The 2nd International Workshop on Cognitive Information Processing
14-15-16 June, 2010
Elba Island • Tuscany • Italy

DISTRIBUTED AND SEQUENTIAL SENSING FOR COGNITIVE RADIO NETWORKS

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Driven to Discover℠

Outline

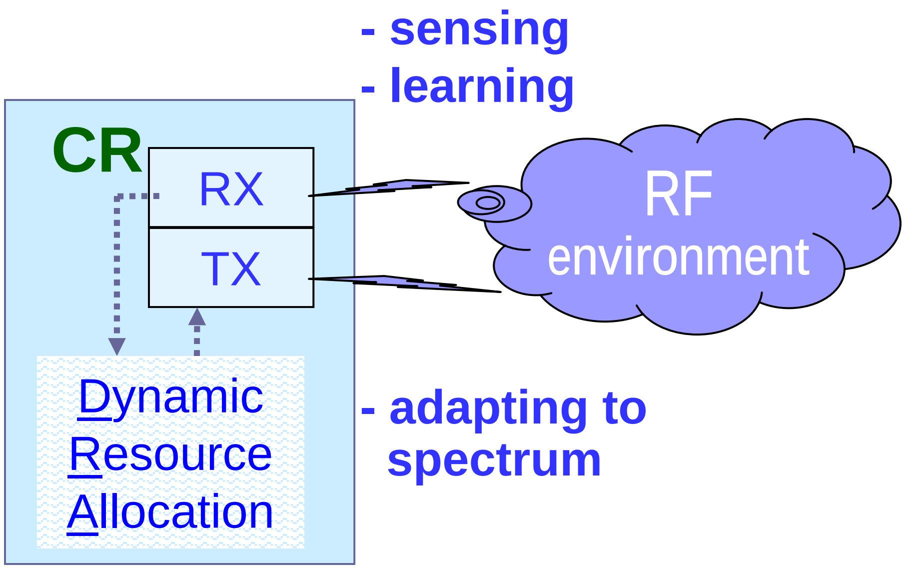
- Cognitive radios (CRs) and spectrum sharing
  - Motivation and context

- Collaborative and distributed CR sensing
  - RF interference spectrum cartography
  - Channel gain cartography

- Sequential CR sensing
  - … if time allows …

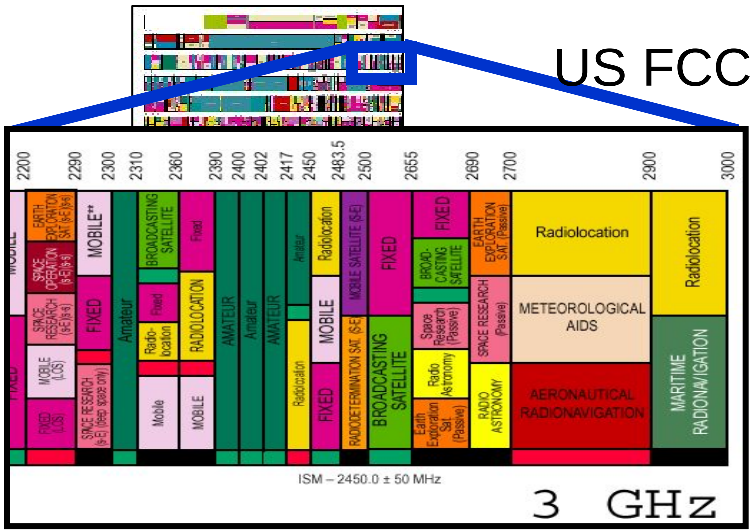
What is a cognitive radio ?

- Fixed radio
  - *policy-based*: parameters set by operators
- Software-defined radio (SDR)
  - *programmable*: can adjust parameters to intended link
- Cognitive radio (CR)
  - *intelligent:* can *sense* the environment & *learn* to *adapt* [Mitola'00]



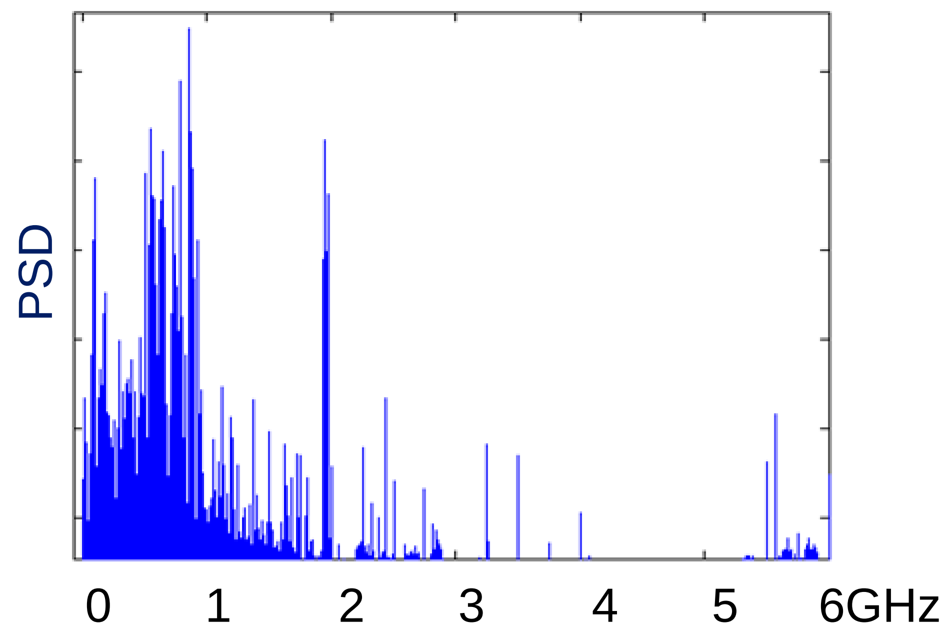
- *Cognizant receiver*: sensing
- *Agile transmitter*: adaptation
- *Intelligent DRA*: decision making
  - radio reconfiguration decisions
  - spectrum access decisions

Spectrum scarcity problem



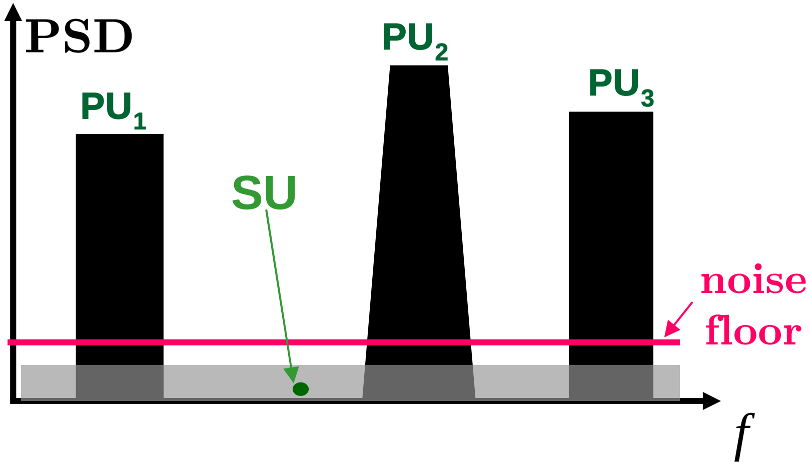
- ☹ fixed spectrum access policies have useful radio spectrum pre-assigned

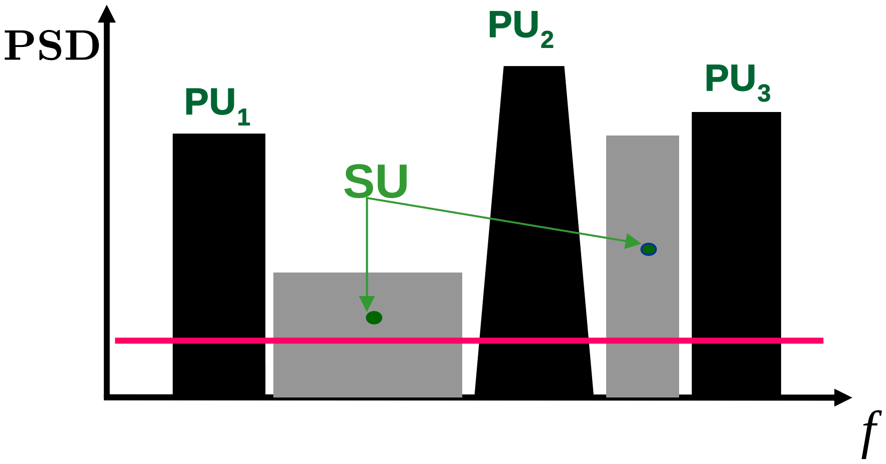
inefficient occupancy



Dynamical access under user hierarchy

- Primary Users (PUs) versus secondary users (SUs/CRs)





- Spectrum underlay
  - restriction on transmit-power levels
  - operation over ultra wide bandwidths
- Spectrum overlay
  - constraints on when and where to transmit
  - avoid interference to PUs via sensing and adaptive allocation

Motivating applications

- Future pervasive networks: efficient spectrum sharing

Licensed networks

Cellular, PCS band

Improved spectrum efficiency

Improved capacity

Secondary markets

Public safety band

Voluntary agreements between licensees and third party

Limited QoS

Third party access in licensed networks

TV bands (400-800 MHz)

Non-voluntary third party access

Licensee sets a protection threshold

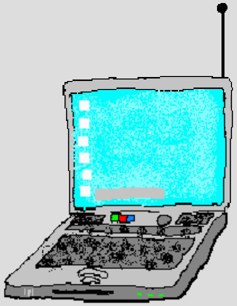
Unlicensed networks

ISM, UNII, Ad-hoc

Automatic frequency coordination

Interoperability

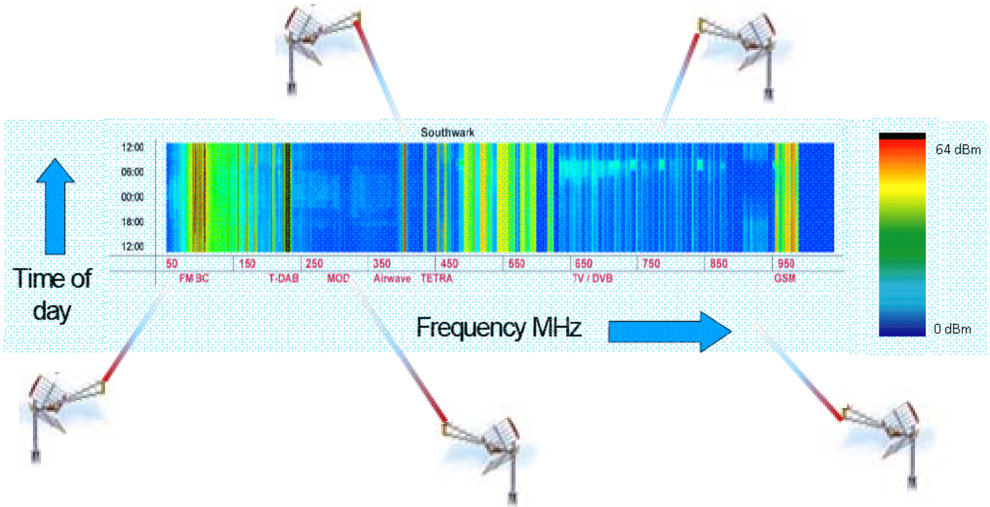
Co-existence



√ **more users/services** √ **higher rates** √ **better quality** √ **less interference**

Efficient sharing requires sensing

- Multiple CRs jointly detect the spectrum [Ganesan-Li'06 Ghasemi-Sousa'07]



Source: Office of Communications (UK)

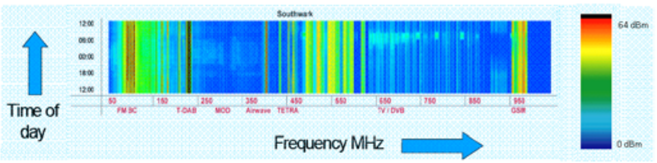
- Benefits of cooperation
  - spatial diversity gain mitigates multipath fading/shadowing
  - reduced sensing time and local processing
  - ability to cope with hidden terminal problem

- Limitation: existing approaches do not exploit spatial dimension

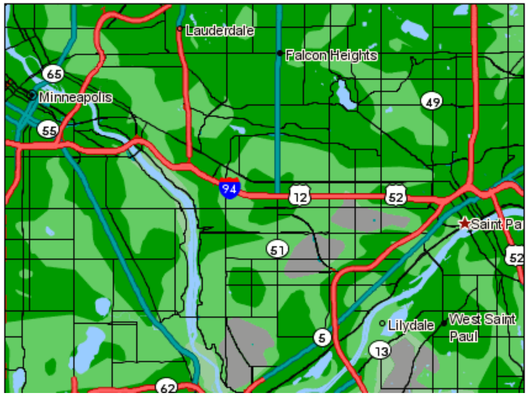
Cooperative PSD cartography

- Idea: collaborate to form a spatial map of the RF spectrum

Goal: Find PSD map $\Phi(\boldsymbol{x}, f)$ across space $\boldsymbol{x} \in \mathbb{R}^2$ and frequency $f \in \mathbb{R}$



- Specifications: coarse approx. suffices
- Approach: basis expansion of $\Phi(\boldsymbol{x}, f)$



J. A. Bazerque and G. B. Giannakis, "Distributed spectrum sensing for cognitive radio networks by exploiting sparsity," *IEEE Transactions on Signal Processing,* vol. 58, no. 3, pp. 1847-1862, March 2010.

Modeling

- Transmitters

  $\text{Tx}_s, \; s = 1, \ldots, N_s$

- Sensing CRs

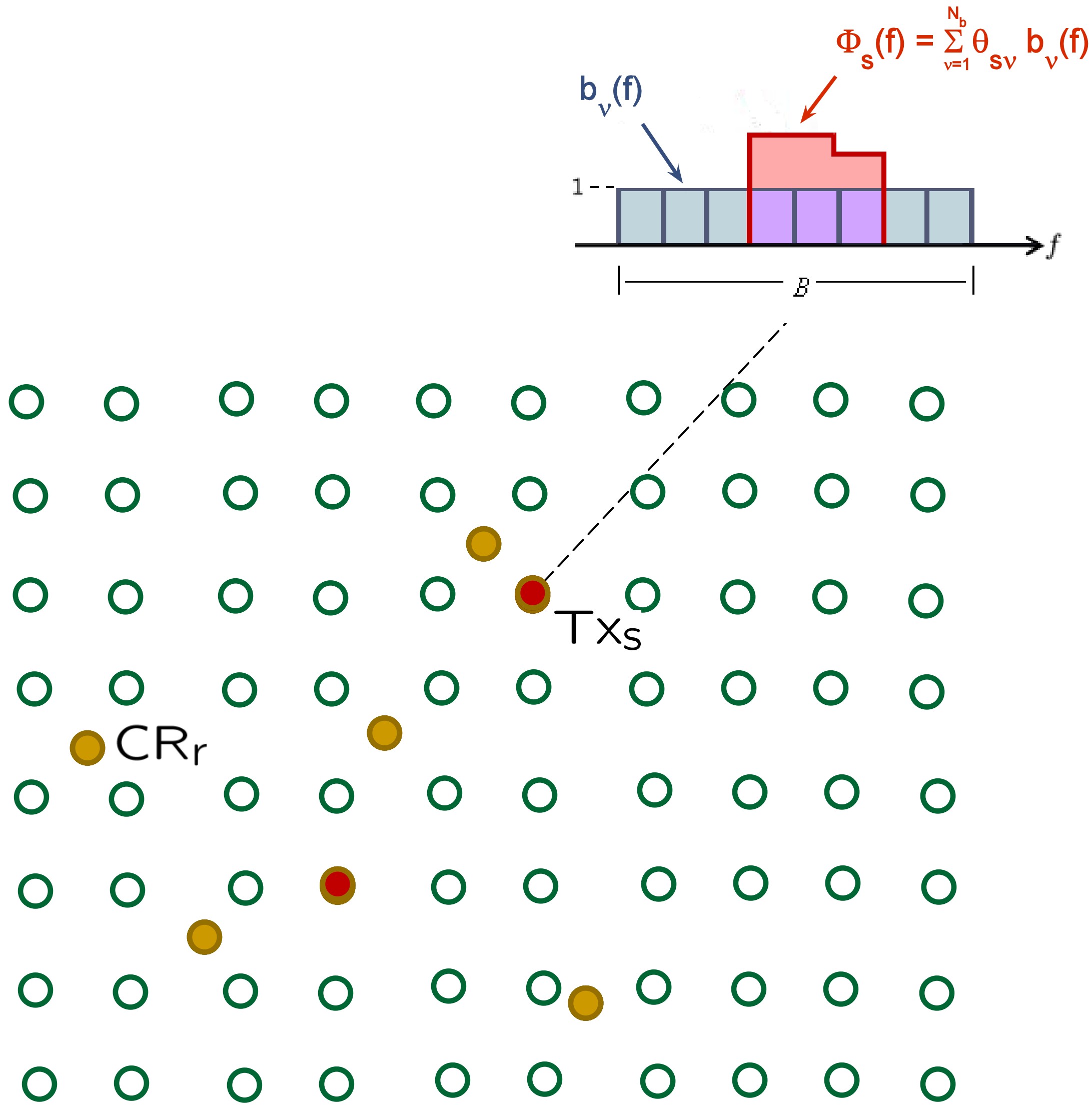
  $\text{CR}_r, \; r = 1 : N_r$

- Frequency bases

  $b_\nu(f), \; \nu = 1 : N_b$

- Sensed frequencies

  $f_k, \; k = 1 : K$



➢ Sparsity present in space and frequency

Space-frequency basis expansion

- Superimposed Tx spectra measured at CR r

$$\Phi_r(f) = \sum_{s=1}^{N_s} \gamma_{sr}\Phi_s(f) + \sigma_r^2 = \sum_{s=1}^{N_s} \gamma_{sr} \sum_{\nu=1}^{N_b} \theta_{s\nu} b_\nu(f) + \sigma_r^2$$

  - Average path-loss $\gamma_{sr} = \mathbb{E}(|H_{sr}(f)|^2) = \gamma_0 \left(\frac{d_0}{||\boldsymbol{x}_s - \boldsymbol{x}_r||}\right)^{-\alpha}, \ \alpha \in [2,5)$
  - Frequency bases $b_\nu(f) = \text{rect}(f - f_\nu)$

- Linear model in $\theta_{s\nu}$

$$\phi = \begin{pmatrix} \Phi_1(f_1) \\ \vdots \\ \Phi_1(f_K) \\ \Phi_2(f_1) \\ \vdots \\ \Phi_2(f_K) \\ \vdots \\ \vdots \\ \Phi_{N_r}(f_1) \\ \vdots \\ \Phi_{N_r}(f_K) \end{pmatrix} = \begin{pmatrix} b_1(f_1)\gamma_{11} & \cdots & b_{N_b}(f_1)\gamma_{N_s1} \\ & & \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ & & \\ b_1(f_K)\gamma_{1N_r} & \cdots & b_{N_b}(f_K)\gamma_{N_sN_r} \end{pmatrix} \begin{pmatrix} \theta_{11} \\ \theta_{21} \\ \vdots \\ \theta_{N_s1} \\ \theta_{12} \\ \vdots \\ \theta_{N_s2} \\ \vdots \\ \vdots \\ \theta_{N_sN_b} \end{pmatrix} = \boldsymbol{B\theta}$$

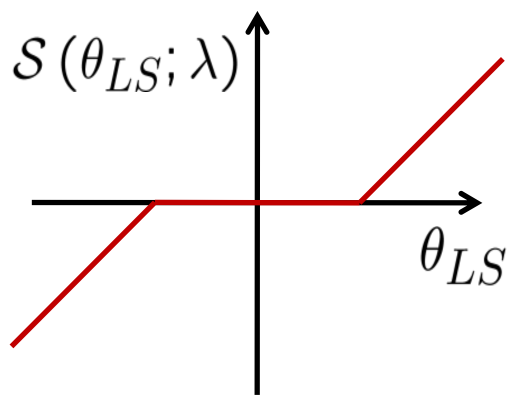
Sparse linear regression

- Seek a sparse $\boldsymbol{\theta}$ to capture the spectrum measured at CR_r

Lasso

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \ ||\boldsymbol{\varphi} - \boldsymbol{B}\boldsymbol{\theta}||_2^2 + \lambda||\boldsymbol{\theta}||_1$$

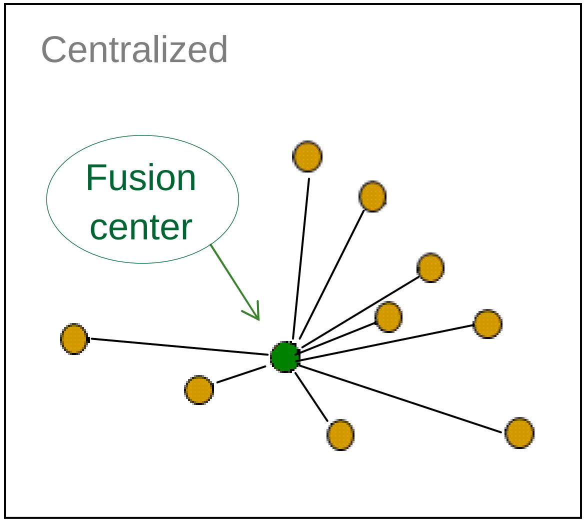
  - Soft threshold shrinks noisy estimates to zero



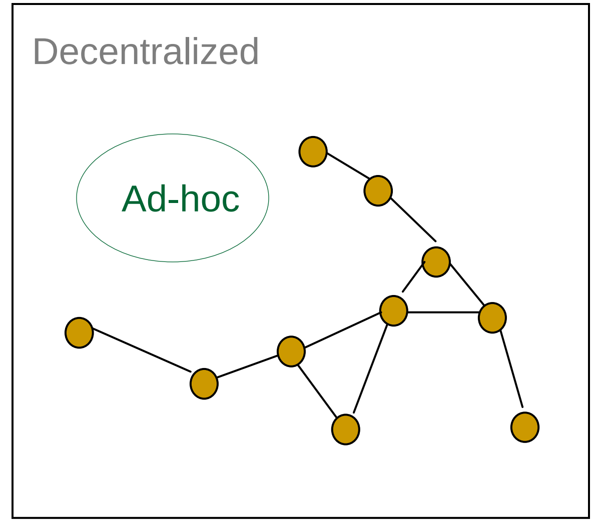
- Effects sparsity and variable selection
- Improves LS performance by incorporating a priori information

R. Tibshirani, ``Regression Shrinkage and Selection via the Lasso," *Journal of the Royal Statistical Society,* Series B, vol. 58, no. 1, pp. 267-288, 1996.

Distributed recursive implementation



Scalability
Robustness
Lack of infrastructure



- Consensus-based approach
  - solve locally

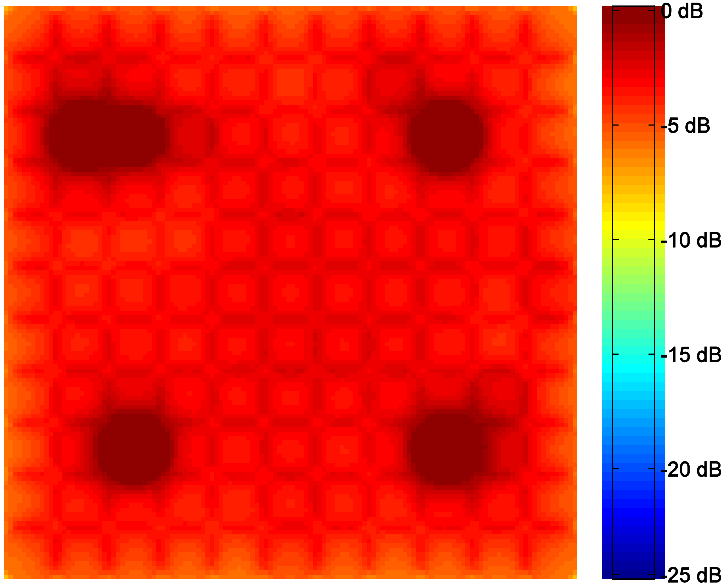
$$\boxed{\begin{aligned} \hat{\boldsymbol{\theta}} = \quad & \arg\min_{\boldsymbol{\theta}_r \geq 0} \; ||\boldsymbol{\varphi}_{rt} - \boldsymbol{B}_r\boldsymbol{\theta}_r||_2^2 + \frac{\lambda}{M}||\boldsymbol{\theta}_r||_1 \\ & \text{s.to} \qquad \boldsymbol{\theta}_r = \boldsymbol{\theta}_{r'}, \quad \forall r' \in \mathcal{N}_r \end{aligned}}$$

Constrained optimization using the alternating-direction method of multipliers (AD-MoM)

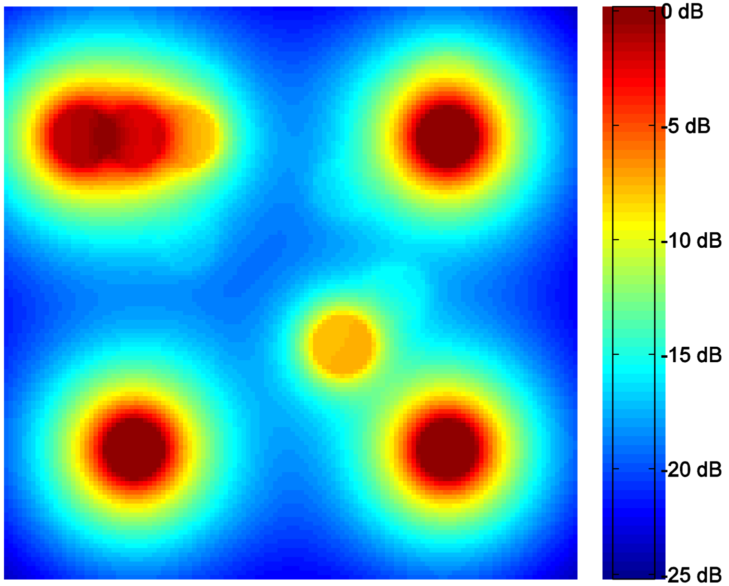
Exchange of local $\boldsymbol{\theta}_r$ estimates

RF spectrum cartography

- 5 sources
- $N_s = 121$ candidate locations, $N_r = 50$ cognitive radios



NNLS

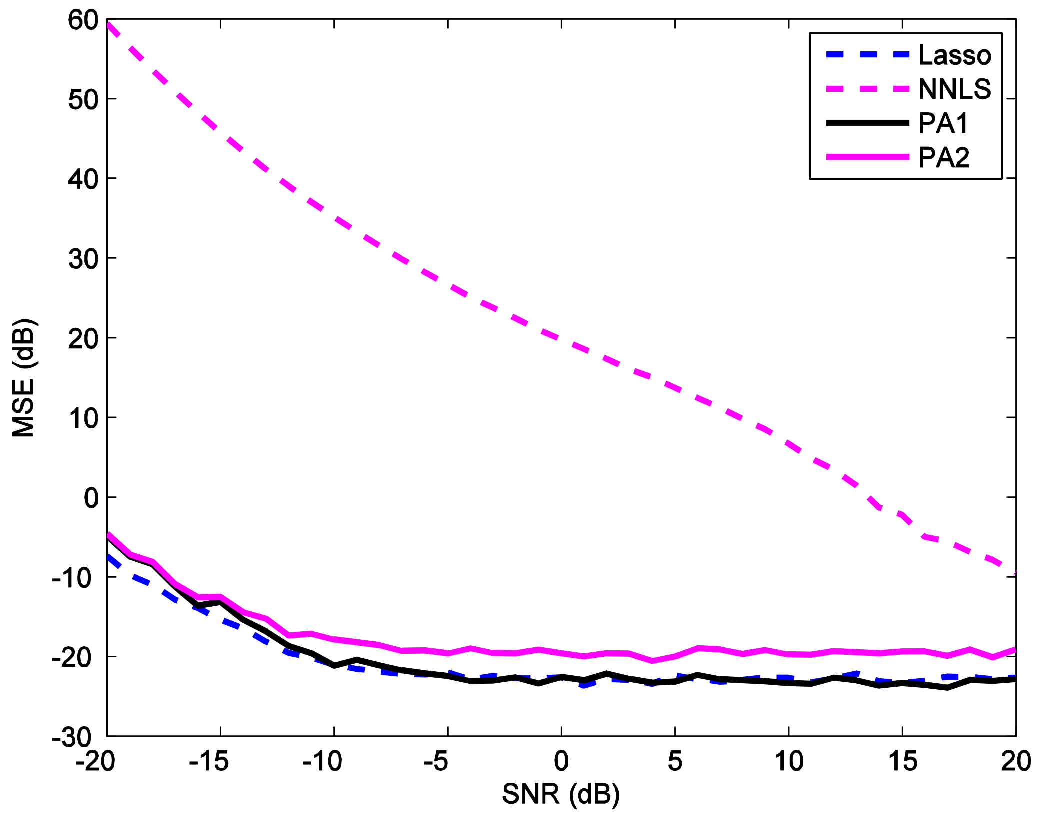


Lasso

- As a byproduct, Lasso localizes all sources via variable selection

MSE performance

- Error between estimate $\hat{\boldsymbol{\theta}}$ and $\boldsymbol{\theta}$
- Monte Carlo MSE versus analytical approximations

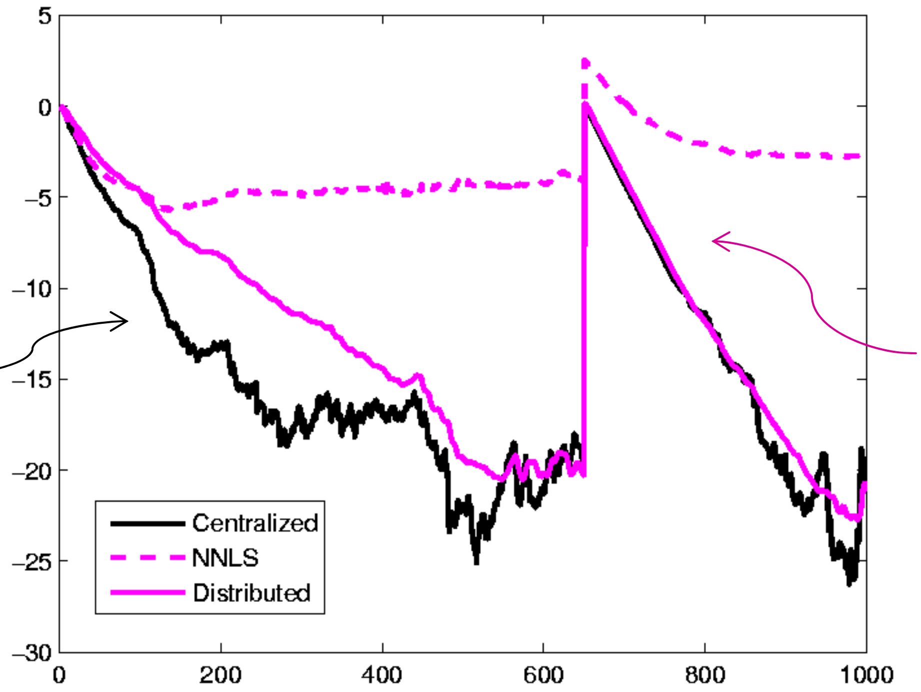


- PA1 with known C_e
- PA2 with bounds for C_e

Tracking performance

- Normalized error $||\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}||/||\boldsymbol{\theta}||$

batch solutions
one per time-slot

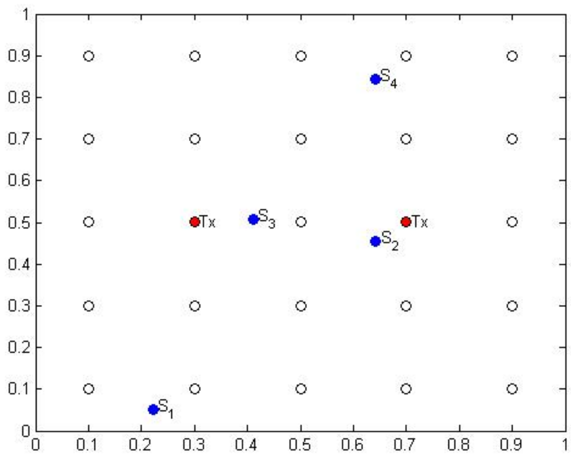


time-slot t

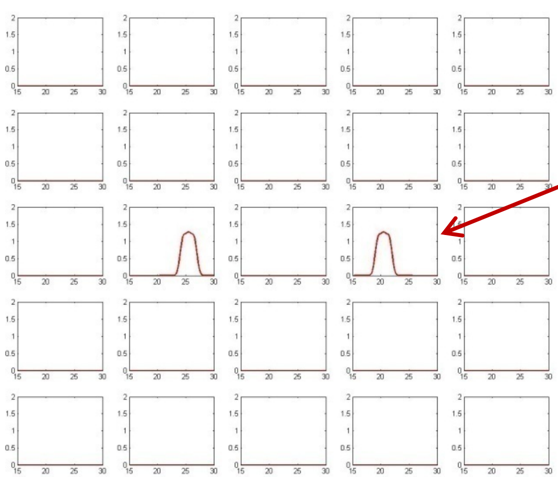
path of distributed
online updates

- Non-stationarity: one Tx exits at time-slot t=650

Simulation: PSD map estimation



- Centralized sensing
- No fading
- *I=25*
- *J=15*

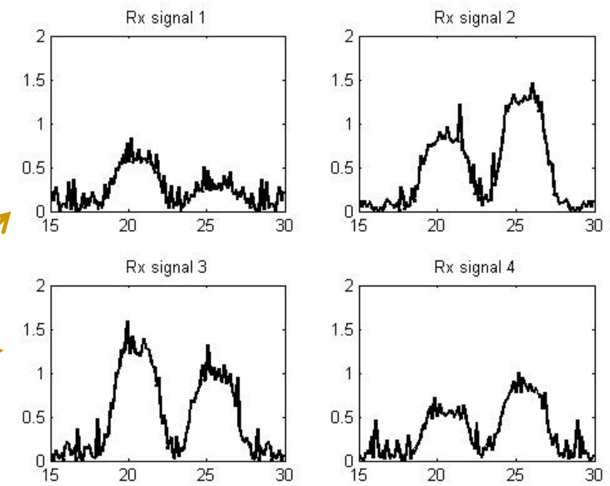


transmitters

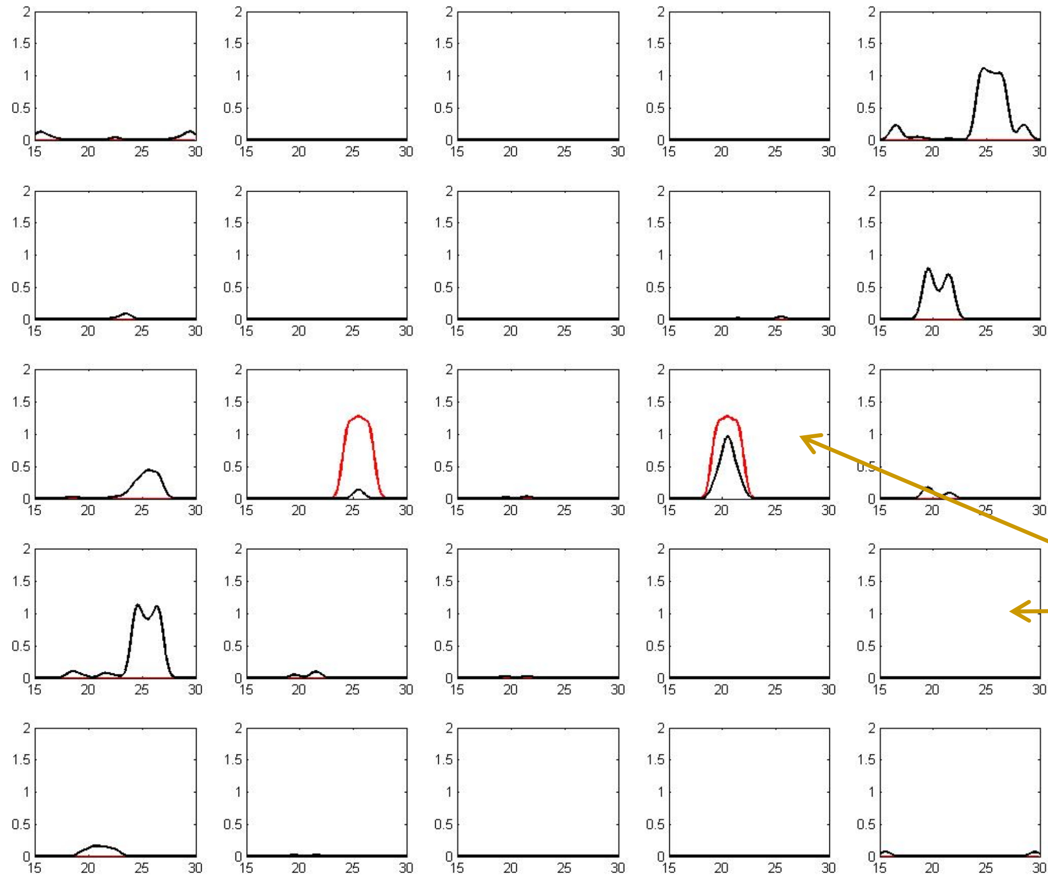
$\Phi_i(f)$

$\Phi_m(f)$

sensors



Centralized sensing without fading



“true” Tx spectrum

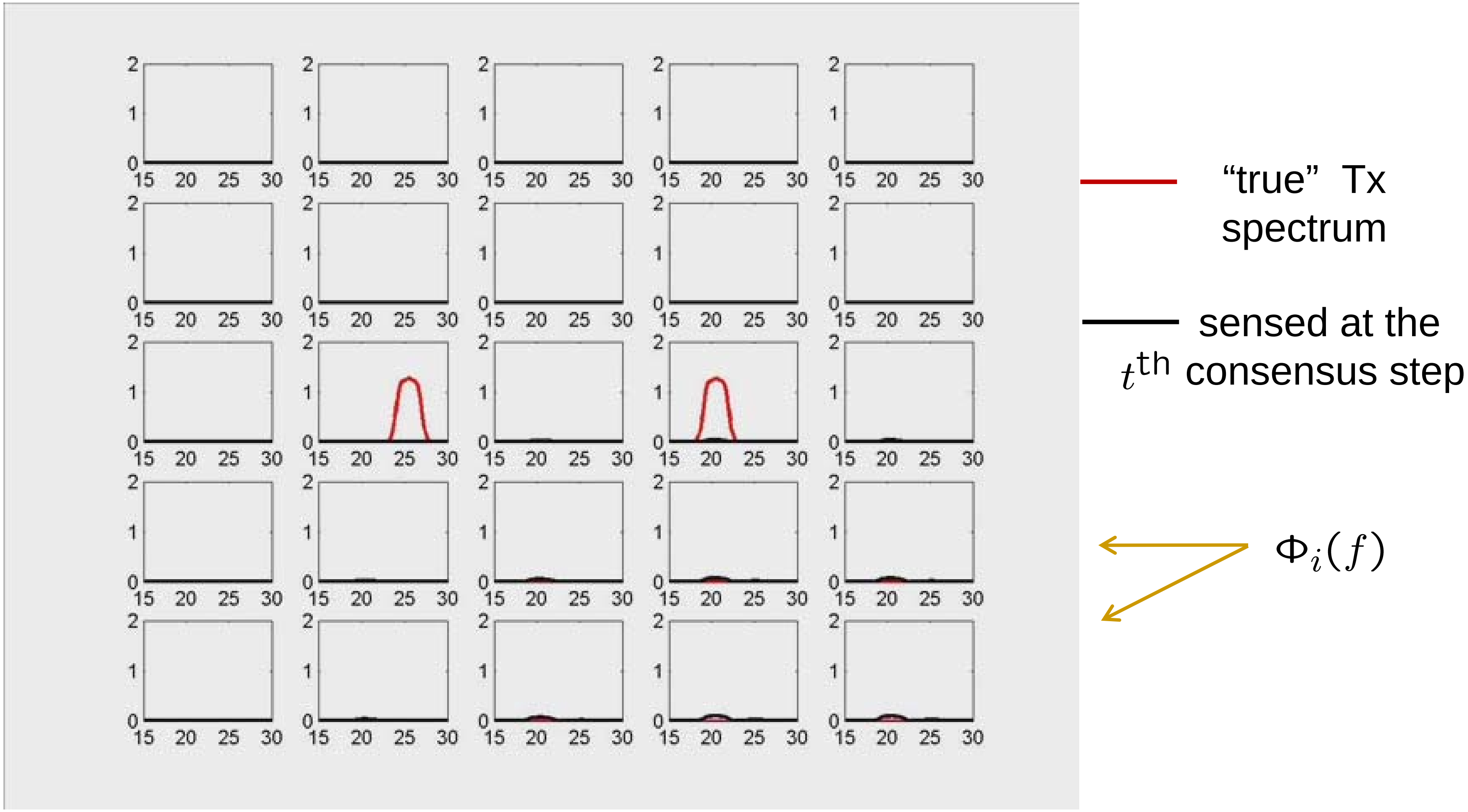
BP solution

$\Phi_i(f)$

- LS solution yields spurious peaks

- L_1 norm minimization yields a sparse solution

Distributed consensus with fading



- Starting from a local estimate, sensors reach consensus

Spline-based PSD cartography

Q: How about shadowing? **A1:** Basis expansion w/ coefficient-functions

$$\boxed{\Phi(\mathbf{x}, f) = \sum_{\nu=1}^{N_b} g_\nu(\mathbf{x}) b_\nu(f)}$$

- $b_\nu(f)$: known bases accommodate prior knowledge
  - overcomplete expansions allow for uncertainty on Tx parameters
- $g_\nu(\mathbf{x})$: unknown dependence on spatial variable $\mathbf{x}$
  - learn shadowing effects from periodograms at spatially distributed CRs

J. A. Bazerque, G. Mateos, and G. B. Giannakis, ``Group-Lasso on Splines for Spectrum Cartography,'' *IEEE Transactions on Signal Processing,* submitted June 2010.

Smooth and sparse coefficient functions

- Twofold regularization of variational LS estimator

$$\min_{\{g_\nu \in \mathcal{S}\}} \frac{1}{N_r N} \sum_{r=1}^{N_r} \sum_{n=1}^{N} \left(\varphi_{rn} - \sum_{\nu=1}^{N_b} g_\nu(\mathbf{x}_r) b_\nu(f_n) \right)^2 \qquad \textbf{(P1)}$$

$$+\lambda \sum_{\nu=1}^{N_b} \int_{\mathbb{R}^2} ||\nabla^2 g_\nu(\mathbf{x})||_F^2 d\mathbf{x} + \mu \sum_{\nu=1}^{N_b} \left\| \left[g_\nu(\mathbf{x}_1), \ldots, g_\nu(\mathbf{x}_{N_r}) \right]' \right\|_2 .$$

Smoothing penalty

sparsity enforcing penalty

Proposition: optimal $g_\nu(\mathbf{x})$ admits kernel expansion

$$g_\nu(\mathbf{x}) = \sum_{r=1}^{N_r} \zeta_{\nu r} k(\mathbf{x} - \mathbf{x}_r)$$

parameters

$k(\Delta \mathbf{x}_r) := ||\Delta \mathbf{x}_r||^2 \log(||\Delta \mathbf{x}_r||)$

Estimating kernel parameters

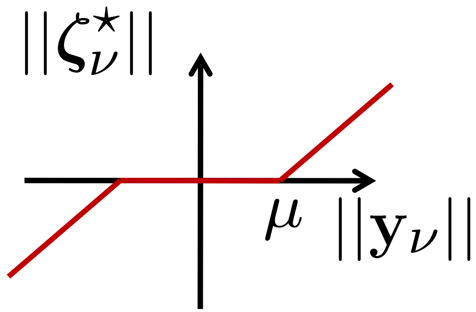
- Need $\zeta_\nu = (\zeta_{\nu 1}, \ldots, \zeta_{\nu N_r}), \;\; \nu = 1, \ldots, N_b$
- Group Lasso on (P1) equivalent

$$\min_{\zeta} \frac{1}{2} \|\mathbf{y} - \mathbf{X}\zeta\|_2^2 + \mu \sum_{\nu=1}^{N_b} \|\zeta_\nu\|_2$$

  - ***X*** depends on kernels and bases

- Case $\mathbf{X} = \mathbf{I}$ admits closed-form solution

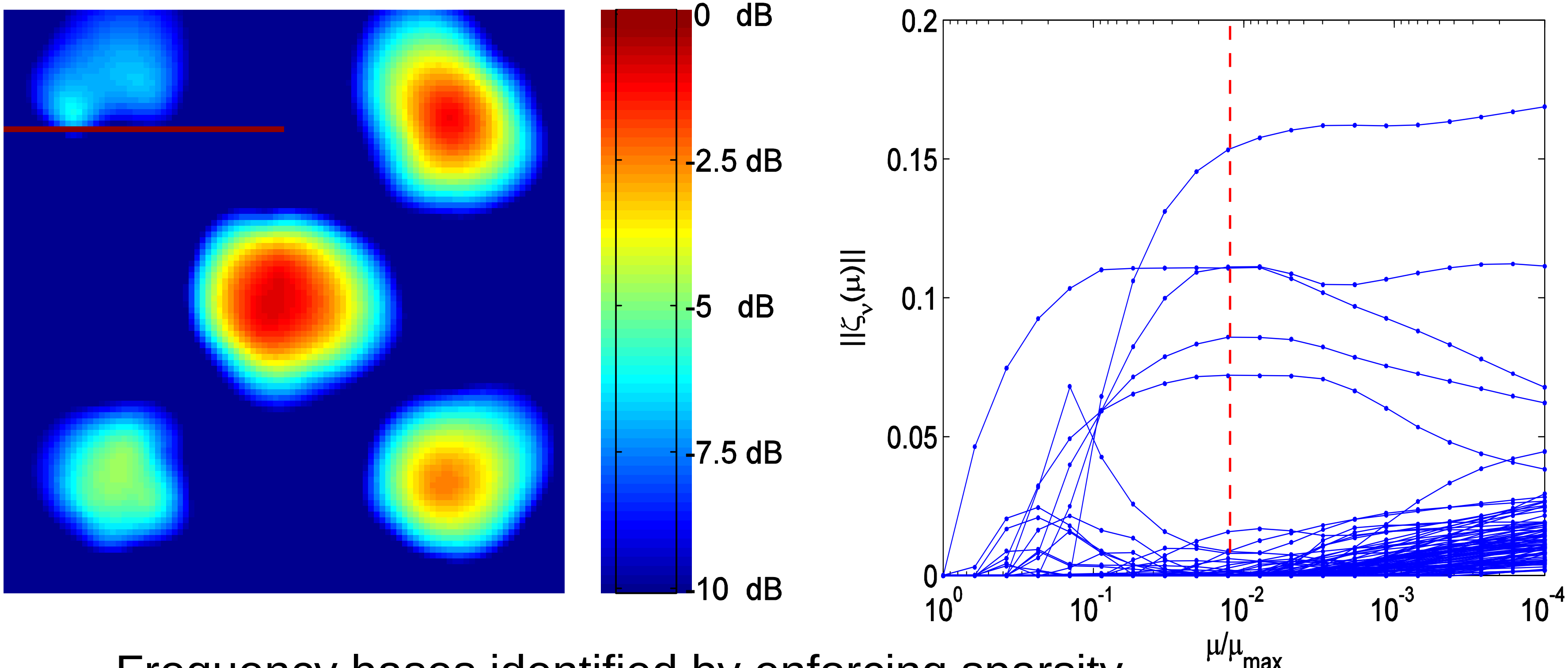
$$\zeta_\nu^\star = \frac{\mathbf{y}_\nu}{\|\mathbf{y}_\nu\|_2} (\|\mathbf{y}_\nu\|_2 - \mu)_+ \Longrightarrow$$



- $\zeta_\nu = \mathbf{0} \Longrightarrow g_\nu(\mathbf{x}) = 0 \; \forall \mathbf{x} \Longrightarrow b_\nu(f)$ not included

Simulation: PSD atlas

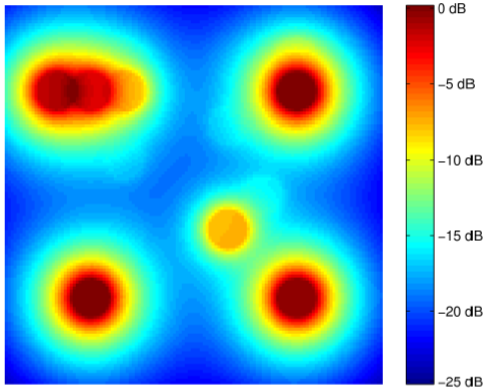
- N_r=100 CRs, N_b=90 bases (raised cosines), N_s=5 Tx PUs



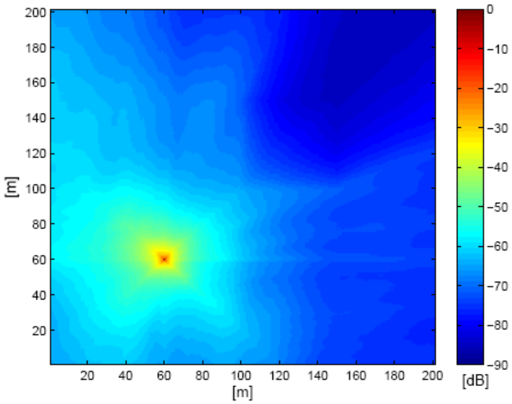
- Frequency bases identified by enforcing sparsity
- Power distribution across space revealed by promoting smoothness

Cartography for CR sensing

- Power spectral density (PSD) maps
  - Capture ambient power in space-time-frequency
  - Can identify “crowded” regions to be avoided



- Channel gain (CG) maps
  - Time-freq. channel *from any-to-any point*
  - CRs adjust Tx power to min. PU disruption



Cooperative CG cartography

- Wireless CG (in dB)

$$G_{\mathbf{x}\to\mathbf{y}}(t) = \underbrace{G_0}_{\text{gain}} \underbrace{-10\gamma\log_{10}(||\mathbf{x}-\mathbf{y}||_2)}_{\text{path loss}} + \underbrace{s_{\mathbf{x}\to\mathbf{y}}(t)}_{\text{shadowing}}$$

- TDMA-based training yields CR-to-CR shadow fading measurements

$$\breve{s}_{\mathbf{x}_j\to\mathbf{x}_r}(t) = s_{\mathbf{x}_j\to\mathbf{x}_r}(t) + \epsilon_{\mathbf{x}_j\to\mathbf{x}_r}(t)$$

$$\breve{\mathbf{s}}_r(t) \triangleq [\breve{s}_{\mathbf{x}_1\to\mathbf{x}_r}(t), \ldots, \breve{s}_{\mathbf{x}_{r-1}\to\mathbf{x}_r}(t), \breve{s}_{\mathbf{x}_{r+1}\to\mathbf{x}_r}(t), \ldots, \breve{s}_{\mathbf{x}_{N_r}\to\mathbf{x}_r}(t)]^T$$

- Goal: Given $\{\breve{\mathbf{s}}_r(\tau)\}\forall r, \tau \geq 1$, estimate $s_{\mathbf{x}\to\mathbf{y}}(t)$ and $G_{\mathbf{x}\to\mathbf{y}}(t)$ *for any* $\mathbf{x}, \mathbf{y} \in \mathcal{A}$

S.-J. Kim, E. Dall'Anese, and G. B. Giannakis, ``Cooperative Spectrum Sensing for Cognitive Radios using Kriged Kalman Filtering," *IEEE J. of Selected Topics in Signal Proc.,* Feb. 2011.

Dynamic shadow fading model

- Shadowing in dB is Gaussian distributed
- *Spatial loss field-based* shadowing model [Agrawal et al. '09]

$$s_{\mathbf{x}\to\mathbf{y}}(t) = \frac{1}{\|\mathbf{x}-\mathbf{y}\|^{\frac{1}{2}}} \int_{\mathbf{x}}^{\mathbf{y}} \ell(\mathbf{u},t)d\mathbf{u}$$

- Spatio-temporal loss-field evolution [Mardia '98] [Wikle et at. '99]

$$\ell(\mathbf{x},t) = \bar{\ell}(\mathbf{x},t) + \tilde{\ell}(\mathbf{x},t)$$

$$\bar{\ell}(\mathbf{x},t) = \int_{\mathcal{A}} w(\mathbf{x},\mathbf{u})\bar{\ell}(\mathbf{u},t-1) + \eta(\mathbf{x},t)$$

$\bar{\ell}(\mathbf{x},t)$: spatio-temporally colored
$\tilde{\ell}(\mathbf{x},t)$: temporally white and spatially colored
$w(\mathbf{x},\mathbf{u})$: known, captures interaction between $\bar{\ell}(\mathbf{x},t)$ and $\bar{\ell}(\mathbf{u},t-1)$
$\eta(\mathbf{x},t)$: zero-mean Gaussian, spatially colored, and temporally white

K. V. Mardia, C. Goodall, E. J. Redfern, and F. J. Alonso, “The Kriged Kalman filter,” *Test*, vol. 7, no. 2, pp. 217–285, Dec. 1998.

State-space model

- Basis-expansion representation for $\bar{\ell}(\mathbf{x}, t)$ and $w(\mathbf{x}, \mathbf{u})$

$$\bar{\ell}(\mathbf{x}, t) = \sum_{k=1}^{\infty} \alpha_k(t)\psi_k(\mathbf{x}) \qquad w(\mathbf{x}, \mathbf{u}) = \sum_{k=1}^{\infty} \beta_k(\mathbf{x})\psi_k(\mathbf{u})$$

- Retain *K* terms and sample at $\{\mathbf{x}_r \in \mathcal{A}\}_{r=1}^{N_r}$

  - state equation $\boxed{\boldsymbol{\alpha}(t) = \mathbf{T}\boldsymbol{\alpha}(t-1) + \boldsymbol{\Psi}^{\dagger}\boldsymbol{\eta}(t)}$

- Recall loss field model

$$\bar{s}_{\mathbf{x}\to\mathbf{y}}(t) = \sum_{k=1}^{\infty} \underbrace{\left[\frac{1}{||\mathbf{x}-\mathbf{y}||^{1/2}} \int_{\mathbf{x}}^{\mathbf{y}} \psi_k(\mathbf{u})d\mathbf{u}\right]}_{\triangleq \phi_{\mathbf{x}\to\mathbf{y},k}} \alpha_k(t) \approx \boldsymbol{\phi}_{\mathbf{x}\to\mathbf{y}}^T \boldsymbol{\alpha}(t)$$

  - measurement equation $\boxed{\breve{\mathbf{s}}(t) = \boldsymbol{\Phi}\boldsymbol{\alpha}(t) + \tilde{\mathbf{s}}(t) + \boldsymbol{\epsilon}(t)}$

Tracking via Kriged Kalman Filtering

Idea: estimate $\boldsymbol{\alpha}(t)$ (and hence $\bar{s}_{\mathbf{x}\to\mathbf{y}}(t)$) via Kalman filtering (KF) spatially interpolate with Kriging (KKF) to account for $\tilde{s}(\mathbf{x}, t)$

- Conditioned on $\check{\mathbf{s}}_{1:t} \triangleq \{\check{\mathbf{s}}(\tau)\}_{\tau=1}^{t}$ $s_{\mathbf{x}\to\mathbf{y}}(t)$ $\mathbf{x}, \mathbf{y} \in \mathcal{A}$ is Gaussian

$$\hat{s}_{\mathbf{x}\to\mathbf{u}}(t) \triangleq \mathbb{E}\{s_{\mathbf{x}\to\mathbf{u}}(t)|\check{\mathbf{s}}_{1:t}\} = \phi_{\mathbf{x}\to\mathbf{u}}^T\hat{\boldsymbol{\alpha}}(t|t) + \mathbf{c}_{\tilde{s}}^T(\mathbf{x},\mathbf{u})\boldsymbol{\Sigma}^{-1}\left[\check{\mathbf{s}}(t) - \boldsymbol{\Phi}\hat{\boldsymbol{\alpha}}(t|t)\right]$$

$$\mathrm{var}\{s_{\mathbf{x}\to\mathbf{u}}(t)|\check{\mathbf{s}}_{1:t}\} = \sigma_{\tilde{s}}^2 - \mathbf{c}_{\tilde{s}}^T(\mathbf{x},\mathbf{u})\boldsymbol{\Sigma}^{-1}\mathbf{c}_{\tilde{s}}(\mathbf{x},\mathbf{u}) +$$
$$+\left[\phi_{\mathbf{x}\to\mathbf{u}}^T - \mathbf{c}_{\tilde{s}}(\mathbf{x},\mathbf{u})\boldsymbol{\Sigma}^{-1}\boldsymbol{\Phi}\right]\mathbf{P}(t|t)\left[\phi_{\mathbf{x}\to\mathbf{u}} - \boldsymbol{\Phi}^T\boldsymbol{\Sigma}^{-1}\mathbf{c}_{\tilde{s}}(\mathbf{x},\mathbf{u})\right]$$

$$\mathbf{c}_{\tilde{s}}(\mathbf{x},\mathbf{y}) \triangleq \mathbb{E}\{\tilde{\mathbf{s}}(t)\tilde{s}_{\mathbf{x}\to\mathbf{y}}(t)\}$$

- Estimated CG map: $\hat{G}_{\mathbf{x}\to\mathbf{y}}(t) = G_0 - 10\gamma\log_{10}(\|\mathbf{x}-\mathbf{y}\|) + \hat{s}_{\mathbf{x}\to\mathbf{y}}(t)$

Distributed implementation

- Prediction step locally but correction step collaboratively

$$\hat{\boldsymbol{\alpha}}(t|t) = \hat{\boldsymbol{\alpha}}(t|t-1) + \mathbf{P}(t|t)\boxed{\boldsymbol{\Phi}^T \boldsymbol{\Sigma}^{-1}\left[\breve{\mathrm{s}}(t) - \boldsymbol{\Phi}\hat{\boldsymbol{\alpha}}(t|t-1)\right]}$$

$$\{\boldsymbol{\chi}_r(t)\}_{r=1}^{N_r} = \arg\min_{\{\boldsymbol{\chi}_r\}} \sum_{r=1}^{N_r} \|\boldsymbol{\chi}_r - N_r \mathbf{H}_r \mathbf{y}_r(t)\|^2$$

$$\text{subject to } \boldsymbol{\chi}_r = \boldsymbol{\chi}_\varrho, \quad \forall \varrho \in \mathcal{N}_r, \quad r = 1, \ldots, N_r$$

$\boldsymbol{\chi}_r(t)$: local copy of $\boldsymbol{\chi}(t)$ at CR *r*

$$\mathbf{y}_r(t) \triangleq \breve{\mathrm{s}}_r(t) - \boldsymbol{\Phi}_r \boldsymbol{\alpha}(t|t-1)$$

$$\boldsymbol{\chi}(t) = \sum_{r=1}^{N_r} \mathbf{H}_r \mathbf{y}_r(t)$$

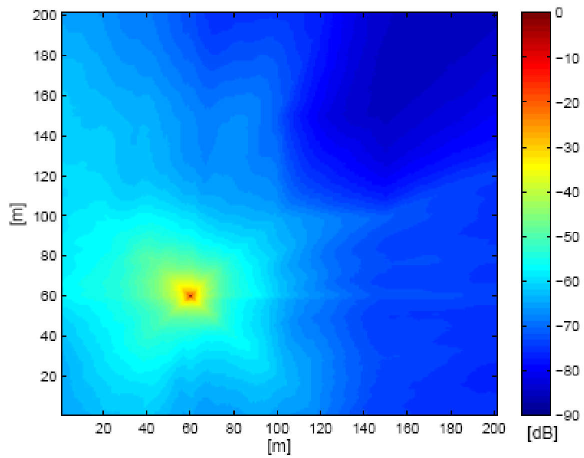
($\mathbf{H}_r$ proper sub-matrix of $\boldsymbol{\Phi}^T \boldsymbol{\Sigma}^{-1}$)

- Distributed solution via alternating direction method of multipliers (AD-MoM)

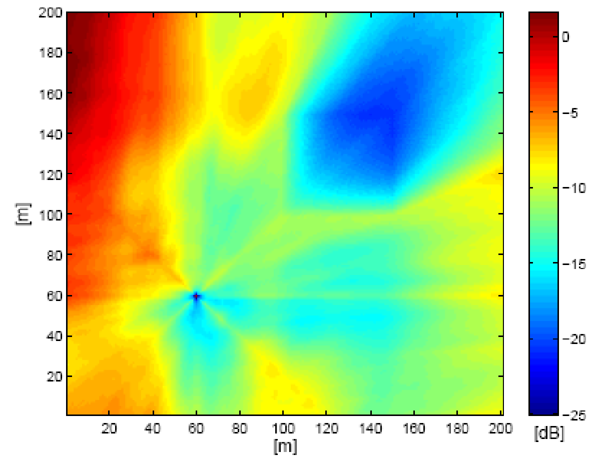
- Kriging can be distributed likewise via AD-MoM and consensus

E. Dall'Anese, S.-J. Kim, and G.B. Giannakis, "Channel Gain Map Tracking via Distributed Kriging," *IEEE Trans. on Vehicular Tech*nology, June 2010 (submitted).

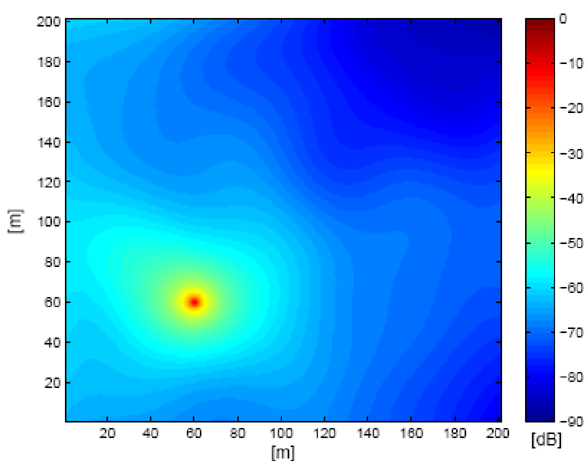
Simulation: map estimation performance



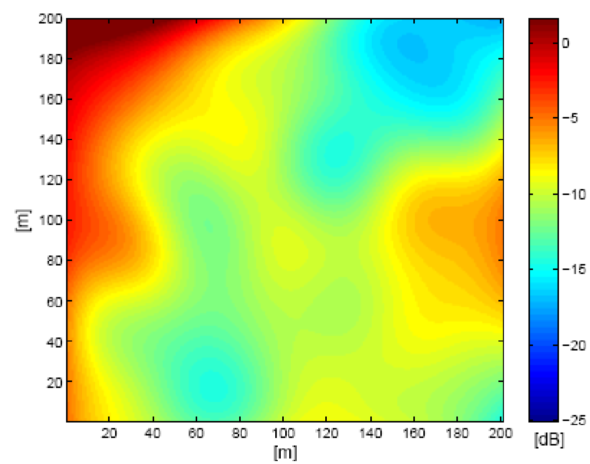
(a) True CG map.



(b) True shadow fading map.



(c) Estimated CG map.



(d) Estimated shadow fading map.

Tracking of PU power and position

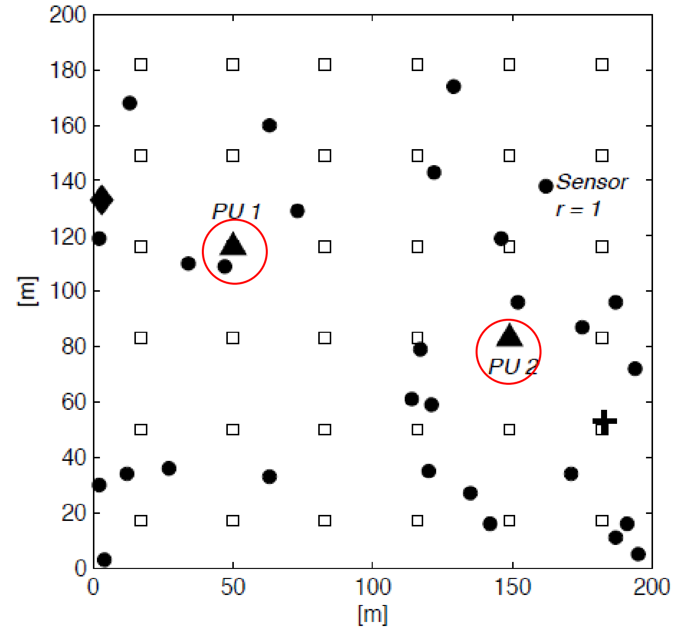
- **Given** maps $\mathbf{g}_r(t) \triangleq [g_{\mathbf{x}_1 \to \mathbf{x}_r}(t) \ \ldots \ g_{\mathbf{x}_{N_s} \to \mathbf{x}_r}(t)]^T$, $\{\mathbf{x}_s \in \mathcal{A}\}_{s=1}^{N_s}$ *candidate* PU positions

$$\pi_r(t) = \mathbf{g}_r^T(t)\mathbf{p}(t) + z_r(t)$$

- **Estimate** *sparse* power vector

$$\mathbf{p}(t) \triangleq [p_1(t) \ \ldots \ p_{N_s}(t)]$$

- Sparse regression for tracking [Kim-Dall'Anese-GG'09]

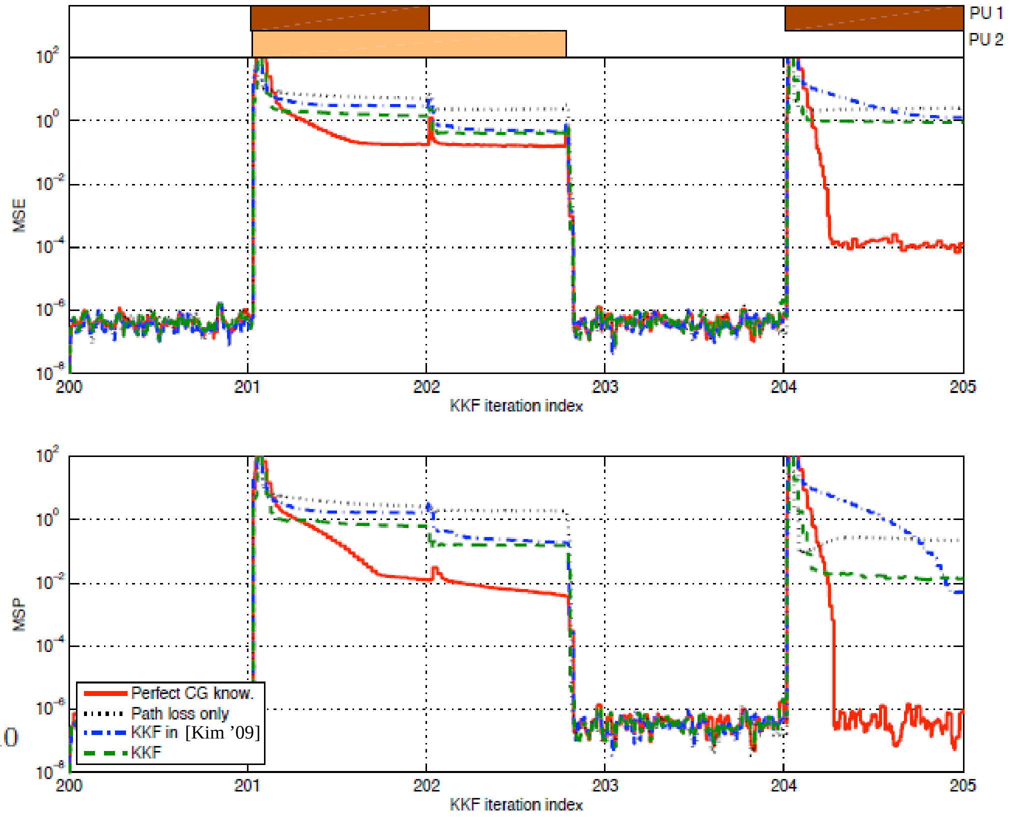


$$\hat{\mathbf{p}}(t) = \arg\min_{\mathbf{p} \succeq \mathbf{0}} J_t(\mathbf{p}), \quad J_t(\mathbf{p}) \triangleq \left[\frac{1}{2}\sum_{\tau=1}^{t} \mu^{t-\tau} \sum_{r=1}^{N_r} \left(\pi_r(\tau) - \hat{\mathbf{g}}_r^T(\tau)\mathbf{p}\right)^2 + \lambda_t \|\mathbf{p}\|_1\right]$$

Simulation: PU power tracking

- Average tracking performance
  - Power MSE (avg. over all grid points) across time (KKF iterations)
  - Mean spurious power (avg. over all grid except PU points) vs. time
  - Area 200m x 200m
  - Parameters

$$N_s = 36,\ N_r = 20\ \mathrm{CR},\ d_{\mathrm{comm}} = 125\mathrm{m}$$
$$\mathrm{var}\{\epsilon_{\mathbf{x}_j \to \mathbf{x}_r}(t)\} = 10,\ \mathrm{var}\{z_r(t)\} = 10^{-10}$$



- Shadowing: 0-mean, std. dev. 10 dB

CG maps for resource allocation

- After having located the PU at $\mathbf{x}_s$ with tx-power P_s (dB); and rx-PU power $\Pi(\mathbf{x})$ at any ***x***

- PU coverage probability: $P_{\text{cov}}(\mathbf{x}) \triangleq \Pr\{\Pi(\mathbf{x}) \geq \Pi_{\min}\}$

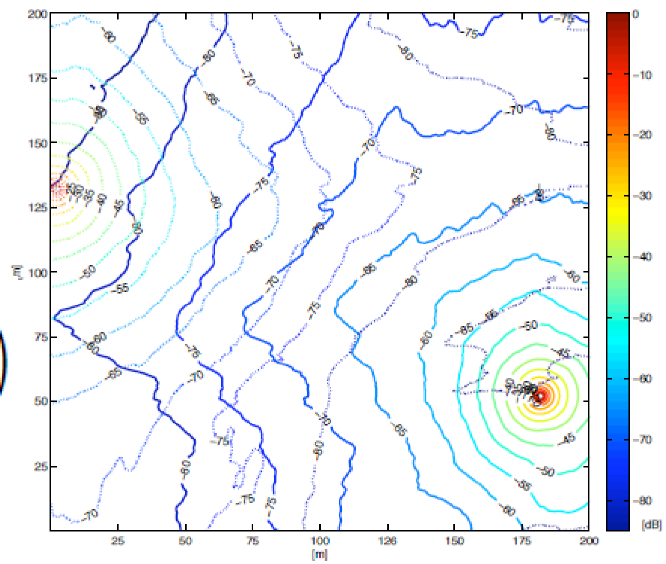
$$P_{\text{cov}}(\mathbf{x}) = Q\left(\frac{\Pi_{\min} - P_s - G_0 + 10\gamma \log_{10} \|\mathbf{x}_s - \mathbf{x}\| - \hat{s}_{\mathbf{x}_s \to \mathbf{x}}}{\sigma_{s_{\mathbf{x}_s \to \mathbf{x}}}}\right)$$

  - Coverage region not a disc (due to shadowing)

- CR interf. probability $P_{\text{int}}(\mathbf{x}) \triangleq \Pr\{\Pi^{\text{CR}}(\mathbf{x}) \geq I_{\max}\}$

$$P_{\text{int}}(\mathbf{x}) = Q\left(\frac{I_{\max} - P_r - G_0 + 10\gamma \log_{10} ||\mathbf{x}_r - \mathbf{x}||_2 - \boxed{\hat{s}_{\mathbf{x}_r \to \mathbf{x}}}}{\sigma_{s_{\mathbf{x}r \to \mathbf{x}}}}\right)$$

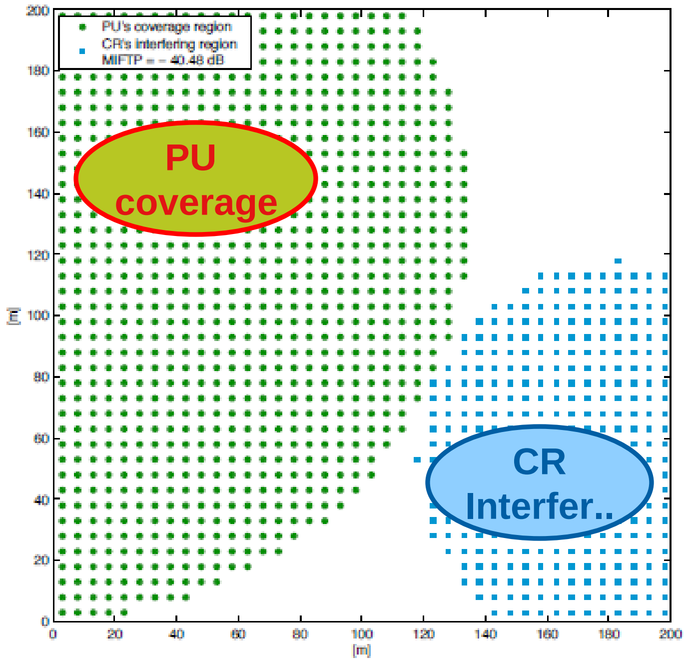
  - Interference regions not discs either

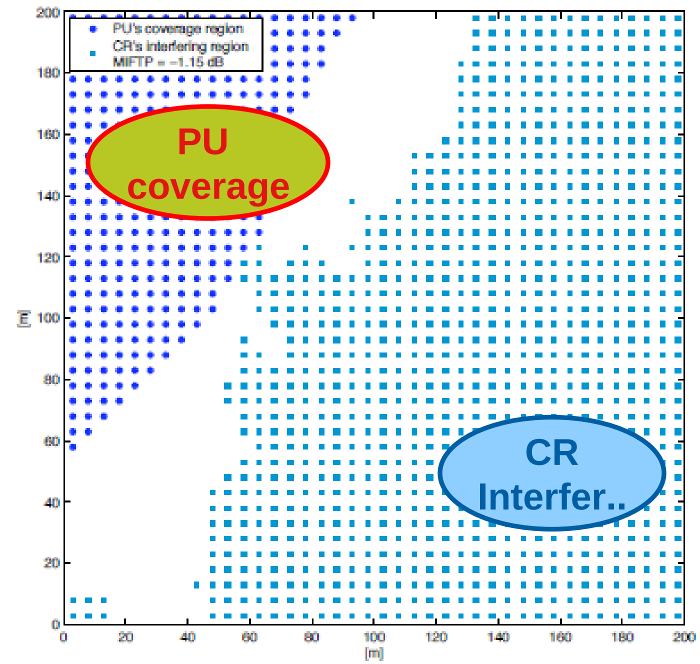


Coverage and interference maps

$\mathcal{C}_s \triangleq \{\mathrm{x} \in \mathcal{A} | P_{\mathrm{cov}}(\mathrm{x}) \geq 0.4\}$, $\mathcal{C}_I \triangleq \{\mathrm{x} \in \mathcal{A} | P_{\mathrm{int}}(\mathrm{x}) \geq 0.01\}$

$P_s = 0\mathrm{dBW}$, $\Pi_{\mathrm{min}} = -60\mathrm{dBW}$, $I_{\mathrm{max}} = -40\mathrm{dBW}$





Path loss-only

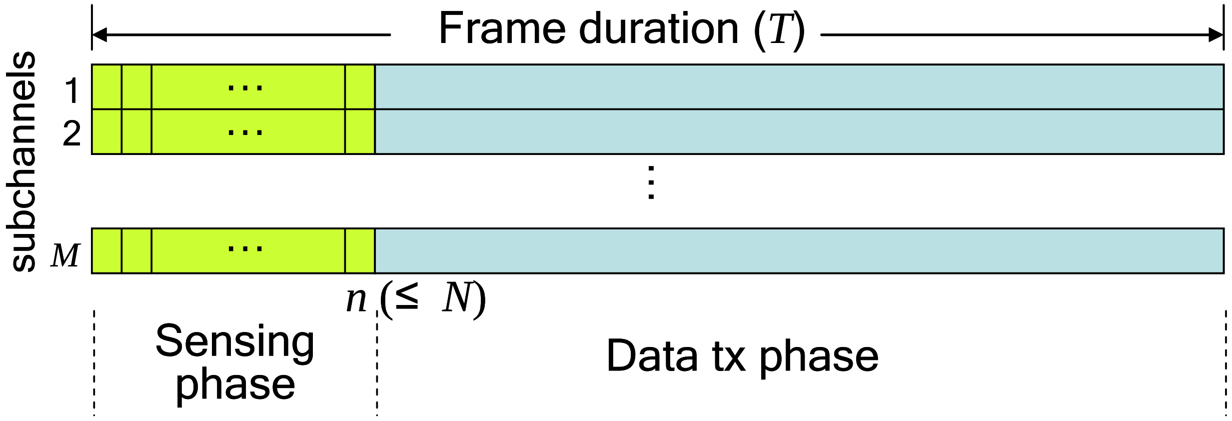
KKF-based

- disc-shaped and time-invariant

- captures spatial macro-diversity and spatio-temporal variations

Sequential sensing for multi-channel CRs

- Extra samples help detection/sensing but lower rate/throughput
  - Sensing-throughput tradeoff in batch single-channel [Liang et al'08]
  - Single-channel sequential CR sensing [Chaudhuri et al'09]
  - Multi-channel (e.g., OFDM) CR sensing [Kim-Giannakis'09]



Joint sensing-throughput optimization

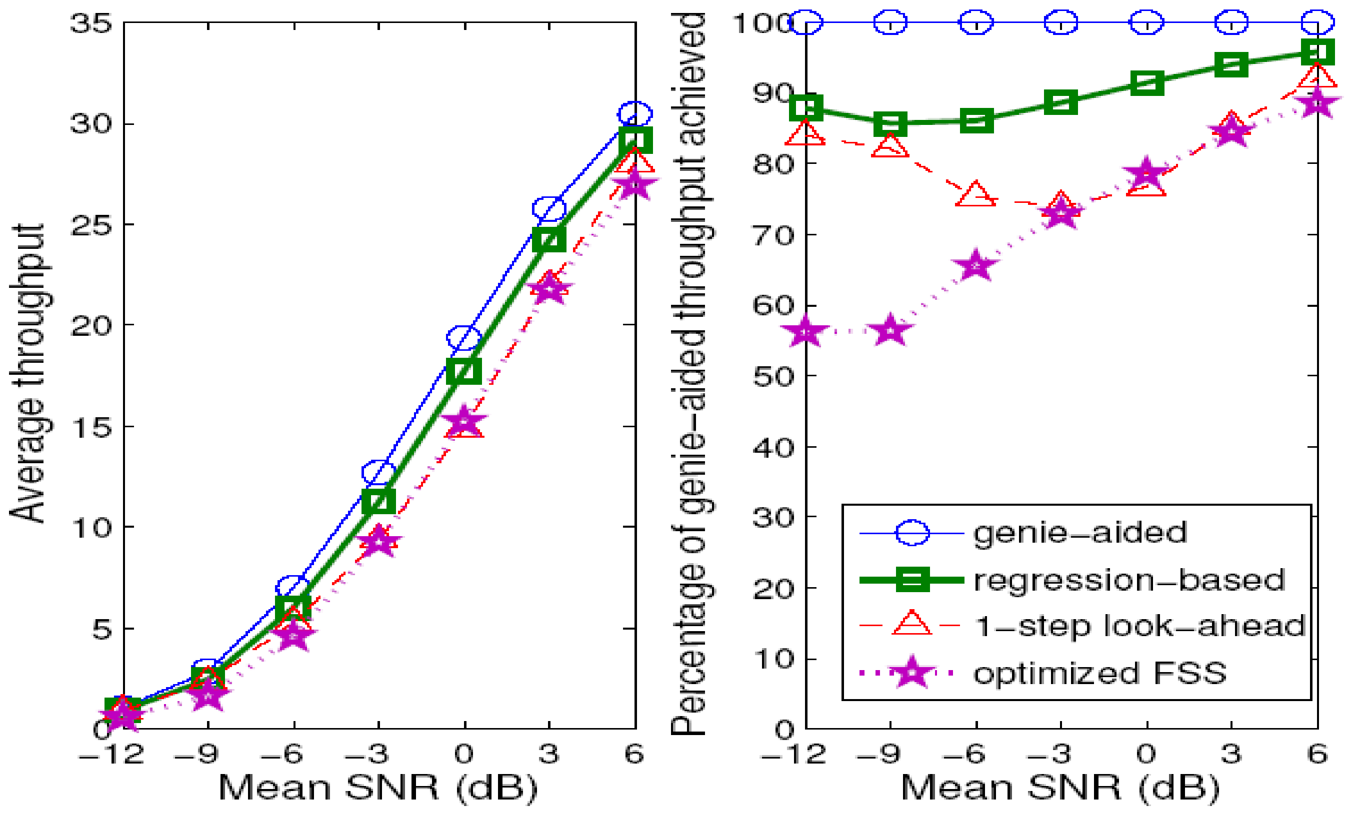
- Features
  - Sense bands in parallel; stop sensing simultaneously (half-duplex constraint)
  - Throughput-optimal sequential sensing terminates when confident

- Basic approach: maximize avg. throughput under collision probability constraints to control Tx-CR interference to PUs (due to miss-detection)
  - Admits a constrained Dynamic Programming (DP) formulation
  - Reduces to an optimum stopping time problem
  - Optimum access: LR test w/ thresholds dependent on Lagrange multipliers

S.-J. Kim and G. Giannakis, "Sequential and Cooperative Sensing for Multichannel Cognitive Radios," *IEEE Transactions on Signal Processing*, 2010.

Simulated test case

- M = 10, N = T = 100, chi-square distributed channel gains
- Average performance over 20,000 runs per operating SNR



Concluding remarks

- Power spectrum density cartography
  - Space-time-frequency view of interference temperature
  - PU/source localization and tracking
- Channel gain cartography
  - Space-time-frequency links from any-to-any point
  - KF for tracking and Kriging for interpolation
- Parsimony via sparsity and distribution via consensus
  - Lasso, group Lasso on splines, and method of multipliers
- Vision: use atlas to enable spatial re-use, hand-off, localization, Tx-power tracking, resource allocation, and routing

Acknowledgements: National Science Foundation
J.-A. Bazerque, E. Dall'Anese, S.-J. Kim, G. Mateos

Thank You!