_n$$

The sum of the MSEs of $\hat{H}_n$, $n \in \mathcal{D}$, can be expressed as

$$\bar{\gamma} := \sum_{n\in\mathcal{D}}\gamma_n = \operatorname{Tr}\left\{\mathbf{W}_{\mathcal{D}}\boldsymbol{\Sigma}_{\hat{\boldsymbol{h}}}\mathbf{W}_{\mathcal{D}}^H\right\} \tag{5}$$

4. OPTIMAL DESIGN FOR FULLY LOADED OFDM

In this case, $\mathcal{A} = \mathcal{N}$. Before we address the issue of pilot design, it is worth pointing out that pilot design dramatically affects estimation performance. Figure 1 illustrate this by depicting the average (over the data carriers) of γ_n and its standard deviation (from the mean) for different pilot carrier placements (the power was evenly split between the pilots).

4.1. Minimizing the MSE on channel estimation

Since equalization is carried out in the frequency domain, we propose to minimize the total (or average) MSE of the frequency-domain channel estimates at the data carriers, unlike [7] and [8] where $\operatorname{Tr}\{\boldsymbol{\Sigma}_{\hat{\boldsymbol{h}}}\}$ was minimized. Thus, we design the pilot carriers as

$$\begin{aligned}\{\boldsymbol{\rho}^o, \mathcal{P}^o\} &= \arg\min_{\boldsymbol{\rho},\mathcal{P}} \bar{\gamma} \\ \bar{\gamma} &= \operatorname{Tr}\left\{\mathbf{W}_d\left(\mathbf{R}_h^{-1} + \frac{1}{\sigma_v^2}\mathbf{W}_p^H\mathbf{D}_{\boldsymbol{\rho}}\mathbf{W}_{\mathcal{P}}\right)^{-1}\mathbf{W}_d^H\right\}\end{aligned}$$

under the constraints

$$\mathcal{P} \subseteq \mathcal{A} \qquad \text{and} \qquad \sum_{n=1}^{L}\rho_n = \sigma_p^2 \qquad \textbf{(C1)}$$

For any $(L \times L)$ positive-definite matrix, $\mathbf{B} = \{b_{k,\ell}\}_{k,\ell=0}^{L-1}$, the following result was derived in [8]

$$\operatorname{Tr}\left\{\mathbf{B}^{-1}\right\} \geq \sum_{\ell=0}^{L-1}\frac{1}{b_{\ell,\ell}}$$

with equality iff $\mathbf{B}$ is diagonal. Applying this result to

$$\mathbf{B} = \left((\mathbf{W}_d^H\mathbf{W}_d)^{-1}\left[\mathbf{R}_h^{-1} + \frac{1}{\sigma_v^2}\mathbf{W}_{\mathcal{P}}^H\mathbf{D}_{\boldsymbol{\rho}}\mathbf{W}_{\mathcal{P}}\right]\right)^{-1} \tag{6}$$

and using the fact that $\mathbf{R}_h$ is diagonal and the diagonal of $\mathbf{W}_{\mathcal{P}}^H\mathbf{D}_{\boldsymbol{\rho}}\mathbf{W}_{\mathcal{P}}$ is $\sigma_p^2\mathbf{I}_L$ and that of $(\mathbf{W}_d^H\mathbf{W}_d)$ is $(N-L)\mathbf{I}_L$, we obtain

$$\bar{\gamma} \geq (N-L))\sum_{\ell=0}^{L-1}\frac{\sigma_v^2\,\sigma_{h_\ell}^2}{\sigma_v^2 + \sigma_p^2\sigma_{h_\ell}^2}$$

with equality if both $\mathbf{W}_{\mathcal{P}}^H\mathbf{D}_{\boldsymbol{\rho}}\mathbf{W}_{\mathcal{P}} = \sigma_p^2\mathbf{I}_L$ and $(\mathbf{W}_d^H\mathbf{W}_d) = (N-L)\mathbf{I}_L$. The optimal design is given next:
Result 1 *The total MSE $\bar{\gamma}$ is minimized iff*

$$\boldsymbol{\rho}^o = \frac{\sigma_p^2}{L}[1,\cdots,1]^T$$

and

$$\mathcal{P}^o = \begin{cases}\mathcal{P}_1^o := \{t+iQ,\ i=0,\cdots,L-1\} & \text{if } Q := \frac{N}{L} \text{ integer} \\ \mathcal{P}_2^o := \{0,\cdots,N-1\} - \mathcal{P}_1^o & \text{if } Q := \frac{N}{N-L} \text{ integer}\end{cases}$$

where t is arbitrary integer from $[0, Q-1)$. The minimum MSE is

$$\bar{\gamma}^o = (N-L)\sum_{\ell=0}^{L-1}\frac{\sigma_v^2\,\sigma_{h_\ell}^2}{\sigma_v^2 + \sigma_p^2\sigma_{h_\ell}^2}$$

The following results are readily obtained.

- The above optimal design also minimizes $\operatorname{Tr}\{\boldsymbol{\Sigma}_{\hat{\boldsymbol{h}}}\}$ which was considered in [8].
- The above optimal design also minimizes the individual MSEs of the $(N-L)$ frequency response estimates $\hat{H}_n$, $n \in \mathcal{D}$.
- The minimum individual MSEs are all equal i.e.

$$\gamma_n^o = \frac{\bar{\gamma}^o}{N-L} = \sum_{\ell=0}^{L-1}\frac{\sigma_v^2\,\sigma_{h_\ell}^2}{\sigma_v^2 + \sigma_p^2\sigma_{h_\ell}^2}$$

This is illustrated in Figure 1 by zero std of γ_n.

None of the above results holds if the OFDM system is not fully loaded as we will see next.

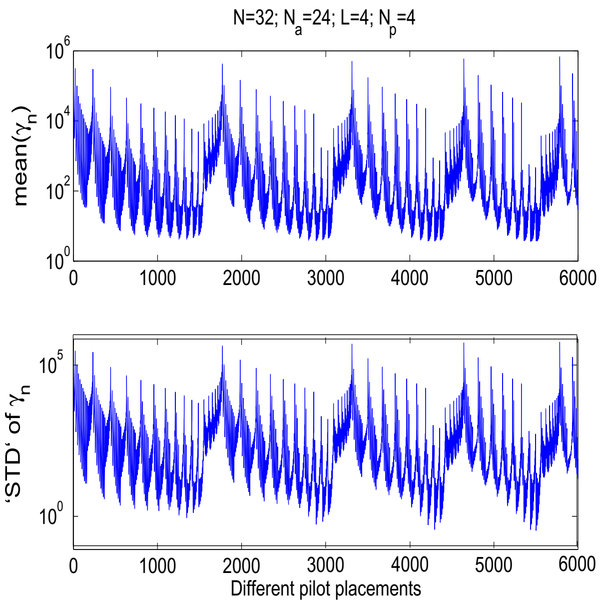

Fig. 2. Mean and 'std' of γ_n for different pilot placements in the presence of virtual carriers.

4.2. Power allocation

To complete the pilot design, σ_p^2 should be optimized for a fixed total transmit power, $\sigma_t^2 := \sigma_p^2 + \sigma_s^2$ where σ_s^2 is the total data power. This was addressed in the case of fully-loaded systems in [8].

5. OPTIMAL DESIGN FOR OFDM WITH VIRTUAL CARRIERS

Here, $\mathcal{A} \neq \mathcal{N}$. Figure 2 displays the total MSE and 'standard deviation' for different pilot placements (the pilots were equipowered).

5.1. Minimizing the MSE on channel estimation

Here, we wish to optimize the total MSE wrt to both $\boldsymbol{\rho}$ and $\mathcal{P}$. Since matrix $\mathbf{B}$ in eq. (6) cannot be diagonal in the presence of virtual carriers, the inequality used in the previous section is not useful in this case. Numerical optimization of $\bar{\gamma}$ over $\{\boldsymbol{\rho}, \mathcal{P}\}$ is possible but computationally inefficient because $\boldsymbol{\rho}$ is a continuous-valued vector, and finding the global optimum is not guaranteed. To simplify the optimization problem, we consider the following alternatives

5.1.1. Equipowered pilots

As in [4], the magnitude of the ρ_n's are here set to be equal and optimization is carried out over $\mathcal{P}$ only using an exhaustive search over all L-element subsets of $\mathcal{A}$, i.e.

$$\begin{aligned} \breve{\mathcal{P}}^o &= \arg\min_{\mathcal{P}} \breve{\bar{\gamma}} \\ \breve{\bar{\gamma}} &= \mathrm{Tr}\left\{ \left(\mathbf{R}_h^{-1} + \frac{\sigma_p^2}{L\sigma_v^2} \mathbf{W}_{\mathcal{P}}^H \mathbf{W}_{\mathcal{P}} \right)^{-1} \mathbf{W}_{\mathcal{D}}^H \mathbf{W}_{\mathcal{D}} \right\} \end{aligned}$$

where $\breve{}$ indicate quantities derived under the assumption of uniform pilot power distribution. Analytically minimizing the above is not tractable. However numerical optimization over $\mathcal{P}$ is computationally efficient since it is a discrete search.

5.1.2. High-SNR approach

Here, we approximate the MSEs of the MMSE estimators by those of the LS estimators, i.e. we set $\mathbf{R}_{\boldsymbol{h}}^{-1} = 0$. The optimization problem can be re-expressed as

$$\{\tilde{\boldsymbol{\rho}}^o, \tilde{\mathcal{P}}^o\} = \arg\min_{\boldsymbol{\rho}, \mathcal{P}} \sum_{n=1}^{L} \frac{\psi_{n,n}}{\rho_n}$$

under (**C1**) where

$$\boldsymbol{\Psi} := \mathbf{W}_{\mathcal{P}}^{-1\mathcal{H}} \mathbf{W}_{\mathcal{D}}^{\mathcal{H}} \mathbf{W}_{\mathcal{D}} \mathbf{W}_{\mathcal{P}}^{-1}$$

In the above, we use $\tilde{}$ to indicate high SNR (or LS-)based quantities. Minimizing the above wrt to $\boldsymbol{\rho}$ under $\sum \rho_n = \sigma_p^2$ gives

$$\tilde{\rho}_n^o = \sigma_p^2 \frac{\sqrt{\psi_{n,n}}}{\sum_{i=1}^{L} \sqrt{\psi_{i,i}}}, \quad n = 1, \cdots, L$$

Using $\tilde{\boldsymbol{\rho}}^o$, the optimal pilot placement is found to be:

$$\tilde{\mathcal{P}}^o = \arg\min_{\mathcal{P} \subset \mathcal{A}} \left(\sum_{n=1}^{L} \sqrt{\psi_{n,n}} \right)^2 \tag{7}$$

Hence, the dimension of the optimization problem is reduced from $(\boldsymbol{\rho}, \mathcal{P})$ to $\mathcal{P}$. As in the previous subsection, an exhaustive search over all L-point subsets of $\mathcal{A}$ is required to find $\tilde{\mathcal{P}}^o$.

It is worth pointing out that ρ_n^o decreases when the nth pilot is close to the virtual carrier zone. This can be explained by the fact that there are less data carriers in this zone and therefore when the criteria is the total MSE, one should assign more power to the pilots that are surrounded by more data carriers.

5.1.3. Equispaced pilots

Here, we assume that $Q := N/L$ is an integer and the number of virtual carriers, $N - N_a$, to be smaller than Q. The subset $\mathcal{P}$ is chosen as in result 1 with t judiciously chosen to make sure all pilot carriers are active carriers. The number of such subsets depends on Q and N_a. For fixed $N - N_a$, this number increases with Q. We next derive the optimal pilot power distribution for such pilot placements.

Here, we also assume that the channel paths have the same power, i.e. $\mathbf{R}_h = \sigma_h^2 \mathbf{I}$. The total MSE, $\bar{\gamma}$, can be expressed as

$$\bar{\gamma} = \sigma_v^2 \sum_{n=1}^{L} \frac{\psi_{n,n}}{\rho_n + \sigma_v^2/\sigma_H^2}$$

where $\sigma_H^2 = \mathrm{Tr}\{\mathbf{R}_h\}$. The optimum pilot power distribution is thus given by

$$\acute{\rho}_n^o = (\sigma_p^2 + L\sigma_v^2/\sigma_H^2) \frac{\sqrt{\psi_{n,n}}}{\sum_{n=1}^{L} \sqrt{\psi_{n,n}}} - \sigma_v^2/\sigma_H^2$$

where $\acute{}$ indicates quantities obtained using the equispaced pilot assumption. Using the above, the optimum equispaced pilot placement is obtained as in (7) where $\mathcal{P}$ is restricted due to the equispaced pilot assumption.

Again, here ρ_n^o decreases when the nth pilot is close to the virtual carrier zone for the reasons mentioned in the previous subsection.

5.1.4. Equipowered and Equispaced pilots

Here all the above equispaced pilots yields the same total MSE. Further, the γ_n's are identical.

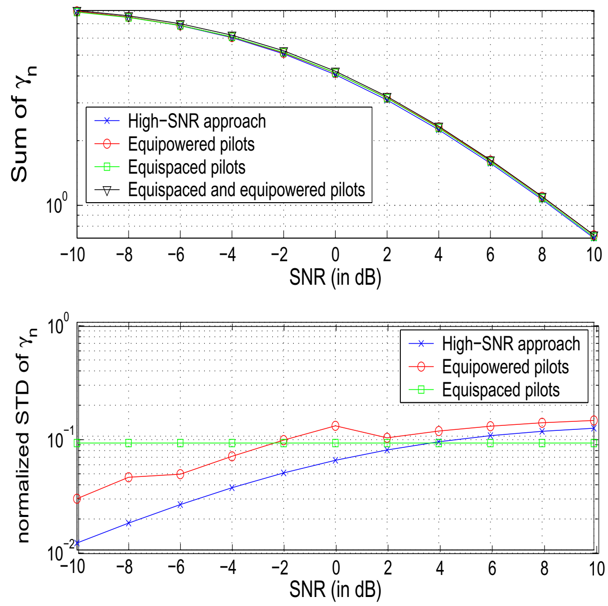

Fig. 3. Mean and 'std' of γ_n vs SNR for different design solutions.

5.1.5. *Comparisons*

An an example, we set $N = 16$, $L = 4$ and $N_a = 13$ and $\sigma_p^2 = 0.35\sigma_t^2$ where σ_t^2 is the total transmit power. The sum of the γ_n's, $n \in \mathcal{D}$, for the above designs are depicted in Figure 3. All four designs provide almost the same total MSE. The normalized (wrt the mean) 'stds' of the γ_n's are also depicted in Figure 3; note that the std for the fourth pilot design is identically zero. All the normalized stds are too small to have any real impact on the bit error rate or capacity of different data carriers. Note however that the third and fourth designs assumes $Q \geq (N - N_a)$, which may be restrictive in some scenarios.

5.2. Power allocation

Since we cannot use the MSE of the MMSE estimator to derive the optimum power allocation, we use the ergodic capacity as a design metric. However, deriving the exact capacity in the presence of channel estimation error is untractable. Hence, as in [3][6] and [8], we maximize a lower bound on the ergodic capacity.

For given $\mathcal{P}$ and $\boldsymbol{\rho}$, using the orthogonality property of the MMSE estimator, the ergodic capacity (in bits per symbol) is lower bounded by

$$\underline{C} = \frac{1}{N+L-1} E\left\{ \sum_{n\in\mathcal{D}} \log\left(1 + \frac{(\sigma_H^2 - \gamma_n)\sigma_s^2(n)|g|^2}{\sigma_s^2(n)\gamma_n + \sigma_v^2}\right)\right\} \tag{8}$$

where g is a complex zero-mean and unit variance Gaussian variable, and $\sigma_s^2(n) = E\left\{|s_n|^2\right\}$. Note that the data power distribution is not forced to be uniform. If the γ_n are not equal, it can be shown that uniform data power distribution is not optimum, i.e. it does not maximize $\underline{C}$. Indeed, if the data carriers experience different channel estimation MSEs, a non-uniform power loading at the transmitter will provide better overall system performance. The question is: how much performance gain can be achieved with optimum over uniform data power loading? This will depend on how much the γ_n's deviate from each other. Since we have shown in the previous subsection that these deviations are small, the performance gain will not be significant. Hence, using one of the optimal designs in the previous subsection and approximating the γ_n's by their mean value, say γ^o, the lower bound can be approximated by

$$\underline{C} \approx \frac{N-L}{N+L-1} E\left\{ \log\left(1 + \frac{(\sigma_H^2 - \gamma^o)\sigma_s^2|g|^2}{\sigma_s^2\gamma^o + (N-L)\sigma_v^2}\right)\right\} \tag{9}$$

where we set $\sigma_s^2(n) = \sigma_s^2/(N-L)$. The optimum power allocation can therefore be derived along the same lines as in [8].

6. CONCLUSIONS

We investigated the effect of virtual carriers on optimal pilot design in OFDM systems. Assuming an Rayleigh fading channels with uncorrelated scattering and using the MSE of the MMSE estimator, different design schemes were presented. It was shown that the optimal pilot power distribution is not uniform; less power should be assigned to the pilots that are close to the virtual carriers. However, this optimal power distribution did not seem to have a significant reduction of the total MSE wrt to uniform distribution. Further, it was shown that in the presence of virtual carriers, the data carriers experience different channel estimation MSE, unlike the case of fully-loaded systems. However, the differences between these MSEs were not significant enough to support a non-uniform power loading at the transmitter. Finally, it was shown that if the number of virtual carriers is smaller than N/L (where N is the total number of carriers and L is the length of the channel), then equispaced and equipowered pilot design gives almost the same total MSE as the other optimal schemes.

7. REFERENCES

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